Recent Advances in Code-based Signatures

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Rudolf Mößbauer Tenure Track Professorship:
Symposium "Selected Topics in Science and Technology"

March 22, 2023

Outline

1. **Code-based Cryptography**
   - **Introduction to Coding Theory**
   - **Hard Problems from Coding Theory**
   - **Previous Work**

2. **Code-based Signature Schemes**
   - **Idea and Previous Work**
   - **FuLeeca**
   - **Restricted Errors**

3. **Future Research**
   - **Rank-metric Decoding**
   - **Quantum Codes**
   - **Further Research Directions**

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Motivation

- Quantum computers: break all currently used asymmetric cryptosystems

$\rightarrow$ Need quantum-secure alternatives

- Candidates for post-quantum cryptography: Systems based NP-hard problems

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- PKE/KEM: 1 lattice-based, round 4: 3 code-based
- Signature schemes: 1 hash-based and 2 based on ideal lattices

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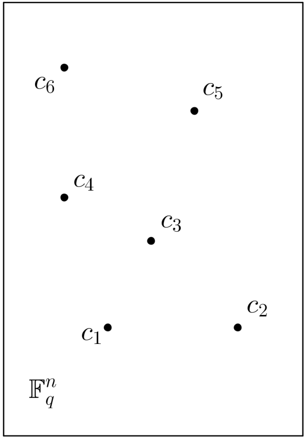
2022 NIST reopened standardization call for signature schemes

Coding Theory



Set Up

- *Code* $\mathcal{C} \subseteq \mathbb{F}_q^n$ linear k-dimensional subspace
- $c \in \mathcal{C}$ *codeword*
- $G \in \mathbb{F}_q^{k \times n}$ *generator matrix*
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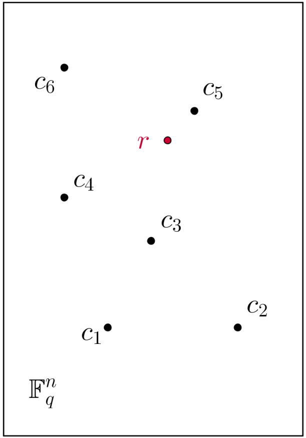

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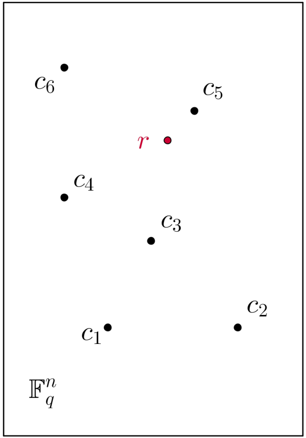

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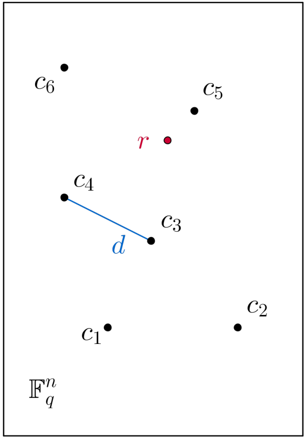
Coding Theory



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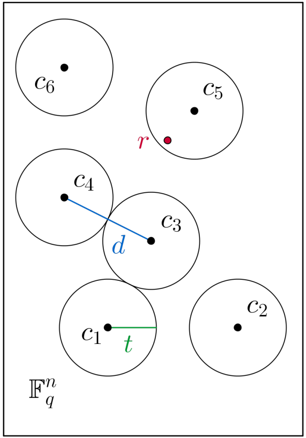

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- *error-correction capacity*: $t = (d(\mathcal{C}) - 1)/2$

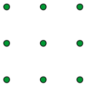
Hard Problems from Coding Theory

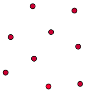
Algebraic structure

(Reed-Solomon, Goppa,..)

