What is going on in the on ramp call?

Violetta Weger

Young Cryptographers in Genova 2024

November 28, 2024

Post-quantum Cryptography

Asymmetric

Public-key

Post-quantum Cryptography

Asymmetric

- RSA signature, encryption
- DH, DSA
- ECDH, ECDSA

Public-key

→ Integer factorization

→ Discrete logarithm over F_p

→ Discrete logarithm over ell. curves

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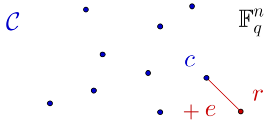
Code-based



- $C = \quad G \quad \mathbb{F}_q^n$ linear subspace
- Decode: $r = mG + e$ find closest $c = mG$
- $\mathrm{wt}_H(e) = |\{i : e_i = 0\}|$

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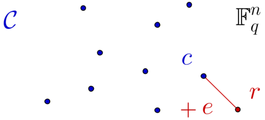

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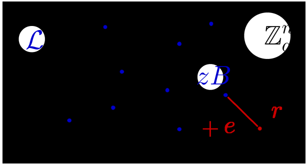
Lattice-based



- $L = \{\sum z_i b_i \mid z_i \in Z\} = \langle B \rangle \subset Z_q^n$
- SVP: $r = zB + e$ find closest zB
- $||e||_2 = \sqrt{\sum e_i^2}$, $||e||_\infty = \max\{|e_i|\}$

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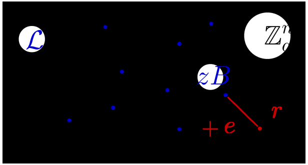

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Multivariate

- $P = (p_1, \ldots, p_m) \quad \mathbb{F}_q[x_1, \ldots, x_n]$
- Given $P(m) = c$ find m
- $P = S \circ F \circ T$, F quadr., S, T affine

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Isogeny-based

- E, E ell. curves over $\mathbb{F}_q$
- find isogeny $\phi : E \to E$

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Post-quantum crypto

Code-based

- Multivariate

Hash-based

- Lattice-based
- Isogeny-based

Timeline

2016	NIST standardization call	for post-quantum PKE/KEM and signatures

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	Standardized KEM:	KYBER
	4th round:	BIKE, Classic McEliece, HQC
2022	Standardized signatures:	DILITHIUM, FALCON, SPHINCS+

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necessary: EUF-CMA, attackers $\geq 2^{64}$ signatures, security levels breaking AES

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Example: Level 1: AES-128: 2^{157} quantum / 2^{143} classical gates

Timeline

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Standardized KEM: KYBER

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On ramp announcement — only signatures

lattice-based: need to outperform DILITHIUM, FALCON

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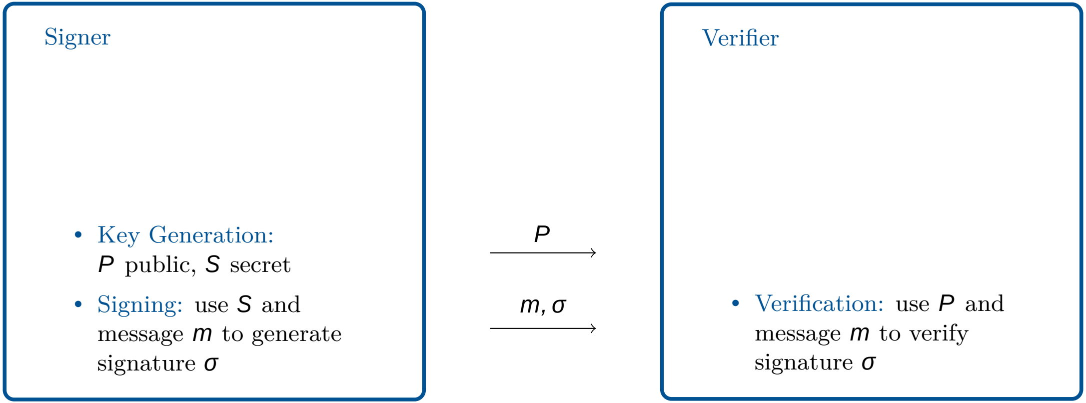
necessary: EUF-CMA, attackers $\geq 2^{64}$ signatures, security levels   breaking AES

nice to haves: side-channel resistant, BUFF, multi-key attacks, well-understood math

Idea of Signature Schemes



Idea of Signature Schemes

Signer

- Key Generation: P public, S secret
- Signing: use S and message m to generate signature σ

EUF-CMA

small P

small σ

Verifier

fast verification

- Verification: use P and message m to verify signature σ

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Approaches for signatures:

- Hash-and-Sign
- ZK Protocol
- ZK + MPC

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	On ramp announcement	
2023	1st round candidates:	40 submissions

1st round Candidates

Code-based: 6

- CROSS
- Enhan. pqsigRM
- FuLeeca
- LESS
- MEDS
- Wave

Lattice-based: 7

- EagleSign
- EHT
- HAETAE
- Hawk
- HuFu
- Raccoon
- Squirrels

MPCitH: 7

- Biscuit
- MIRA
- MiRitH
- MQOM
- PERK
- RYDE
- SDitH

Other: 5

- ALTEQ
- eMLE-Sig
- KAZ-SIGN
- Preon
- Xifrat1-Sign.I

Isogeny: 1

- SQISign

Multivariate: 10

- 3wise
- DME-Sign
- HPPC
- MAYO
- PROV
- QRUOV
- SNOVA
- TUOV
- UOV
- VOX

Symmetric: 4

- AIMer
- Ascon-Sign
- FAEST
- SPHINCS

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Basics

Code $C \quad \mathbb{F}_q^n$ linear subspace

G generator matrix ! $c = mG$

Basics

Code C $\mathbb{F}_q^n$ linear subspace

H parity-check matrix ! $cH^{>} = 0$

Basics

Code $\mathcal{C} \subseteq \mathbb{F}_q^n$ linear subspace

H parity-check matrix $\rightarrow$ $rH^\top = eH^\top = s$

Hamming weight: $\mathrm{wt}_H(e) = |\{i \mid e_i \neq 0\}|$

Basics

algebraic structure

e.g. RS, Goppa codes

! e cient decoders

Basics

random code

decoding is NP-hard

! Information set decoding

Syndrome Decoding Problem (SDP)

Given H, s, weight t, nd e s.t.

