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Signature scheme with restricted errors

Violetta Weger

3. PQC Update
Fraunhofer AISEC

May 13, 2024



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Timeline

2016

NIST standardization call

for post-quantum PKE/KEM and signatures



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2022

Standardized signatures:

Dilithium, FALCON, SPHINCS+



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	1st round candidates:	40 submissions 11 code-based



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Implementation

- optimized AVX2
- memory-optimized
- constant worst-case runtime

fast < 1 MCycle (NIST cat. I)
fits on Cortex-M4 microcontroller
no signature rejection



Ingredients

- Restricted Syndrome Decoding
- Zero-Knowledge protocol

→ compact objects & efficient arithmetic
→ NP-hard problem
→ simple and well-studied
→ EUF-CMA security
→ standard optimizations



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Basics

message

channel

received

m



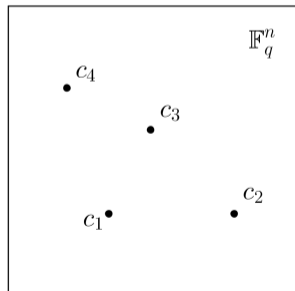
$m + e$



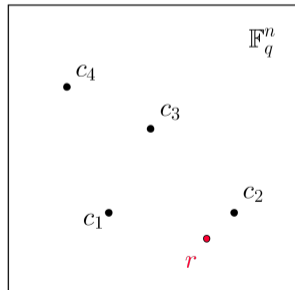
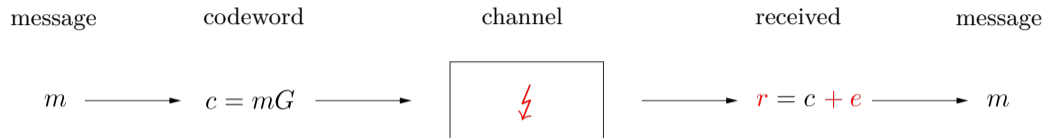
message

channel

received

 m  $m + e$ 

- Code $\mathcal{C} \subseteq \mathbb{F}_q^n$ linear k -dimensional subspace
- G generator matrix $\rightarrow c = mG$
- H parity-check matrix $\rightarrow cH^T = 0$



- Code $\mathcal{C} \subseteq \mathbb{F}_q^n$ linear k -dimensional subspace
- G generator matrix $\rightarrow c = mG$
- H parity-check matrix $\rightarrow rH^\top = eH^\top = s$
- Hamming weight: $\text{wt}(e) = |\{i \mid e_i \neq 0\}|$



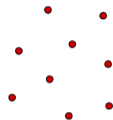
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Hard Problems

Algebraic structure
(Reed-Solomon, Goppa,...)
→ efficient decoders

\mathcal{C}



\mathcal{C}'

random code

→ how hard to decode?



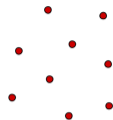
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Syndrome Decoding Problem (SDP)

Given p.c. matrix H , syndrome s , target weight t , find e s.t.

1. $s = eH^T$

2. $\text{wt}(e) \leq t$



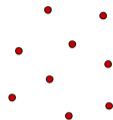
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Given p.c. matrix H , syndrome s , target weight t , find e s.t.

lin. constraint

$$1. s = eH^T$$

$$2. wt(e) \leq t$$

non-lin. constraint

- SDP is NP-hard
- ISD: exponential cost



E. Berlekamp, R. McEliece, H. Van Tilborg. "On the inherent intractability of certain coding problems", IEEE TIT, 1978.



E. Prange. "The use of information sets in decoding cyclic codes", IRE TIT, 1962.



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Zero-Knowledge Protocol

Prover

 secret

Verifier

 public





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Zero-Knowledge Protocol

Prover

Ⓡ secret



Interaction



Verifier

Ⓡ public





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Zero-Knowledge Protocol

signature scheme

Prover

\mathbb{F} secret

Fiat-Shamir



Verifier

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♀ e of $\text{wt}(e) = t$

♀ H, s, t

1. ✓ / 2. ✓



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signature scheme

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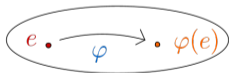
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J. Stern. "A new identification scheme based on syndrome decoding", Annual Int. Cryptology Conf., 1993.