$\rightarrow$ efficient decoders

$\langle G \rangle$





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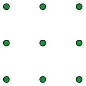
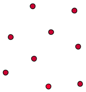
$\langle G \rangle$ 

 $\langle \widetilde{G} \rangle$

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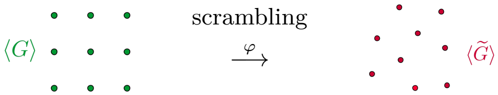

Seemingly random code

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- Decoding random linear code is NP-hard
- First code-based cryptosystem based on this problem

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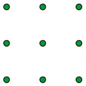
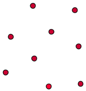
$\langle G \rangle$ 

scrambling
$\xrightarrow{\varphi}$

 $\langle \widetilde{G} \rangle$

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A. Becker, A. Joux, A. May, A. Meurer "Decoding random binary linear codes in $2^{n/20}$: How 1+ 1= 0 improves information set decoding", Eurocrypt, 2012.

Previous Work

Lee Metric

For $x, y \in \mathbb{Z}/p^s\mathbb{Z}^n$

- *Lee weight*: $\mathrm{wt}_L(x) = \sum_{i=1}^n \mathrm{wt}_L(x_i) = \sum_{i=1}^n \min\{x_i, \mid p^s - x_i \mid\}$
- *Lee distance*: $d_L(x, y) = \mathrm{wt}_L(x - y).$

$\rightarrow$ $d_L(\mathcal{C})$ much larger than $d_H(\mathcal{C})$

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- Decoding random linear code in Lee-metric is NP-hard
- Fastest solvers: Lee-metric ISD, exponential time
- Behaviour of random ring-linear codes

V.W., K. Khathuria, A.-L. Horlemann, M. Battaglioni, P. Santini, E. Persichetti. "On the hardness of the Lee syndrome decoding problem", Advances in Mathematics of Communications, 2021.

J. Bariffi, K. Khathuria, **V.W.** "Information Set Decoding for Lee-Metric Codes using Restricted Balls", CBCrypto, 2022.

E. Byrne, A.-L. Horlemann, K. Khathuria, **V.W.** "Density of free modules over finite chain rings", Linear Algebra and its Applications, 2022.

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Idea of Signature Schemes

Signer **Verifier**

Key Generation

Secret key $\mathcal{S}$, public key $\mathcal{P}$

$\xrightarrow{\mathcal{P}}$

Signing

Message m, signature σ

$\xrightarrow{m,\sigma}$

Verification

Verify σ

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- Hash-and-sign
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$\rightarrow$ large public key sizes

$\rightarrow$ our solution: FuLeeca

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Hash-and-Sign

N. Courtois, M. Finiasz, N. Sendrier. "How to achieve a McEliece-based digital signature scheme", Asiacrypt, 2001.

- Following idea of McEliece:

$\rightarrow$ start with structured code $\rightarrow$ large public key sizes

$\rightarrow$ publish scrambled code

- $\mathrm{Hash}(m) = eH^\top$, $\mathrm{wt}_H(e) \leq t$ $\rightarrow$ slow signing
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How to reduce public key sizes/ thwart statistical attacks?
How to speed-up signing?

FuLeeca

S. Ritterhoff, G. Maringer, S. Bitzer, **V.W.**, P. Karl, T. Schamberger, J. Schupp, A. Wachter-Zeh, G. Sigl. "FuLeeca: A Lee-based Signature Scheme", Preprint, 2023.

Secret key	Quasi-cyclic, low Lee weight generators
Public key	Systematic form, scrambled generator matrix
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	public key size	signature size	total size
Falcon	897 B	666 B	1563 B
Dilithium	1312 B	2420 B	3732 B
Sphincs+	32 B	7856 B	7888 B
FuLeeca	389 B	276 B	665 B

$\rightarrow$ Can beat all standardized signature schemes in total size

Code-based ZK Protocols

ZK protocol $\xrightarrow{\text{Fiat-Shamir}}$ Signature scheme

Syndrome Decoding Problem

Given parity-check matrix H, syndrome s, weight t, find e s.t. 1. $s = eH^\top$ 2. $\mathrm{wt}_H(e) \leq t$

P.-L. Cayrel, P. Véron, S. El Yousfi Alaoui. "A zero-knowledge identification scheme based on the q-ary syndrome decoding problem", Selected Areas in Cryptography, 2011.