1. $s = eH^{>}$
2. $wt_H(e) = t$

Basics

Code $C \subseteq \mathbb{F}_{q^m}^n$ linear subspace

H parity-check matrix $\rightarrow$ $rH^\top = eH^\top = s$

Rank weight: $\mathrm{wt}_R(e) = \dim(\langle e_1, \ldots, e_n \rangle_{\mathbb{F}_q})$

Rank SDP

Given H, s, weight t, find e s.t.

1. $s = eH^\top$
2. $\mathrm{wt}_R(e) = t$

$\mathrm{wt}_R(e) = \dim_{\mathbb{F}_q}(E)$

Basics

Code C $\mathbb{F}_q^{m}$ ^{n} linear subspace

$G_1; \ldots; G_k$! $C = \mathrm{P}\ {}_i G_i$;

Rank weight: $\mathrm{wt}_R(E) = \mathrm{rk}(E)$

MinRank

Given C $\mathbb{F}_q^{m}$ ^{n}, R, t, nd E s.t.

1. R $E \in C$
2. $\mathrm{rk}(E) = t$

basis of $\mathbb{F}_{q^m} / \mathbb{F}_q$: $\mathrm{wt}_R(e) = \mathrm{rk}((e))$ basis

Classical Approach: Hash and Sign

structured code

e cient decoding

random code

hard to decode

Idea McEliece: use Goppa code as secret code

trapdoor

encryption

messages

ciphertexts

Classical Approach: Hash and Sign

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Hash

signature

signatures

messages

repeat

Classical Approach: Hash and Sign

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Disadvantage: slow signing, large public key

Advantage: small signatures, fast veri cation

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Idea McEliece: use Goppa code as secret code

trapdoor

Disadvantage: slow signing, large public key

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Wave: $(u; u + v)$ ternary code and t large

Zero-Knowledge Protocol

Signature Scheme

Signer

Veri er

secret

public

$\longrightarrow$

Zero-Knowledge Protocol

ZK Protocol

Prover		Veri er
secret		public
	⟵	
Commitments c	Interaction	
	⟶	Challenge
		recover c X
Responser		

Zero-Knowledge Protocol

Signature Scheme

Signer

Veri er

secret

public

Commitments c

Fiat-Shamir

Challenge= Hash(m; c) $\longrightarrow$

X

Responser

Zero-Knowledge Protocol

Signature Scheme

Impersonator

Veri er

secret

public

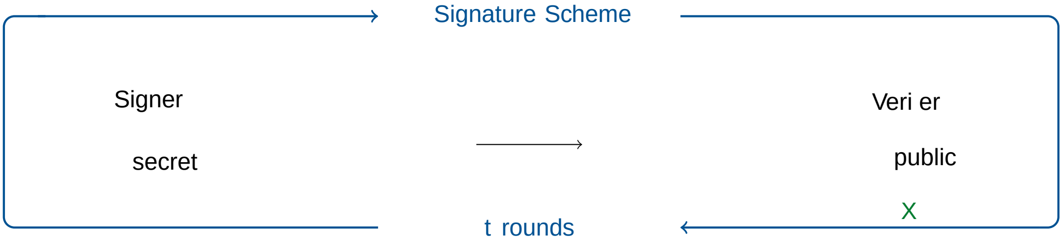
cheating prob.

Fiat-Shamir

$\longrightarrow$

X

Zero-Knowledge Protocol



Zero-Knowledge Protocol

ZK Protocol

Prover

secret

$\longleftarrow$

Interaction

$\longrightarrow$

Verifier

public

X

Isomorphism Problems Given O, O^0, find φ s.t. $\varphi(O) = O^0$

φ

O, O^0

$1: \varphi_1(O) = \tilde{O}$ X /

$2: \varphi_2(O^0) = \tilde{O}$ X

Zero-Knowledge Protocol

ZK Protocol

Prover

secret

Veri er

public

X

$\longleftarrow$

Interaction

$\longrightarrow$

Isomorphism Problems Given O, O^0, nd ' s.t. ' (O) = O^0

'

O, O^0

1: ' $_1$(O) = Ơ X /

2: ' $_2$(O^0) = Ơ X

! MEDS, LESS

Code Equivalence

Code equivalence

Given $G, G' \in \mathbb{F}_q^{k \times n}$ find isometry φ s.t.

