How to Sign using Restricted Errors

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Nordic Congress of
Mathematicians with EMS

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Antonia Wachter-Zeh

Motivation

2016 NIST standardization call for post-quantum PKE/KEM and signatures

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- PKE/KEM: 1 lattice-based, round 4: 3 code-based
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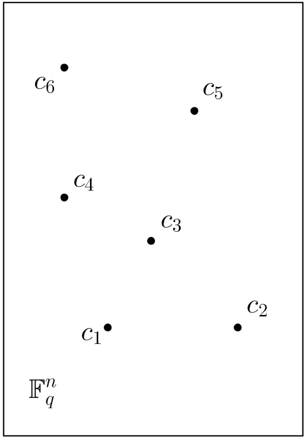
Coding Theory



Set Up

- *Code* $\mathcal{C} \subseteq \mathbb{F}_q^n$ linear k-dimensional subspace
- $c \in \mathcal{C}$ *codeword*
- $G \in \mathbb{F}_q^{k \times n}$ *generator matrix* $\mathcal{C} = \{xG \mid x \in \mathbb{F}_q^k\}$
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- $s = eH^\top$ *syndrome*

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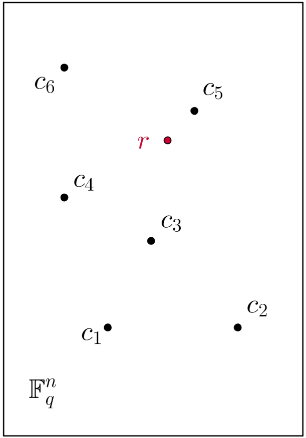

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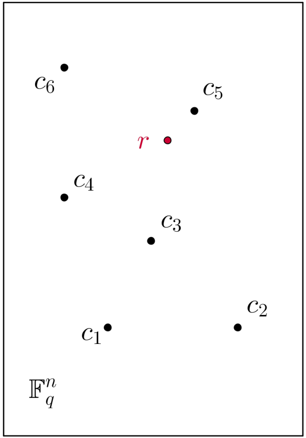

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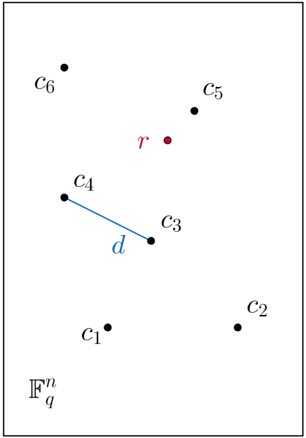

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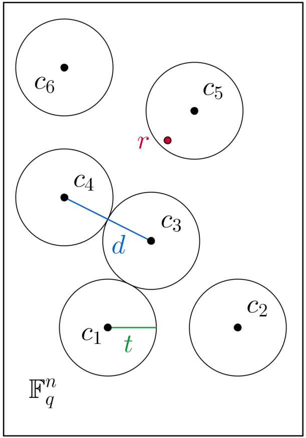
Coding Theory



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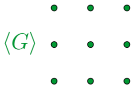
- *error-correction capacity*: $t = \lfloor (d(\mathcal{C}) - 1)/2 \rfloor$

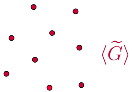
Hard Problems from Coding Theory

Algebraic structure

(Reed-Solomon, Goppa,..)

$\rightarrow$ efficient decoders





random code

$\rightarrow$ how hard to decode?

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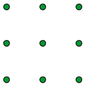
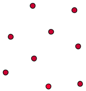
$\langle G \rangle$



- Decoding random linear code is NP-hard



$\langle \widetilde{G} \rangle$

random code

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E. Berlekamp, R. McEliece, H. Van Tilborg. "On the inherent intractability of certain coding problems ", IEEE Trans. Inf. Theory, 1978.

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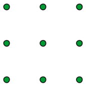
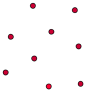
$\langle G \rangle$ 

scrambling
$\xrightarrow{\varphi}$

 $\langle \widetilde{G} \rangle$

Seemingly random code

$\rightarrow$ how hard to decode?

- Decoding random linear code is NP-hard
- First code-based cryptosystem based on this problem

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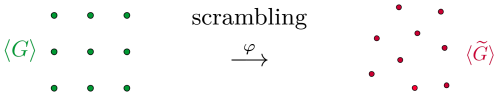

Seemingly random code
$\rightarrow$ how hard to decode?