- Random H, e of weight t, compute $s = eH^\top$ $\rightarrow$ small public key sizes
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- Verifier challenges either 1. or 2. by asking for transformation φ or transformed secret $\varphi(e)$
- Large cheating probability $\rightarrow$ many rounds, large signature size, CVE: 40 KB
- Recent improvements through in the head computations $\rightarrow$ smaller signature sizes, 10 KB

T. Feneuil, A. Joux, M. Rivain "Shared permutation for syndrome decoding: New zero-knowledge protocol and code-based signature", Designs, Codes and Cryptography, 2022.

T. Feneuil, A. Joux, M. Rivain "Syndrome decoding in the head: shorter signatures from zero-knowledge proofs", Crypto, 2022.

Restricted Errors

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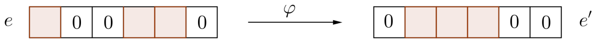

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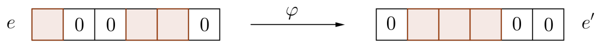

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$$\downarrow$$

Restricted Syndrome Decoding Problem

Given $H \in \mathbb{F}_q^{(n-k)\times n}$, syndrome $s \in \mathbb{F}_q^{n-k}$, $E \subseteq \mathbb{F}_q^\star$, find $e \in E^n$ such that $s = eH^\top$.

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M. Baldi, S. Bitzer, A. Pavoni, P. Santini, A. Wachter-Zeh, **V.W.** "Zero Knowledge Protocols and Signatures from the Restricted Syndrome Decoding Problem ", Preprint, 2023

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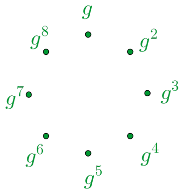

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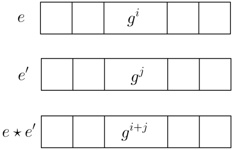

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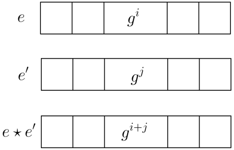

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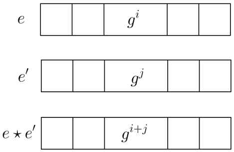

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Open Question

Can we exploit the commutativity of the restricted transformations?

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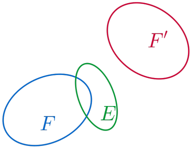
Future Research: Rank-metric Decoding



- For $x \in \mathbb{F}_{q^m}^n$: *Rank metric*: $wt_R(x) = \dim(\langle x_1, \ldots, x_n \rangle_{\mathbb{F}_q})$
- Rank Syndrome Decoding Problem: no NP-hard reduction
- Hamming-metric decoders have cost in $\mathcal{O}\left(q^{nc}\right)$ for some constant c
- Rank-metric decoders have cost in $\mathcal{O}\left(q^{n^2c'}\right)$ for some constant c'

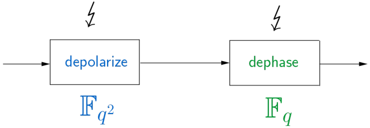
  $\rightarrow$ Small key sizes

$\rightarrow$ **Goal: Improve decoders**

- *Error support* $E = \langle e_1, \ldots, e_n \rangle_{\mathbb{F}_q}$
- candidate supersupports F, F'

TUM	Antonia Wachter-Zeh
International	Alberto Ravagnani (TU/e)

Future Research: Quantum Codes



- Quantum error-corrections:
  (1) depolarizing channel,
  (2) dephasing channel
- Introduced errors:
  (1) Z and X-errors,
  (2) only Z-errors
- X-errors are in $\mathbb{F}_{q^2} \setminus \mathbb{F}_q$
  Z-errors are in $\mathbb{F}_q \setminus \{0\}$

$\rightarrow$ Errors in base field more likely

$\rightarrow$ New metric:
$wt_\lambda(x) = \lambda$ if $x \in \mathbb{F}_{q^2} \setminus \mathbb{F}_q$
$wt_\lambda(x) = 1$ if $x \in \mathbb{F}_q \setminus \{0\}$

$\rightarrow$ **Goal: New bounds and constructions**

TUM	Robert König
International	Markus Grassl (ICTQT)

Further Research Directions

- **Quantum-Private Information Retrieval**
  - Retrieve file from database managed by untrusted server
  - without revealing to the server which file was requested
  - single server: only number-theoretic solutions: not quantum-secure

  → **Goal:** code-based quantum-private information retrieval

TUM	Antonia Wachter-Zeh
International	Camilla Hollanti (Aalto University)

- **Locally Recoverable Codes**

  → **Goal:** New constructions

  TUM Gregor Kemper

- **Isogeny-based Cryptography**

  → **Goal:** New systems

  TUM Christian Liedtke

Questions?