$$\varphi(\langle G \rangle) = \langle G' \rangle$$

Hamming isometries $\varphi \in (\mathbb{F}_q^{\star})^n \rtimes S_n$

$\rightarrow$ LESS

Rank isometries $\varphi \in GL_m(\mathbb{F}_q) \times GL_n(\mathbb{F}_q)$

$\rightarrow$ MEDS

Disadvantages: medium/large public keys

Advantages: medium/small signatures

Zero-Knowledge Protocol

ZK Protocol

Prover		Veri er
	⟵	
	Interaction	
secret	⟶	public
		X

SDP Given H, s, t, nd e s.t. 1. $s = eH^{\top}$, 2. $\mathrm{wt}_H(e) = t$

Zero-Knowledge Protocol

ZK Protocol

Prover	Interaction	Veri er
secret		public
		X

SDP Given H, s, t, nd e s.t. 1. $s = eH^{>}$, 2. $\mathrm{wt}_H(e) = t$

e of $\mathrm{wt}_H(e) = t$

$H; s, t$

1. X / 2. X

Zero-Knowledge Protocol

ZK Protocol

Prover ⟵ Veri er

secret Interaction public

⟶

X

SDP	Given H, s, t, nd e s.t.	1. $s = eH^{>}$,	2. $wt_H(e) = t$

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$H; s, t$

' : 1. X / ' (e): 2. X

Zero-Knowledge Protocol

ZK Protocol

Prover ⟵ Veri er

Interaction

secret ⟶ public

X

SDP	Given H , s, t, nd e s.t.	1. $s = eH^{>}$,	2. $wt_H(e) = t$

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H; s , t

' : 1. X / ' (e): 2. X

1. Problem   cheating prob. $\frac{1}{2}$! many rounds

Zero-Knowledge Protocol

ZK Protocol

Prover ← Veri er

secret Interaction public

→ X

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e of $\mathrm{wt}_H(e) = t$

$H; s$, t

' : 1. X / ' (e): 2. X

1. Problem cheating prob. $\frac{1}{2}$! many rounds

! Solution MPCitH: change protocol

MPC in-the-head

ZK Protocol

Prover		Verifier
secret S		public
$(N \quad 1)$-private MPC:	$c_i ; \quad _i$ $\longrightarrow$	
Split S into N shares: s_i	ℓ $\longleftarrow$	Challenge $\ell \in \{1, \ldots, N\}$
Commitments c_i for s_i	s_i for $i \notin \ell$ $\longrightarrow$	Check $c_i ; \quad _i$ for $i \notin \ell$ ✓
Broadcasts $\quad _i = f(s_i)$		

$(N \quad 1)$-private MPC: Secret S split into N shares s_i

$N \quad 1$ many s_i $\Rightarrow$ no info. on S — broadcasts $\quad _i$ to check validity of S

Example $e = \sum_{i=1}^{N} e^{(i)}$, $f(e^{(i)}) = e^{(i)} H^{\top} = s^{(i)}$ $\Rightarrow$ can check $\sum_{i=1}^{N} s^{(i)} = s$

MPC in-the-head

Prover | ZK Protocol | Verifier

secret S

public

$c_i ; \ {}_i$ $\longrightarrow$

ℓ $\longleftarrow$

s_i for $i \neq \ell$ $\longrightarrow$

Challenge $\ell \in \{1, \dots, N\}$

Check $c_i ; \ {}_i$ for $i \neq \ell$ X

MPC in-the-head

Prover

ZK Protocol

Verifier

secret S

public

$c_i ;\ \ _i$

ℓ

Challenge $\ell \in \{1, \dots, N\}$

s_i for $i \neq \ell$

Check $c_i ;\ \ _i$ for $i \neq \ell$ ✓

MPC in-the-head

Prover	ZK Protocol	Verifier
secret S		public
	c_i ; $\;_i$ →	
	← ℓ	Challenge $\ell \in \{1, \ldots, N\}$
	s_i for $i \neq \ell$ →	Check c_i ; $\;_i$ for $i \neq \ell$ ✓

→ New cheating probability: $1/N$

MPC in-the-head

ZK Protocol

Prover		Veri er
secret S		public
m-private MPC:	c_i ; $_i$ →	
Split S into N shares: s_i	← I	Challenge $|I| = m$
Commitments c_i for s_i	s_i for $i \in I$ →	Check c_i ; $_i$ for $i \in I$ ✓
Broadcasts $_i = f(s_i)$		

! New cheating probability: $1/\binom{N}{m}$

MPC in-the-head

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$\rightarrow$ New cheating probability: $1/N$

t/N rounds, but more computations

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t/N rounds, but more computations

Disadvantages: slow

Advantages: small sizes

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$\to$ New cheating probability: $1/N$

t/N rounds, but more computations

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Using rank SDP $\to$ RYDE

Using MinRank $\to$ MIRA, MiRitH

More novel problems

d-split SDP

Given H, s, t, find $(e_1; e_2)$ s.t.

1. $s = eH^\top$
2. $\mathrm{wt}_H(e_i) = t/2$

Subcode equivalence

Given $G \in \mathbb{F}_q^{k \times n}$; $G' \in \mathbb{F}_q^{k' \times n}$ find P s.t.

$$\langle GP \rangle \subseteq \langle G' \rangle$$

$\rightarrow$ SDitH

$\rightarrow$ PERK

More novel problems

d-split SDP

Given H, s, t, nd $(e_1; e_2)$ s.t.

1. $s = eH^{>}$
2. $wt_H(e_i) = t{=}2$

Permuted Kernel

Given G 2 $F_q^{k \quad n}$; H^0 2 $F_q^{n \quad k^0 \quad n}$ nd P s.t.

$$H^0(GP)^{>} = 0$$

! SDitH

! PERK

More novel problems

d-split SDP

Given H, s, t, nd $(e_1; e_2)$ s.t.

1. $s = eH^{>}$
2. $wt_H(e_i) = t=2$

Relaxed permuted kernel problem

Given G 2 $F_q^{k \; n}$; H^0 2 $F_q^{n \; k^0 \; n}$ nd x; P :

$$H^0(xGP)^{>} = 0$$

! SDitH

! PERK

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$H; s$, t

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2. Problem

1 round: large commun. cost

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H; s , t

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2. Problem

1 round: large commun. cost

$S = f\, wt_H(e) = t g$

' : S ! S linear, transitive

! j ' j large

' 2 $(F_q^?)^n$ o S_n

j' j t $\log_2(n(q \quad 1))$

Zero-Knowledge Protocol

SDP Given H, s, t, nd e s.t. 1. $s = eH^{>}$, 2. $\mathrm{wt}_H(e) = t$

e of $\mathrm{wt}_H(e) = t$

$H; s, t$

$' :$ 1. X / $'(e)$: 2. X

2. Problem

1 round: large commun. cost

$S = f\,\mathrm{wt}_H(e) = t g$

$'$: S ! S linear, transitive

! j ' j large

$' \; 2 \; (\mathbb{F}_q^?)^n$ o S_n

$j' j \quad t \log_2(n(q \quad 1))$

! Solution

change underlying problem

! CROSS

Hard Problems

Syndrome Decoding Problem Given p.c. matrix H, syndrome s, weight t, nd e s.t.

lin. constraint 1. $s = eH^{>}$ 2. $\mathrm{wt}_H(e) = t$ non-lin. constraint

Hard Problems

Restricted SDP (R-SDP) Given p.c. matrix H, syndrome s, restriction E, find e s.t.

lin. constraint 1. $s = eH^\top$ 2. $e \in E^n$ non-lin. constraint

$E = \{g^i \mid i \in \{1, \ldots, z\}\} < \mathbb{F}_q^*$ $g \in \mathbb{F}_q^*$ of prime order z