- Decoding random linear code is NP-hard
- First code-based cryptosystem based on this problem
- Fastest solvers: ISD, exponential time

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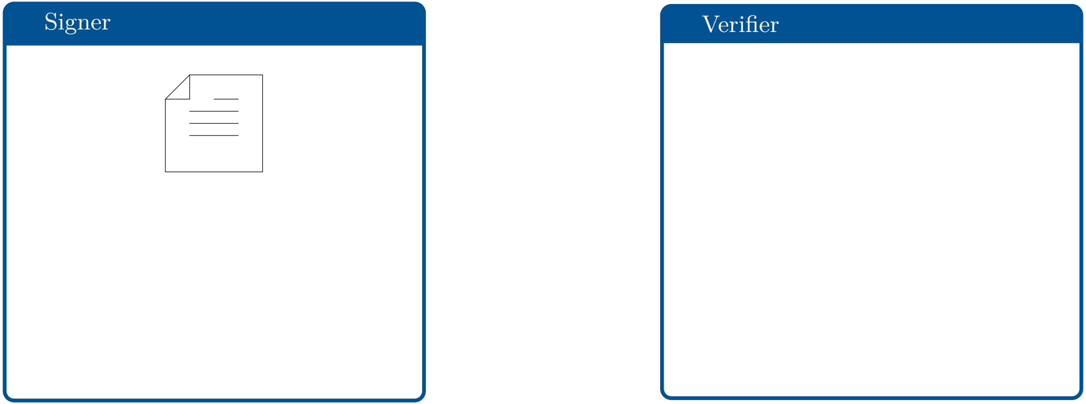
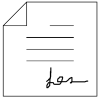
Idea of Signature Schemes



Idea of Signature Schemes

Signer



Verifier

Idea of Signature Schemes

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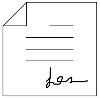
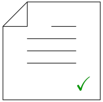


$\longrightarrow$

Verifier



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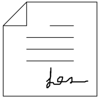


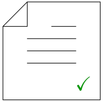
- Key Generation:
  $\mathcal{P}$ public, $\mathcal{S}$ secret
- Signing: use $\mathcal{S}$ and message m to generate signature σ

$\longrightarrow$

$\xrightarrow{\mathcal{P}}$

$\xrightarrow{m,\ \sigma}$

Verifier



- Verification: use $\mathcal{P}$ and message m to verify signature σ

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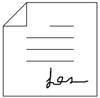

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small $\mathcal{P}$

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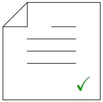

fast verification

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Idea of Signature Schemes

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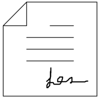

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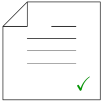

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2 Approaches for signatures:

- Hash-and-Sign
- Through ZK protocol

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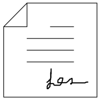

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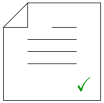

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2 Approaches for signatures:

Idea and Problem

- Hash-and-Sign

Main Topic

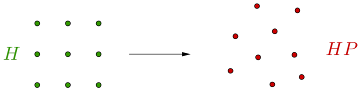
- Through ZK protocol

Hash-and-Sign

Following idea of McEliece

N. Courtois, M. Finiasz, N. Sendrier. "How to achieve a McEliece-based digital signature scheme", Asiacrypt, 2001.

- $\to$ start with structured code H
- $\to$ publish scrambled code HP



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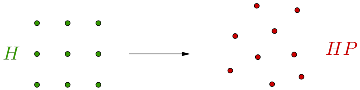

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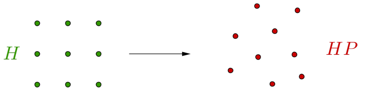
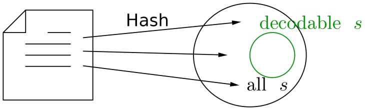


→ $\text{Hash}(m) = eH^\top$, $\text{wt}_H(e) \leq t$

→ signature $\sigma = eP$



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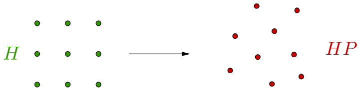
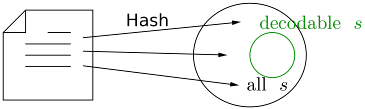