Thank you!

Hash-and-Sign: CFS

PROVER		VERIFIER
KEY GENERATION		
$\mathcal{S} = H$ parity-check matrix $\mathcal{P} = (t, HP)$ permuted H		
SIGNING		
Choose message m $s = \text{Hash}(m)$ Find e: $s = eH^\top = eP(HP)^\top$, and $\text{wt}(e) \leq t$	$\xrightarrow{m, eP}$	
		VERIFICATION
		Check if $\text{wt}(eP) \leq t$ and $eP(HP)^\top = \text{Hash}(m)$

Hash-and-Sign: CFS

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	VERIFICATION
	Check if $\text{wt}(eP) \leq t$
	and $eP(HP)^\top = \text{Hash}(m)$

Problem: Distinguishability

Hash-and-Sign: CFS

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KEY GENERATION		
$\mathcal{S} = H$ parity-check matrix $\mathcal{P} = (t, HP)$ permuted H		
SIGNING		
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		VERIFICATION
		Check if $\mathrm{wt}(eP) \leq t$ and $eP(HP)^\top = \mathrm{Hash}(m)$

Not any s is syndrome of low weight e

ZKID

PROVER		VERIFIER
		VERIFICATION
commitments c_0, c_1	$\xrightarrow{c_0, c_1}$	
	$\xleftarrow{b}$	$b \in \{0, 1\}$
response r_b	$\xrightarrow{r_b}$	
		Verify c_b using $r_b, \mathcal{P}$

SIGNING		
Choose message m		
Construct signature s from $\mathcal{S}$, m		
	$\xrightarrow{m, s}$	
		VERIFICATION
		Verify signature s using $\mathcal{P}$, m

Signature Scheme

ZKID

PROVER		VERIFIER
		VERIFICATION
commitments c_0, c_1	$\xrightarrow{c_0, c_1}$	
	$\xleftarrow{b}$	$b \in \{0, 1\}$
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Fiat-Shamir $\downarrow$

SIGNING		
Choose message m		
Construct signature s from $\mathcal{S}$, m		
	$\xrightarrow{m, s}$	
		VERIFICATION
		Verify signature s using $\mathcal{P}$, m

Signature Scheme

Fiat-Shamir

PROVER		VERIFIER
KEY GENERATION		
Given $\mathcal{P}, \mathcal{S}$ of some ZKID and message m		
SIGNING		
Choose commitment c $b = \text{Hash}(m, c)$ Compute response r_b Signature $s = (b, r_b)$		
	$\xrightarrow{m,s}$	
		VERIFICATION
		Using $r_b, \mathcal{P}$ construct c check if $b = \text{Hash}(m, c)$

CVE

PROVER		VERIFIER
KEY GENERATION		
Choose e with $\mathrm{wt}(e) \leq t$		
H parity-check matrix		
Compute $s = eH^\top$	$\xrightarrow{\mathcal{P}=(H,s,t)}$	
		VERIFICATION
Choose $u \in \mathbb{F}_q^n$, $\sigma \in \mathcal{S}_n$		
Set $c_1 = \mathrm{Hash}(\sigma, uH^\top)$		
Set $c_2 = \mathrm{Hash}(\sigma(u), \sigma(e))$	$\xrightarrow{c_1, c_2}$	
	$\xleftarrow{z}$	Choose $z \in \mathbb{F}_q^\times$
Set $y = \sigma(u + ze)$	$\xrightarrow{y}$	
$r_1 = \sigma$	$\xleftarrow{b}$	Choose $b \in \{1, 2\}$
$r_2 = \sigma(e)$	$\xrightarrow{r_b}$	$b = 1$: $c_1 = \mathrm{Hash}(\sigma, \sigma^{-1}(y)H^\top - zs)$
		$b = 2$: $\mathrm{wt}(\sigma(e)) = t$
		and $c_2 = \mathrm{Hash}(y - z\sigma(e), \sigma(e))$