Hard Problems

Restricted SDP (R-SDP) Given p.c. matrix H, syndrome s, restriction E, find e s.t.

lin. constraint 1. $s = eH^\top$ 2. $e \in E^n$ non-lin. constraint

$E = \{g^i \mid i \in \{1, \ldots, z\}\} < \mathbb{F}_q^\star$ $g \in \mathbb{F}_q^\star$ of prime order z

$\mathbb{F}_q^\star$ $\mathbb{F}_q^\star$ $\mathbb{F}_q^\star$ $\rightarrow$ g^{i_1} g^{i_2} g^{i_n}

NP-hard

adaption of ISD: exponential cost

R-SDP

Benefits

restriction $\mathbb{E} = \{g^i \mid i \in \{1, \ldots, z\}\}$

rest. vectors $e = (g^{i_1}, \ldots, g^{i_n}) \in \mathbb{F}_q^n$

R-SDP

Bene ts

restriction $\textcolor{green}{E} = \{ g^i \mid i \in \{1, \ldots, z\}\}$	$\xrightarrow{\ell}$	exponents $\mathbb{F}_z^n$
rest. vectors $e = (g^{i_1}, \ldots, g^{i_n}) \in \mathbb{F}_q^n$		$\ell(e) = (i_1, \ldots, i_n)$

R-SDP

Benefits

restriction $E = \{g^i \mid i \in \{1, \dots, z\}\}$	$\xrightarrow{\ell}$	exponents $\mathbb{F}_z^n$
rest. vectors $e = (g^{i_1}, \dots, g^{i_n}) \in \mathbb{F}_q^n$		$\ell(e) = (i_1, \dots, i_n)$
secret space $S = E^n$; $\varphi : S \to S$	$\xrightarrow{\ell}$	$\lvert e \rvert = \lvert \varphi \rvert = n \log_2(z)$
$\varphi(e) = e' \star e$, $e' = (g^{j_1}, \dots, g^{j_n})$		

R-SDP

Benefits

restriction $E = \{g^i \mid i \in \{1,\dots,z\}\}$	$\xrightarrow{\ell}$	exponents $\mathbb{F}_z^n$
rest. vectors $e = (g^{i_1},\dots,g^{i_n}) \in \mathbb{F}_q^n$		$\ell(e) = (i_1,\dots,i_n)$
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$\varphi(e) = e' \star e$; $e' = (g^{j_1},\dots,g^{j_n})$		$\ell(\varphi(e)) = \ell(e) + \ell(e')$

R-SDP

Benefits	restriction $E = \{g^i \mid i \in \{1, \dots, z\}\}$	$\xrightarrow{\ell}$	exponents $\mathbb{F}_z^n$
	rest. vectors $e = (g^{i_1}, \dots, g^{i_n}) \in \mathbb{F}_q^n$		$\ell(e) = (i_1, \dots, i_n)$
	secret space $S = E^n$; $\varphi : S \to S$	$\xrightarrow{\ell}$	$\lvert e \rvert = \lvert \varphi \rvert = n \log_2(z)$
	$\varphi(e) = e' \star e$; $e' = (g^{j_1}, \dots, g^{j_n})$		$\ell(\varphi(e)) = \ell(e) + \ell(e')$
Example	$E = \{1, 3, 9\} \subseteq \mathbb{F}_{13}$	$\xrightarrow{\ell}$	exponents in $\mathbb{F}_3^4$
	$e = (1, 9, 3, 3)$		$\ell(e) = (0, 2, 1, 1)$
	$\downarrow \star(3, 3, 9, 1)$		$\downarrow +(1, 1, 2, 0)$
	$\tilde{e} = (3, 1, 1, 3)$		$\ell(\tilde{e}) = (1, 0, 0, 1)$

R-SDP(G)

R-SDP Given H, s, E, nd e s.t. 1. $s = eH^{>}$ 2. $e \, 2 \, E^n$ $(E^n ; ?) \, ' \, (F_z^n ; +)$

R-SDP(G)

R-SDP(G) Given H, s, G, find e s.t. 1. $s = eH^{\top}$ 2. $e \in G$ $(G, \star) < (E^n, \star)$

Benefits

$$x_1 = (g^{i_1}, \ldots, g^{i_n})$$

$$\vdots$$

$$x_m = (g^{j_1}, \ldots, g^{j_n})$$

R-SDP(G)

R-SDP(G) Given H, s, G, nd e s.t. 1. $s = eH^{>}$ 2. $e \, 2 \, G$ $(G; ?) < (E^n; ?)$

Bene ts

$$x_1 = (g^{i_1}; : : : ; g^{i_n})$$
$$\vdots$$
$$x_m = (g^{j_1}; : : : ; g^{j_n})$$

$$\longrightarrow \quad M = \begin{pmatrix} i_1 & & i_n \\ \vdots & & \vdots \\ j_1 & & j_n \end{pmatrix} \, 2 \, F_z^{m \; n}$$

R-SDP(G)

R-SDP(G) Given H, s, G, find e s.t. 1. $s = eH^\top$ 2. $e \in G$ $\quad G \cong C \quad \mathbb{F}_z^n$

Benefits

$$
\begin{matrix}
x_1 = (g^{i_1}, \ldots, g^{i_n}) \\
\vdots \\
x_m = (g^{j_1}, \ldots, g^{j_n})
\end{matrix}
\quad \xrightarrow{\ \ell\ } \quad
M = \begin{pmatrix} i_1 & \cdots & i_n \\ \vdots & & \vdots \\ j_1 & \cdots & j_n \end{pmatrix} \in \mathbb{F}_z^{m \times n}
$$

$$
e = x_1^{u_1} \star \cdots \star x_m^{u_m} \qquad\qquad \ell(e) = (u_1, \ldots, u_m) M
$$

$$
\varphi : G \to G;\ \varphi(e) = e' \star e \quad \xrightarrow{\ \ell\ } \quad |e| = |\varphi| = m \log_2(z) < 1.5
$$

R-SDP(G)