P.-L. Cayrel, P. Véron, S. El Yousfi Alaoui. "A zero-knowledge identification scheme based on the q -ary syndrome decoding problem", Int. Workshop on Selected Areas in Cryptography, 2011.



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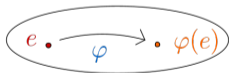
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e

	0	0			0
--	---	---	--	--	---

$\rightarrow \varphi \in (\mathbb{F}_q^*)^n \times S_n$

⚡ permutations are costly



Syndrome Decoding Problem Given p.c. matrix H , syndrome s , weight t , find e s.t.

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Restricted SDP (R-SDP) Given p.c. matrix H , syndrome s , restriction \mathbb{E} , find e s.t.

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1. $s = eH^\top$

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non-lin. constraint

$$\mathbb{E} = \{g^i \mid i \in \{1, \dots, z\}\} < \mathbb{F}_q^*$$

$g \in \mathbb{F}_q^*$ of prime order z



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e

	0	0			0
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\mathbb{F}_q^* \mathbb{F}_q^* \mathbb{F}_q^*

→

e

--	--	--	--	--	--

g^{i_1} g^{i_2} ... g^{i_n}

○ NP-hard

○ adaption of ISD: exponential cost



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R-SDP

Benefits of R-SDP

restriction $\mathbb{E} = \{g^i \mid i \in \{1, \dots, z\}\}$

rest. vectors $e = (g^{i_1}, \dots, g^{i_n}) \in \mathbb{F}_q^n$



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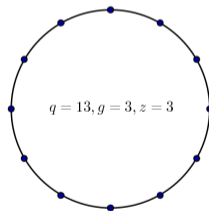
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Example

- $g = 3 \in \mathbb{F}_{13}$ of order $z = 3$
 - $\mathbb{E} = \{1, 3, 9\}$
 - $e = (1, 9, 3, 3) \in \mathbb{E}^4$
- exponent $\ell(e) = (0, 2, 1, 1) \in \mathbb{F}_3^4$





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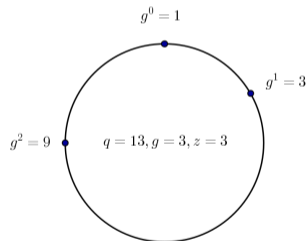
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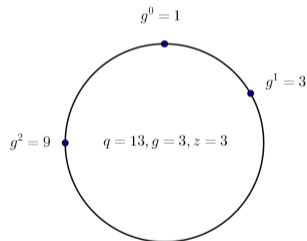
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size $|e|$ SDP: $t \log_2(n) + t \log_2(q - 1)$ R-SDP: $n \log_2(z)$



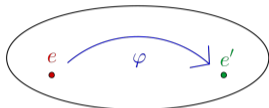
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Benefits of R-SDP

ZK protocols need linear transitive maps $\varphi : S \rightarrow S$



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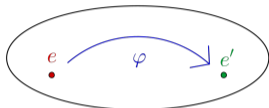
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$$\begin{array}{l} e = (g^{i_1} , \dots , g^{i_n}) \\ \varphi \curvearrowright \\ e' = (g^{j_1} , \dots , g^{j_n}) \end{array}$$

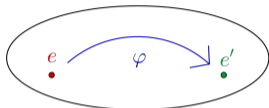


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$$\varphi \left(\begin{array}{l} e = (g^{i_1}, \dots, g^{i_n}) \\ \tilde{e} = (g^{j_1 - i_1}, \dots, g^{j_n - i_n}) \\ e' = (g^{j_1}, \dots, g^{j_n}) \end{array} \right)$$

$$\rightarrow \varphi(e) = e \star \tilde{e}$$

$$\rightarrow \tilde{e} \in \mathbb{E}^n$$

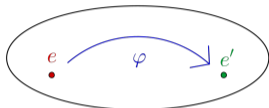


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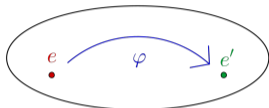


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 + \quad + \\
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 \rightarrow \varphi(e) \text{ is } (\mathbb{F}_z^n, +)
 \end{array}$$