- $\longrightarrow$ $\mathrm{Hash}(m) = eH^\top$, $\mathrm{wt}_H(e) \leq t$
- $\longrightarrow$ signature $\sigma = eP$
- $\longrightarrow$ slow signing



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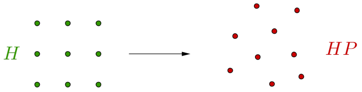

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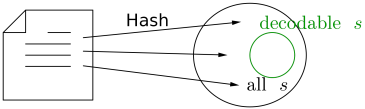
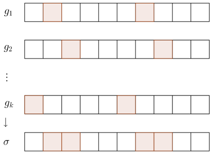


- reduce key sizes:

→ use quasi-cyclic codes

→ use low density generators



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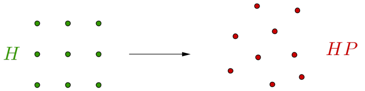

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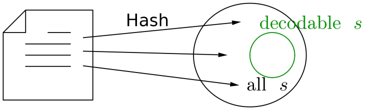
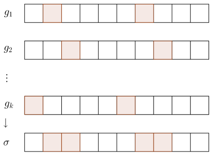


- reduce key sizes:

→ use quasi-cyclic codes

→ use low density generators

→ statistical attacks



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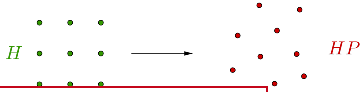
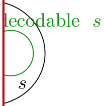


→ Hash(m) =

→ signature σ

→ slow signing



- reduce key

→ use quasi-cyclic codes

→ use low density generators

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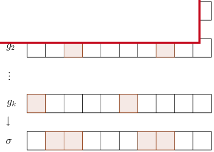

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S. Ritterhoff, G. Maringer, S. Bitzer, **V.W.**, P. Karl, T. Schamberger, J. Schupp, A. Wachter-Zeh, G. Sigl. "FuLeeca: A Lee-based signature scheme", 2023.

Idea of ZK Protocol

Prover		**Verifier**
$\mathcal{S}$: secret $\mathcal{P}$: related public key c: commitments to secret r_b: response to challenge b	$\xrightarrow{\mathcal{P},c}$ $\xleftarrow{b}$ $\xrightarrow{r_b}$	b: challenge Recover c from r_b and $\mathcal{P}$

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Idea of ZK Protocol

Prover

$\mathcal{S}$: secret
$\mathcal{P}$: related public key
c: commitments to secret
b: Hash of message, c
r_b: response to challenge b

Fiat-Shamir

$\xrightarrow{\mathcal{P},(b,r_b)}$

Verifier

Recover c from r_b and $\mathcal{P}$
Verify $b = \mathrm{Hash}(m, c)$

A. Fiat, A. Shamir. "How to prove yourself: Practical solutions to identification and signature problems.", Proceedings on Advances in cryptology-CRYPTO, 1986.

Idea of ZK Protocol

N

Prover

$\mathcal{S}$: secret
$\mathcal{P}$: related public key
c: commitments to secret
b: Hash of message, c
r_b: response to challenge b

$\xrightarrow{\mathcal{P},(b,r_b)}$

Verifier

Recover c from r_b and $\mathcal{P}$
Verify $b = \text{Hash}(m, c)$

- α cheating probability, λ bit security level
- *Rounds*: have to repeat ZK protocol N times: $2^{\lambda} < (1/\alpha)^{N}$

A. Fiat, A. Shamir. "How to prove yourself: Practical solutions to identification and signature problems.", Proceedings on Advances in cryptology-CRYPTO, 1986.

Code-based ZK Protocols

P.-L. Cayrel, P. Véron, S. El Yousfi Alaoui. "A zero-knowledge identification scheme based on the q-ary syndrome decoding problem", Selected Areas in Cryptography, 2011.

Syndrome Decoding Problem

Given parity-check matrix H, syndrome s, weight t, find e s.t.