CVE

PROVER		VERIFIER
KEY GENERATION		Recall SDP: (1) $s = eH^\top$ (2) $\mathrm{wt}(e) \leq t$
Choose e with $\mathrm{wt}(e) \leq t$		
H parity-check matrix		
Compute $s = eH^\top$	$\xrightarrow{\mathcal{P}=(H,s,t)}$	
		VERIFICATION
Choose $u \in \mathbb{F}_q^n$, $\sigma \in \mathcal{S}_n$		
Set $c_1 = \mathrm{Hash}(\sigma, uH^\top)$		
Set $c_2 = \mathrm{Hash}(\sigma(u), \sigma(e))$	$\xrightarrow{c_1, c_2}$	
	$\xleftarrow{z}$	Choose $z \in \mathbb{F}_q^\times$
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CVE

PROVER		VERIFIER
KEY GENERATION		
Choose e with $\mathrm{wt}(e) \leq t$		
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		$b = 2$: $\mathrm{wt}(\sigma(e)) = t$
		and $c_2 = \mathrm{Hash}(y - z\sigma(e), \sigma(e))$

$\boxed{\text{Problem: big signature sizes}}$

Cheating Probability

- Cheating probability = Probability of impersonator getting accepted
- For security level 2^{λ} want cheating probability $2^{-\lambda}$
- If cheating probability δ, with N rounds $\rightarrow$ cheating probability δ^{N}

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Cheating Probability

- Cheating probability = Probability of impersonator getting accepted
- For security level 2^λ want cheating probability $2^{-\lambda}$
- If cheating probability δ, with N rounds $\rightarrow$ cheating probability δ^N
- might need many rounds: large communication cost
- solution: compression technique
- do not send c_0^i, c_1^i in each round i
- before 1. round send $c = \text{Hash}(c_0^1, c_1^1, \ldots, c_0^N, c_1^N)$
- ith round: receiving challenge b prover sends r_b^i, c_{1-b}^i
- end: verifier checks $c = \text{Hash}(c_0^1, c_1^1, \ldots, c_0^N, c_1^N)$

C. Aguilar, P. Gaborit, J. Schrek. "A new zero-knowledge code based identification scheme with reduced communication", IEEE Information Theory Workshop, 2011.

Cheating Probability

- Cheating probability = Probability of impersonator getting accepted
- For security level 2^λ want cheating probability $2^{-\lambda}$
- If cheating probability δ, with N rounds $\rightarrow$ cheating probability δ^N
- might need many rounds: large communication cost
- other solution: MPC in the head
- third party: trusted helper sends commitments $\rightarrow \delta = 0$
- instead prover sends seeds of commitment: not ZK $\rightarrow$ cut and choose
- $x < N$ times send response, $N - x$ times send the seed of commitment
- to compress: use Merkle root or seed tree

T. Feneuil, A. Joux, M. Rivain. " Syndrome decoding in the head: Shorter signatures from zero-knowledge proofs", 2022.

Comparison

	ZKID	Hash-and-Sign
reduction to NP-hard		
low public key size		
low signature size		
fast verification		

Comparison

	ZKID	Hash-and-Sign
reduction to NP-hard	✓	×
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reduction to NP-hard	✓	×
low public key size	✓	×
low signature size		
fast verification		

Comparison

	ZKID	Hash-and-Sign	
reduction to NP-hard	✓	×	
low public key size	CVE: 70 B	WAVE: 3 MB	NIST: 3 KB
low signature size			
fast verification			

Comparison

	ZKID	Hash-and-Sign	
reduction to NP-hard	✓	×	
low public key size	CVE: 70 B	WAVE: 3 MB	NIST: 3 KB
low signature size	∼	✓	
fast verification			

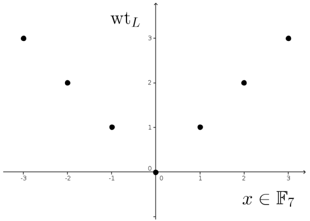
Comparison

	ZKID	Hash-and-Sign	
reduction to NP-hard	$\checkmark$	$\times$	
low public key size	CVE: 70 B	WAVE: 3 MB	NIST: 3 KB
low signature size	CVE: 43 KB	WAVE: 1 KB	NIST: 2 KB
fast verification			