R-SDP(G) Given H, s, G, find e s.t. 1. $s = eH^\top$ 2. $e \in G$ $\quad G \cong C \subseteq \mathbb{F}_z^n$

Benefits

$$x_1 = (g^{i_1}, \dots, g^{i_n}), \quad \vdots \quad x_m = (g^{j_1}, \dots, g^{j_n}) \xrightarrow{\ell} M = \begin{pmatrix} i_1 & \cdots & i_n \\ \vdots & & \vdots \\ j_1 & \cdots & j_n \end{pmatrix} \in \mathbb{F}_z^{m \times n}$$

$$e = x_1^{u_1} \star \cdots \star x_m^{u_m} \qquad \ell(e) = (u_1, \dots, u_m)M$$

$$\varphi : G \to G, \ \varphi(e) = e' \star e \xrightarrow{\ell} |E| = |\varphi| = m \log_2(z) < 1.5$$

Example

$$E = \{1, 3, 9\} \subseteq \mathbb{F}_{13} \xrightarrow{\ell} \text{exponents in } \mathbb{F}_3^4$$

$$x_1 = (3, 1, 1, 3) \qquad M = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 \end{pmatrix}$$

$$x_2 = (1, 3, 9, 1)$$

$$e = x_1^{2} \star x_2^{1} = (9, 3, 9, 9) \qquad \ell(e) = (2, 1, 2, 2) = (2, 1)M$$

Summary

Hash & Sign

Large weight SDP	→	WAVE	large public key

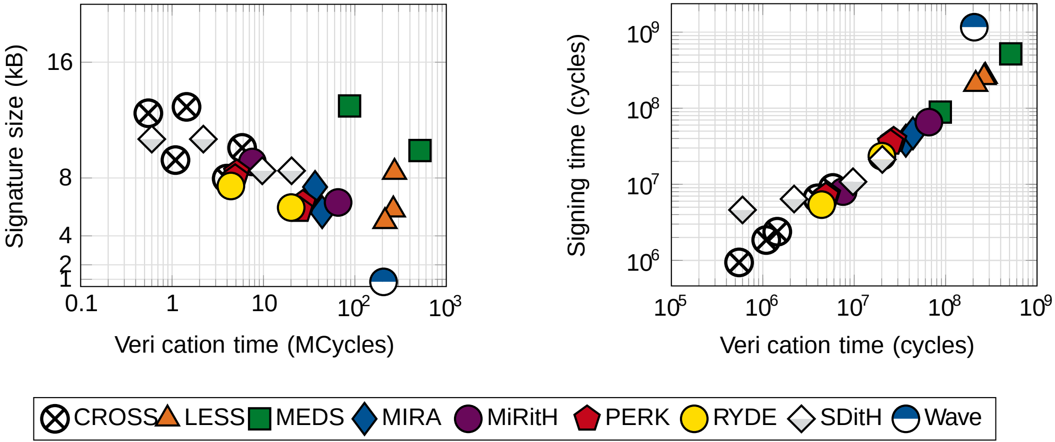
ZK Protocol

Restricted SDP	→	CROSS	
CEP	→	LESS	
Matrix CEP	→	MEDS	large signature

ZK + MPC

d-split SDP	→	SDitH	
Rank SDP	→	RYDE	
MinRank	→	MIRA/MiRitH	
PKP	→	PERK	slow

Comparison



Timings taken from https://pqshield.github.io/nist-sigs-zoo/

Timeline

2016	NIST standardization call	for post-quantum PKE/KEM and signatures	
	Standardized KEM:	KYBER	
	4th round:	BIKE, Classic McEliece, HQC	
2022	Standardized signatures:	DILITHIUM, FALCON, SPHINCS+	
2023	On ramp announcement		
	1st round candidates:	29 survivors	9 code-based
2024			

Timeline

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2022	Standardized signatures:	DILITHIUM, FALCON, SPHINCS+	
2023	On ramp announcement		
	1st round candidates:	29 survivors	9 code-based
2024	2nd round announced	14 schemes	6 code-based

2nd Round Candidates

Code-based: 9

- CROSS
- LESS
- MEDS
- MIRA
- MiRitH
- PERK
- RYDE
- SDitH
- Wave

Other: 1

- Preon

Lattice-based: 5

- HAETAE
- Hawk
- HuFu
- Raccoon
- Squirrels

Symmetric: 4

- AIMer
- Ascon-Sign
- FAEST
- SPHINCS

Multivariate: 9

- Biscuit
- MAYO
- MQOM
- PROV
- QRUOV
- SNOVA
- TUOV
- UOV
- VOX

Isogeny: 1

- SQISign

2nd Round Candidates

Code-based: 6

- CROSS
- LESS
- MEDS
- MiRatH
- PERK
- RYDE
- SDitH
- Wave

Other: 0

- Preon

Lattice-based: 1

- HAETAE
- Hawk
- HuFu
- Raccoon
- Squirrels

Symmetric: 1

- AIMer
- Ascon-Sign
- FAEST
- SHPINCS

Multivariate: 5

- Biscuit
- MAYO
- MQOM
- PROV
- QRUOV
- SNOVA
- TUOV
- UOV
- VOX

Isogeny: 1

- SQISign

2nd Round Candidates

NIST.IR.8528 Status report

1) security   2) cost and performance   3) implementation

Code-based: 6

- CROSS
- LESS
- MiRatH
- PERK
- RYDE
- SDitH

Lattice-based: 1

- Hawk

Symmetric: 1

- FAEST

Isogeny: 1

- SQISign

Multivariate: 5

- MAYO
- MQOM
- QRUOV
- SNOVA
- UOV

2nd Round Candidates

NIST.IR.8528 Status report

1) security 2) cost and performance 3) implementation

a) simplicity b) uniqueness c) elegance

Code-based: 6

- CROSS
- LESS
- MiRatH
- PERK
- RYDE
- SDitH

Lattice-based: 1

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Symmetric: 1

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non-lattice, better performance than SPHINCS

new, improve performance

2nd Round Candidates

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1) security 2) cost and performance 3) implementation

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- RYDE
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Lattice-based: 1