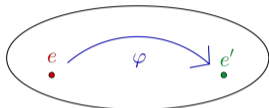
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Example

$$\mathbb{E}^4 = \{1, 3, 9\}^4 \subset \mathbb{F}_{13}^4$$

○ $e = (1, 9, 3, 3)$

$\star(3, 3, 9, 1)$

○ $e' = (3, 1, 1, 3)$

exponents \mathbb{F}_3^4

○ $\ell(e) = (0, 2, 1, 1)$

$+(1, 1, 2, 0)$

○ $\ell(e') = (1, 0, 0, 1)$



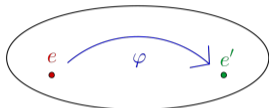
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size $|\varphi|$

SDP: $n \log_2(n) + t \log_2(q - 1)$

R-SDP: $n \log_2(z)$

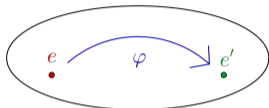


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size $|\varphi|$ **SDP:** $n \log_2(n) + t \log_2(q - 1)$ **R-SDP:** $n \log_2(z)$ $\varphi(e)$ **SDP:** $S_n \times (\mathbb{F}_q^n, \cdot)$ **R-SDP:** $(\mathbb{F}_z^n, +)$



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R-SDP(G)

R-SDP

Given H, s, \mathbb{E} , find e s.t. 1. $s = eH^\top$ 2. $e \in \mathbb{E}^n$



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R-SDP(G)

R-SDP(G) Given H, s, G , find e s.t. 1. $s = eH^\top$ 2. $e \in G$

subgroup $G = \langle x_1, \dots, x_m \rangle < \mathbb{E}^n$

$G = \{e = x_1^{u_1} \star \dots \star x_m^{u_m} \mid u_i \in \mathbb{F}_z\}$



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$$\mathbb{E} = \{1, 3, 9\} \subset \mathbb{F}_{13}$$

- $x_1 = (3, 1, 1, 3)$

- $x_2 = (1, 3, 9, 1)$

- $e = x_1^2 \star x_2^1 = (9, 3, 9, 9)$



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$x_2 = (1, 3, 9, 1)$

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exponents \mathbb{F}_3^4

○ $M_G = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 \end{pmatrix} \in \mathbb{F}_z^{m \times n}$

○ $(2, 1)M_G = (2, 1, 2, 2)$

○ send $(u_1, \dots, u_m) \in \mathbb{F}_z^m$



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○ send $(u_1, \dots, u_m) \in \mathbb{F}_z^m$

$|e| = |\varphi|$

R-SDP: $n \log_2(z)$

R-SDP(G): $m \log_2(z) < 1.5\lambda$



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Attacks

- \mathbb{E}, G have **multiplicative** structure

$$e = (g^{i_1}, \dots, g^{i_n})$$

- $s = eH^\top$ has **additive** structure

$$s_j = \sum_{\ell=1}^n h_{j,\ell} g^{i_\ell} \text{ for } j \in \{1, \dots, n-k\}$$



CROSS

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Attacks

- \mathbb{E}, G have **multiplicative** structure

$$e = (g^{i_1}, \dots, g^{i_n})$$

- Take \mathbb{E} with **no** additive structure

- $s = eH^\top$ has **additive** structure

$$s_j = \sum_{\ell=1}^n h_{j,\ell} g^{i_\ell} \text{ for } j \in \{1, \dots, n-k\}$$



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- \mathbb{E}, G have **multiplicative** structure
 $e = (g^{i_1}, \dots, g^{i_n})$
- Take \mathbb{E} with **no** additive structure
- **good**: $q = 13, g = 3, \mathbb{E} = \{1, 3, 9\}$

Attacks

- $s = eH^\top$ has **additive** structure
 $s_j = \sum_{\ell=1}^n h_{j,\ell} g^{i_\ell}$ for $j \in \{1, \dots, n-k\}$
- **bad**: $q = 13, g = 5, \mathbb{E} = \{1, 5, -1, -5\}$



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Attacks

- \mathbb{E}, G have **multiplicative** structure

$$e = (g^{i_1}, \dots, g^{i_n})$$

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- combinatorial:

ISD algorithms

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S. Bitzer, A. Pavoni, V. Weger, P. Santini, M. Baldi, and A. Wachter-Zeh. “[Generic Decoding of Restricted Errors](#)”, ISIT, 2023.