Comparison

	ZKID	Hash-and-Sign	
reduction to NP-hard	✓	×	
low public key size	CVE: 70 B	WAVE: 3 MB	NIST: 3 KB
low signature size	CVE: 43 KB	WAVE: 1 KB	NIST: 2 KB
fast verification	∼	✓	

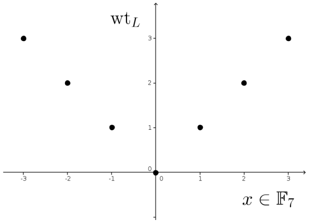
FuLeeca



Set up

- For $x \in \mathbb{F}_p$: $\mathrm{wt}_L(x) = \min\{x, \mid p - x \mid\}$.
  For $x \in \mathbb{F}_p^n$: $\mathrm{wt}_L(x) = \sum_{i=1}^n \mathrm{wt}_L(x_i)$.
- Representing $\mathbb{F}_p = \{-\frac{p-1}{2}, \ldots, 0, \ldots, \frac{p-1}{2}\}$,
  $\mathrm{wt}_L(x) = \mid x \mid$.

FuLeeca



x	-1	-3	1	2

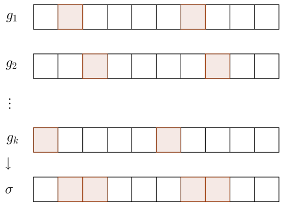
y	-3	1	2	0

$\mathrm{mt}(x, y) = 2$

Set up

- For $x \in \mathbb{F}_p$: $\mathrm{wt}_L(x) = \min\{x, \mid p - x \mid\}$.
  For $x \in \mathbb{F}_p^n$: $\mathrm{wt}_L(x) = \sum_{i=1}^n \mathrm{wt}_L(x_i)$.
- Representing $\mathbb{F}_p = \{-\frac{p-1}{2}, \dots, 0, \dots, \frac{p-1}{2}\}$,
  $\mathrm{wt}_L(x) = \mid x \mid$.
- Number of matches between $x, y \in \mathbb{F}_p^n$
  $\mathrm{mt}(x, y) = \mid \{i \mid \mathrm{sgn}(x_i) = \mathrm{sgn}(y_i)\} \mid$.

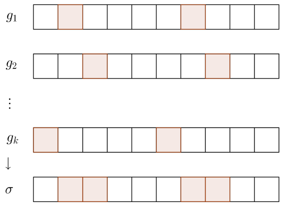
Statistical Attacks



Set up

- Low Hamming weight generators will produce low Hamming weight signatures
- Observing many signatures reveals the support of the secret low Hamming weight generators

Statistical Attacks



g_1	1	1	2	1	3	0
g_2	0	1	1	2	1	3
$\vdots$						
g_k	1	2	1	3	0	1
$\downarrow$						
σ	2	4	4	6	4	4

Set up

- Low Hamming weight generators will produce low Hamming weight signatures
- Observing many signatures reveals the support of the secret low Hamming weight generators

- Low Lee weight generators: $\text{supp}_L(x) = (\text{wt}_L(x_1), \dots, \text{wt}_L(x_n))$
- Signatures have low Lee weight
- Recovering Lee support of secret generators: much harder

FuLeeca

PROVER		VERIFIER
KEY GENERATION		
Secret key: $G = [A \;\; B]$, quasi-cyclic matrix, with low Lee weight		
Public key: $G' = [\mathrm{Id} \;\; A^{-1}B]$	$\xrightarrow{(G',t,\mu)}$	
SIGNING		
Choose message m		
$c = \mathrm{Hash}(m) \in \{\pm 1\}^n$		
Iteratively use G to construct codeword σ with		
$\mathrm{wt}_L(\sigma) \leq t$,		
$\mathrm{mt}(\sigma, c) \geq \mu$	$\xrightarrow{m,\sigma}$	
		VERIFICATION
		Verify that: $(1)\sigma H^\top = 0$,
		$(2)\mathrm{wt}_L(\sigma) \leq t$,
		$(3)\mathrm{mt}(c, \sigma) \geq \mu$