- Hawk

Symmetric: 1

- FAEST

Isogeny: 1

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Multivariate: 5

- MAYO
- MQOM
- QRUOV
- SNOVA
- UOV

non-lattice, better performance than SPHINCS

new, improve performance: threshold, VOLE

2nd Round Candidates

NIST.IR.8528 Status report

1) security 2) cost and performance 3) implementation

a) simplicity b) uniqueness c) elegance

Code-based: 6

- CROSS
- LESS
- MiRatH
- PERK
- RYDE
- SDitH

Lattice-based: 1

- Hawk

Symmetric: 1

- FAEST

Isogeny: 1

- SQISign

Multivariate: 5

- MAYO
- MQOM
- QRUOV
- SNOVA
- UOV

non-lattice, better performance than SPHINCS

complex, technical

2nd Round Candidates

NIST.IR.8528 Status report

1) security 2) cost and performance 3) implementation

a) simplicity b) uniqueness c) elegance

Code-based: 6

- CROSS
- LESS
- MiRatH
- PERK
- RYDE
- SDitH

Lattice-based: 1

- Hawk

Symmetric: 1

- FAEST

Isogeny: 1

- SQISign

Multivariate: 5

- MAYO
- MQOM
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- SNOVA
- UOV

no oating points

new

2nd Round Candidates

NIST.IR.8528 Status report

1) security 2) cost and performance 3) implementation

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Code-based: 6

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- LESS
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Lattice-based: 1

- Hawk

Symmetric: 1

- FAEST

Isogeny: 1

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Multivariate: 5

- MAYO
- MQOM
- QRUOV
- SNOVA
- UOV

non-lattice, better performance than SPHINCS

new, recent attacks

How will the 2nd round go?

Timeline

- Submission deadline: Jan. 17
- 3rd round decision?
- How many schemes?

How will the 2nd round go?

Timeline

Submission deadline: Jan. 17

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How many schemes? nal?

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What's next?

- Will MPC ! VOLE?
- Will SQISign reduce times?
- New attacks?

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Open Problems

- Cost of d-split SDP
- Cost of restricted SDP
- Cost of rank SDP
- Cost of q-ary SDP

How will the 2nd round go?

Timeline

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What's next?

- Will MPC $\rightarrow$ VOLE?
- Will SQISign reduce times?
- New attacks?

Open Problems

- Cost of d-split SDP
- Cost of restricted SDP
- Cost of rank SDP
- Cost of q-ary SDP
- How hard is code equivalence?

Abhi's talk!

How will the 2nd round go?

Timeline

- Submission deadline: Jan. 17
- 3rd round decision? 2026
- How many schemes? final?

What's next?

- Will MPC → VOLE?
- Will SQISign reduce times?
- New attacks?

Open Problems

- Cost of d-split SDP
- Cost of restricted SDP
- Cost of rank SDP
- Cost of q-ary SDP
- How hard is code equivalence?

Slides

Stay tuned!

Thank you

VOLE

Vector Oblivious Linear Transfer

ZK Protocol

Prover	Verifier
secret s	public
v random	Δ eval. point
$f(x) = sx + v$	
	$q = f(\Delta)$

VOLE

Vector Oblivious Linear Transfer

ZK Protocol

Prover			Verifier
	$\xleftarrow{s, v}$	$\xleftarrow{\Delta}$	
secret s			public
v random	$\xleftarrow{\Delta}$	$\xrightarrow{\text{seed}_i}$	Δ eval. point
$f(x) = sx + v$			seed_i for $i = \Delta$
$q = f(\Delta)$			$q = f(\Delta)$

VOLE correlation $q = s\Delta + v = f(\Delta)$

dishonest prover needs to guess Δ before committing to GGM tree: $\mathsf{P} \quad 1/p$

VOLE

Vector Oblivious Linear Transfer

ZK Protocol

Prover			Verifier
	s, v $\longleftarrow$	Δ $\longleftarrow$	
secret s			public
v random	Δ $\longleftarrow$	seed_i $\longrightarrow$	Δ eval. point
$f(x) = sx + v$			seed_i for $i = \Delta$
$q = f(\Delta)$			$q = f(\Delta)$

MPC

$s = \square\, s_i$ MPC $\longleftarrow$ $N - 1$ views

VOLE

$s = \square\, s_i$ GGM $\overset{\Delta}{\longleftarrow}$ $N - 1$ seeds

$v = \square\, i s_i$ $q = \square\, s_i(\Delta - i) = s\Delta + v$

VOLE

Vector Oblivious Linear Transfer

ZK Protocol

Prover				Verifier
	s, v $\longleftarrow$		Δ $\longleftarrow$	
secret s				public
v random	Δ $\longleftarrow$		seed_i $\longrightarrow$	Δ eval. point
$f(x) = sx + v$				seed_i for $i = \Delta$
$q = f(\Delta)$				$q = f(\Delta)$

$$f(x) = \sum_{i=0}^{d} f_i x^i,$$

$$s = f_d$$

$$f_1(x), f_2(x)$$

$$f_1(\Delta) + f_2(\Delta) = (f_1 + f_2)(\Delta)$$

$$f_1(\Delta) f_2(\Delta) = (f_1 f_2)(\Delta)$$

VOLE

Vector Oblivious Linear Transfer

ZK Protocol

Prover				Verifier
	$\xleftarrow{s, v}$		$\xleftarrow{\Delta}$	
secret s				public
v random	$\xleftarrow{\Delta}$		$\xrightarrow{\mathsf{seed}_i}$	Δ eval. point
$f(x) = sx + v$				seed_i for $i = \Delta$
$q = f(\Delta)$				$q = f(\Delta)$

Disadvantages: slow

Advantages: small sizes

Main Features

Implementation

- optimized AVX2 — fast < 1 MCycle (NIST cat. I)
- memory-optimized — fits on Cortex-M4 microcontroller
- constant worst-case runtime — no signature rejection
- available on lib open quantum safe

Ingredients

- Restricted Syndrome Decoding
  - → compact objects & efficient arithmetic
  - → NP-hard problem
- Zero-Knowledge protocol
  - → simple and well-studied
  - → EUF-CMA security
  - → BUFF security
  - → standard optimizations

Future of CROSS

What's next?