M. Baldi, S. Bitzer, A. Pavoni, P. Santini, A. Wachter-Zeh, and V. Weger. “[Zero knowledge protocols and signatures from the restricted syndrome decoding problem](#)”, PKC, 2024.



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Attacks

- o \mathbb{E}, G have **multiplicative** structure

$$e = (g^{i_1}, \dots, g^{i_n})$$

- o Take \mathbb{E} with **no** additive structure

- o **good**: $q = 13, g = 3, \mathbb{E} = \{1, 3, 9\}$

- o combinatorial:

ISD algorithms

- o algebraic attacks:

$e_i^z = 1$ Gröbner basis

- o $s = eH^T$ has **additive** structure

$$s_j = \sum_{\ell=1}^n h_{j,\ell} g^{i_\ell} \text{ for } j \in \{1, \dots, n - k\}$$

- o **bad**: $q = 13, g = 5, \mathbb{E} = \{1, 5, -1, -5\}$



S. Bitzer, A. Pavoni, V. Weger, P. Santini, M. Baldi, and A. Wachter-Zeh. “[Generic Decoding of Restricted Errors](#)”, ISIT, 2023.



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M. Baldi, et al. “[CROSS](#)”, NIST PQC round 1, 2023.



W. Beullens, P. Briaud, M. Øygarden. “[A Security Analysis of Restricted Syndrome Decoding Problems](#)”, 2024.



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Performance

Standard optimizations

- Hash trees
- weighted challenges

NIST cat. I

Problem	q, z	Type	(n, k, m)	rounds	Sign. (kB)	Sign (MCycles)	Verify (MCycles)
R-SDP	(127, 7)	fast	(127, 76, -)	163	19.1	1.28	0.78
		balanced		252	12.9	2.38	1.44
		short		960	10.1	8.96	5.84
R-SDP(G)	(509, 127)	fast	(55, 36, 25)	153	12.5	0.94	0.55
		balanced		243	9.2	1.85	1.09
		short		871	7.9	6.54	3.96

private and public keys < 0.1 kB

key gen. < 0.1 MCycle

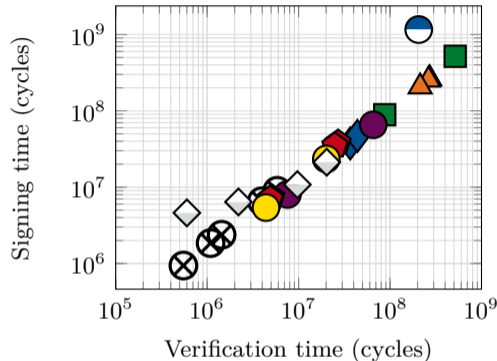
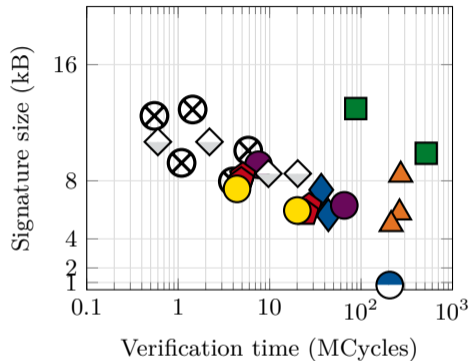
Measurements collected on an AMD Ryzen 5 Pro 3500U, clocked at 2.1GHz. The computer was running Debian GNU/Linux 12



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Comparison



Timings taken from <https://pqshield.github.io/nist-sigs-zoo/>



What's next?

- Hardware implementation
- Side-channel protection
- Worst-case to average-case reduction
- Smaller signatures?



Slides



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Codes & Restricted Objects Signature Scheme
<http://cross-crypto.com/>



Website



What's next?

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Slides



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Website

Thank you!