- Hardware implementation
- Side-channel protection
- Worst-case to average-case reduction
- Smaller signatures: VOLE

Website

CROSS

Codes & Restricted Objects Signature Scheme
http://cross-crypto.com/

Attacks

- E, G have multiplicative structure

  $e = (g^{i_1}, \ldots, g^{i_n})$

- $s = eH^\top$ has additive structure

  $s_j = \sum_{\ell=1}^{n} h_{j,\ell} g^{i_\ell}$ for $j \in \{1, \ldots, n-k\}$

Attacks

- E, G have multiplicative structure
  $e = (g^{i_1}, \ldots, g^{i_n})$
- Take E with no additive structure

- $s = eH^{\top}$ has additive structure
  $s_j = \sum_{\ell=1}^{n} h_{j,\ell}\, g^{i_\ell}$ for $j \in \{1, \ldots, n-k\}$

Attacks

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- $s = eH$ has additive structure
  $s_j = \sum_{i=1}^{n} h_{j,i} g^{i}$ for $j \in \{1, \ldots, n-k\}$
- Take E with no additive structure
- good: $q = 13, g = 3, \mathsf{E} = \{1, 3, 9\}$
- bad: $q = 13, g = 5, \mathsf{E} = \{1, 5, -1, -5\}$

Attacks

- E, G have multiplicative structure

  $e = (g^{i_1}, \dots, g^{i_n})$

- $s = eH^\top$ has additive structure

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- combinatorial:

  ISD algorithms

S. Bitzer, A. Pavoni, V. Weger, P. Santini, M. Baldi, and A. Wachter-Zeh. "Generic Decoding of Restricted Errors", ISIT, 2023.

M. Baldi, S. Bitzer, A. Pavoni, P. Santini, A. Wachter-Zeh, and V. Weger. "Zero knowledge protocols and signatures from the restricted syndrome decoding problem", PKC, 2024.

Attacks

- E, G have multiplicative structure

  $e = (g^{i_1}, \dots, g^{i_n})$

- $s = eH$ has additive structure

  $s_j = \sum_{=1}^{n} h_{j,} g^{i}$ for $j \in \{1, \dots, n-k\}$

- Take E with no additive structure

- good: $q = 13, g = 3, \mathsf{E} = \{1, 3, 9\}$

- bad: $q = 13, g = 5, \mathsf{E} = \{1, 5, -1, -5\}$

- combinatorial:

  ISD algorithms

- algebraic attacks:

  $e_i^z = 1$ Gröbner basis

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M. Baldi, et al. "CROSS", NIST PQC round 1, 2023.

W. Beullens, P. Briaud, M. Øygarden. "A Security Analysis of Restricted Syndrome Decoding Problems", 2024.

Performance

NIST cat. I

Problem	q, z	**Type**	(n, k, m)	**rounds**	**|Sign.|** (kB)	**Sign** (MCycles)	**Verify** (MCycles)
R-SDP	$(127, 7)$	fast	$(127, 76, -)$	163	19.1	1.28	0.78
		balanced		252	12.9	2.38	1.44
		short		960	10.1	8.96	5.84
R-SDP(G)	$(509, 127)$	fast	$(55, 36, 25)$	153	12.5	0.94	0.55
		balanced		243	9.2	1.85	1.09
		short		871	7.9	6.54	3.96

private and public keys < 0.1 kB   key gen. < 0.1 MCycle

Measurements collected on an AMD Ryzen 5 Pro 3500U, clocked at 2.1GHz. The computer was running Debian GNU/Linux 12

CVE

PROVER		VERIFIER
KEY GENERATION		
Choose e with $\mathrm{wt}_H(e) = t$		
H parity-check matrix		
Compute $s = eH$	$\xrightarrow{P=(H,s,t)}$	
		VERIFICATION
Choose u $\mathbb{F}_q^n$, ϕ M_n		
Set $c_1 = \mathrm{Hash}(\phi, uH \quad)$		
Set $c_2 = \mathrm{Hash}(\phi(u), \phi(e))$	$\xrightarrow{c_1, c_2}$	
	$\xleftarrow{z}$	Choose z $\mathbb{F}_q^\times$
Set $y = \phi(u + ze)$	$\xrightarrow{y}$	
$r_1 = \phi$	$\xleftarrow{b}$	Choose b $\{1, 2\}$
$r_2 = \phi(e)$	$\xrightarrow{r_b}$	$b = 1$: $c_1 = \mathrm{Hash}(\phi, \phi^{-1}(y)H \quad - zs)$
		$b = 2$: $\mathrm{wt}_H(\phi(e)) = t$
		and $c_2 = \mathrm{Hash}(y - z\phi(e), \phi(e))$

CVE

PROVER		VERIFIER
KEY GENERATION		
Choose e with $\text{wt}_H(e) = t$		
H parity-check matrix		
Compute $s = eH$	$\xrightarrow{P=(H,s,t)}$	
		VERIFICATION
Choose $u \quad \mathbb{F}_q^n$, $\phi \quad M_n$		
Set $c_1 = \text{Hash}(\phi, uH \quad)$		
Set $c_2 = \text{Hash}(\phi(u), \phi(e))$	$\xrightarrow{c_1, c_2}$	
	$\xleftarrow{z}$	Choose $z \quad \mathbb{F}_q^\times$
Set $y = \phi(u + ze)$	$\xrightarrow{y}$	
$r_1 = \phi$	$\xleftarrow{b}$	Choose $b \quad \{1, 2\}$
$r_2 = \phi(e)$	$\xrightarrow{r_b}$	$b = 1$: $c_1 = \text{Hash}(\phi, \phi^{-1}(y)H \quad - zs)$
		$b = 2$: $\text{wt}_H(\phi(e)) = t$
		and $c_2 = \text{Hash}(y - z\phi(e), \phi(e))$

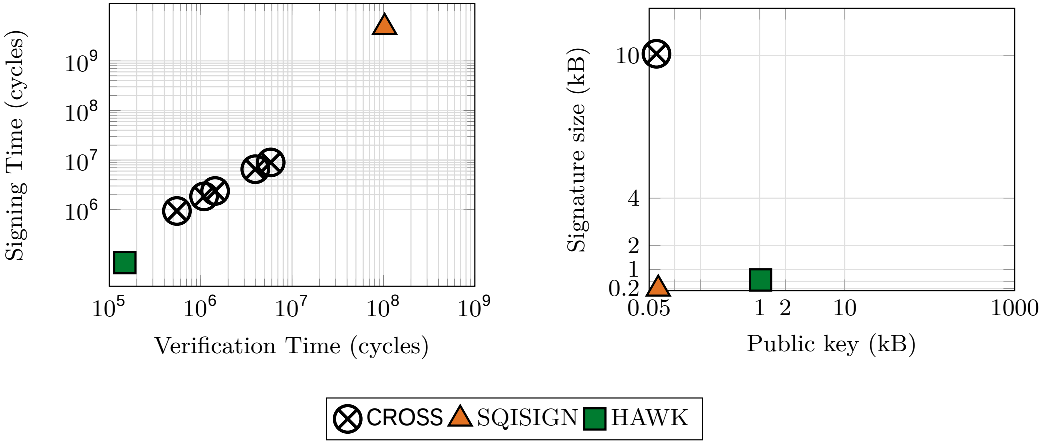
Recall SDP: (1) $s = eH$ (2) $\text{wt}_H(e) = t$

CVE

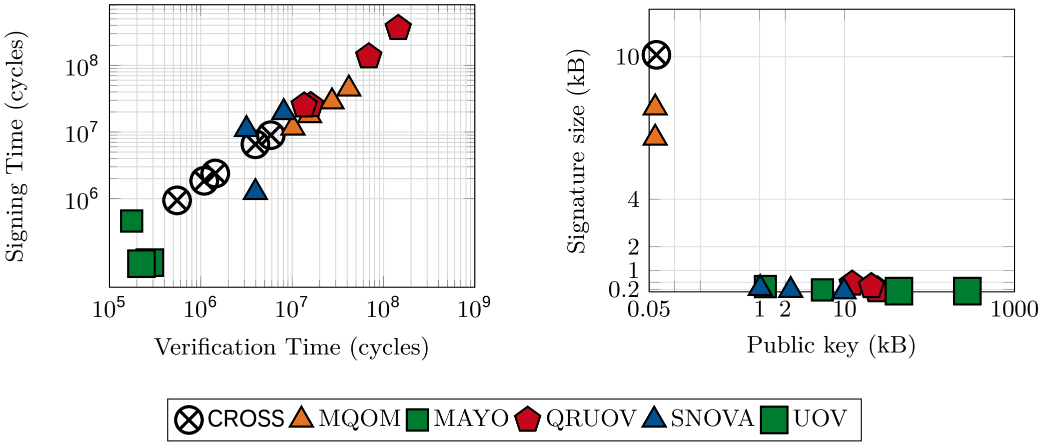
PROVER		VERIFIER
KEY GENERATION		
Choose e with $\mathrm{wt}_H(e) = t$		
H parity-check matrix		
Compute $s = eH$	$\xrightarrow{P=(H,s,t)}$	
		VERIFICATION
Choose $u \quad \mathbb{F}_q^n,\ \phi \quad M_n$		
Set $c_1 = \mathrm{Hash}(\phi, uH \quad)$		
Set $c_2 = \mathrm{Hash}(\phi(u), \phi(e))$	$\xrightarrow{c_1, c_2}$	
	$\xleftarrow{z}$	Choose $z \quad \mathbb{F}_q^\times$
Set $y = \phi(u + ze)$	$\xrightarrow{y}$	
$r_1 = \phi$	$\xleftarrow{b}$	Choose $b \quad \{1, 2\}$
$r_2 = \phi(e)$	$\xrightarrow{r_b}$	$b = 1$: $c_1 = \mathrm{Hash}(\phi, \phi^{-1}(y)H \quad - zs)$
		$b = 2$: $\mathrm{wt}_H(\phi(e)) = t$
		and $c_2 = \mathrm{Hash}(y - z\phi(e), \phi(e))$

Problem: big signature sizes

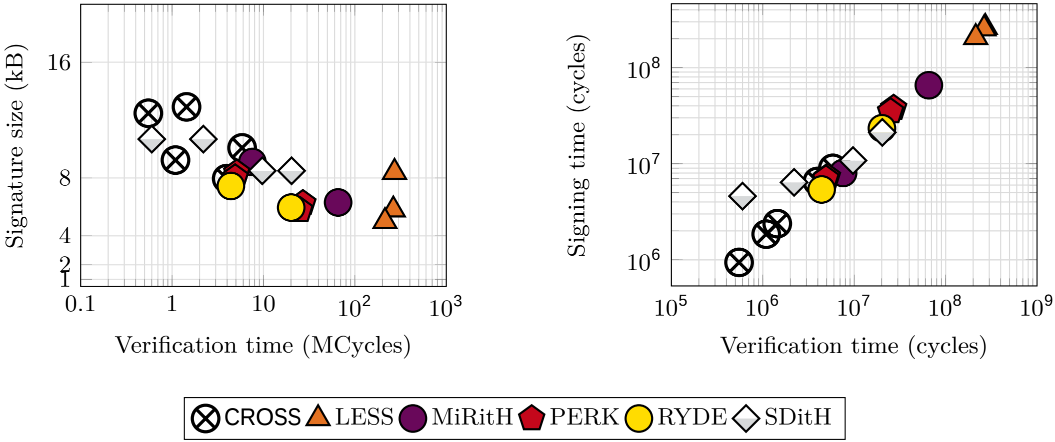
vs: Isogenies and lattices



vs: Multivariate



Comparison



Signature size (kB)
16
8
4
2
1
0.1
1
10
10^2
10^3
Verification time (MCycles)
Signing time (cycles)
10^8
10^7
10^6
10^5
10^6
10^7
10^8
10^9
Verification time (cycles)
CROSS
LESS
MiRitH
PERK
RYDE
SDitH