PROVER	VERIFIER
KEY GENERATION	
Choose e with $\text{wt}(e) \leq t$	
H parity-check matrix	
Compute $s = eH^\top$	$\xrightarrow{\mathcal{P}=(H,s,t)}$
VERIFICATION	
Choose $u \in \mathbb{F}_q^n, \sigma \in \mathcal{S}_n$	
Set $c_1 = \text{Hash}(\sigma, uH^\top)$	
Set $c_2 = \text{Hash}(\sigma(u), \sigma(e))$	$\xrightarrow{c_1, c_2}$
	\xleftarrow{z} Choose $z \in \mathbb{F}_q^\times$
Set $y = \sigma(u + ze)$	\xrightarrow{y}
$r_1 = \sigma$	\xleftarrow{b} Choose $b \in \{1, 2\}$
$r_2 = \sigma(e)$	$\xrightarrow{r_b}$ $b = 1: c_1 = \text{Hash}(\sigma, \sigma^{-1}(y)H^\top - zs)$
	$b = 2: \text{wt}(\sigma(e)) = t$
	and $c_2 = \text{Hash}(y - z\sigma(e), \sigma(e))$

CVE

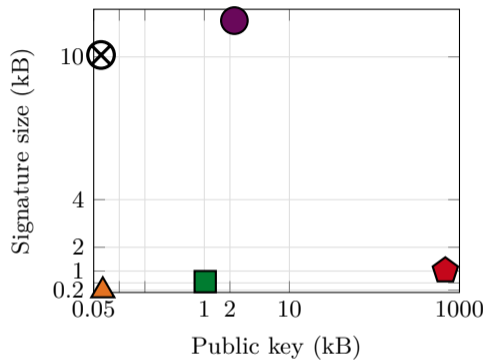
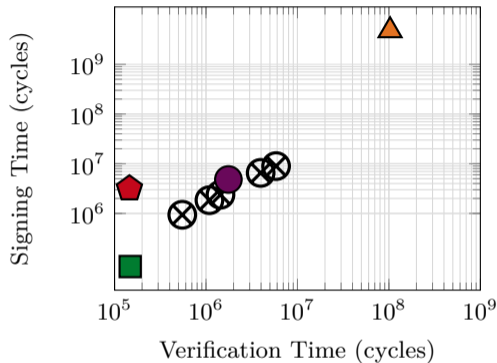
PROVER	VERIFIER
KEY GENERATION	Recall SDP: (1) $s = eH^\top$ (2) $\text{wt}(e) \leq t$
Choose e with $\text{wt}(e) \leq t$ H parity-check matrix Compute $s = eH^\top$	
	VERIFICATION
Choose $u \in \mathbb{F}_q^n$, $\sigma \in \mathcal{S}_n$ Set $c_1 = \text{Hash}(\sigma, uH^\top)$ Set $c_2 = \text{Hash}(\sigma(u), \sigma(e))$	$\xrightarrow{c_1, c_2}$ Choose $z \in \mathbb{F}_q^\times$
Set $y = \sigma(u + ze)$	\xleftarrow{z} \xrightarrow{y}
$r_1 = \sigma$	\xleftarrow{b} Choose $b \in \{1, 2\}$
$r_2 = \sigma(e)$	$\xrightarrow{r_b}$ $b = 1$: $c_1 = \text{Hash}(\sigma, \sigma^{-1}(y)H^\top - zs)$ $b = 2$: $\text{wt}(\sigma(e)) = t$ and $c_2 = \text{Hash}(y - z\sigma(e), \sigma(e))$

PROVER	VERIFIER
KEY GENERATION	
Choose e with $\text{wt}(e) \leq t$	
H parity-check matrix	
Compute $s = eH^\top$	$\xrightarrow{\mathcal{P}=(H,s,t)}$
VERIFICATION	
Choose $u \in \mathbb{F}_q^n, \sigma \in \mathcal{S}_n$	
Set $c_1 = \text{Hash}(\sigma, uH^\top)$	
Set $c_2 = \text{Hash}(\sigma(u), \sigma(e))$	$\xrightarrow{c_1, c_2}$
	\xleftarrow{z}
Set $y = \sigma(u + ze)$	\xrightarrow{y}
$r_1 = \sigma$	\xleftarrow{b}
$r_2 = \sigma(e)$	$\xrightarrow{r_b}$
	<div style="border: 1px solid red; padding: 5px; display: inline-block; color: red;"> Problem: big signature sizes </div>
	Choose $z \in \mathbb{F}_q^\times$
	Choose $b \in \{1, 2\}$
	$b = 1: c_1 = \text{Hash}(\sigma, \sigma^{-1}(y)H^\top - zs)$
	$b = 2: \text{wt}(\sigma(e)) = t$
	and $c_2 = \text{Hash}(y - z\sigma(e), \sigma(e))$



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- vs: Isogenies and lattices





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vs: Multivariate