1. $s = eH^\top$ 2. $\mathrm{wt}_H(e) \leq t$

Prover		**Verifier**
$\mathcal{S}$: e of weight t,		
$\mathcal{P}$: random H, $s = eH^\top$, t	$\xrightarrow{\mathcal{P}, c_1, c_2}$	
c_1: commitment to syndrome equation 1.		$b \in \{1, 2\}$
c_2: commitment to weight 2.	$\xleftarrow{b}$	
response: $r_1 = \varphi$, $r_2 = \varphi(e)$	$\xrightarrow{r_b}$	recover c_b from r_b and $\mathcal{P}$

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Prover **Verifier**

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c_1: commitment to

c_2: commitment to weight 2.

$\longleftarrow$

response: $r_1 = \varphi$, $r_2 = \varphi(e)$ $\xrightarrow{r_b}$ recover c_b from r_b and $\mathcal{P}$

Problem: large cheating probability $\rightarrow$ big signature sizes

Performance of Classical Approach

Classical CVE

- $\lambda = 128$ bit security level $\rightarrow N = 135$ $\rightarrow$ public key size: 832 b
- $q = 31, n = 256, k = 204$ $\rightarrow$ signature size: 43 kB

Performance of Classical Approach

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Recent improvements through in-the-head computations
$\rightarrow$ smaller signature sizes $\sim$ 15 kB

T. Feneuil, A. Joux, M. Rivain "Shared permutation for syndrome decoding: New zero-knowledge protocol and code-based signature", Designs, Codes and Cryptography, 2022.

T. Feneuil, A. Joux, M. Rivain "Syndrome decoding in the head: shorter signatures from zero-knowledge proofs", Crypto, 2022.

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based on knowing we need many rounds

T. Feneuil, A. Joux, M. Rivai … zero-knowledge protocol and code-based signature", Design…

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Problem of Classical Approach

Classical CVE (1 round)

- public key size: seed of H, s; $\log_2(q)(n-k) < 0.1$ kB
- signature size: Hash(m, c) and response: transformation φ or $\varphi(e)$

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Which φ are allowed?

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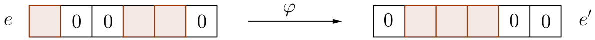
Given $H \in \mathbb{F}_q^{(n-k)\times n}, s \in \mathbb{F}_q^{n-k}$, weight t, find $e \in \mathbb{F}_q^n$ such that $s = eH^\top$ and $\mathrm{wt}_H(e) \le t$.



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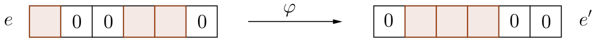


$\rightarrow \varphi$: linear isometries of Hamming metric:
permutation + scalar multiplication

Problem of Classical Approach

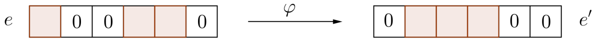
Classical CVE (1 round)

- public key size: seed of H, s; $\log_2(q)(n-k) < 0.1$ kB
- signature size: $\varphi(e) : t\log_2(q-1) + t\log_2(n)$ or $\varphi : n\log_2(q-1) + n\log_2(n)$

Which φ are allowed?

Syndrome Decoding Problem

Given $H \in \mathbb{F}_q^{(n-k)\times n}$, $s \in \mathbb{F}_q^{n-k}$, weight t, find $e \in \mathbb{F}_q^n$ such that $s = eH^\top$ and $\mathrm{wt}_H(e) \leq t$.



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Restricted Errors

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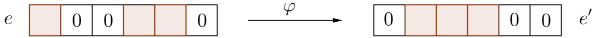

Can we avoid permutations ?

Restricted Errors

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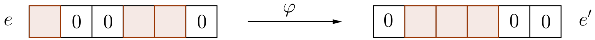

Can we avoid permutations - but keep the hardness of the problem?

$\downarrow$

Syndrome Decoding Problem

Given $H \in \mathbb{F}_q^{(n-k)\times n}$, syndrome $s \in \mathbb{F}_q^{n-k}$, find $e \in \mathbb{F}_q^n$ such that $s = eH^\top$.

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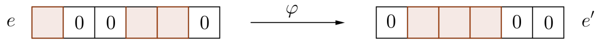

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Restricted Syndrome Decoding Problem

Given $H \in \mathbb{F}_q^{(n-k)\times n}$, syndrome $s \in \mathbb{F}_q^{n-k}$, $\mathbb{E} \subseteq \mathbb{F}_q^\star$, find $e \in \mathbb{E}^n$ such that $s = eH^\top$.

Restricted Errors

Syndrome Decoding Problem

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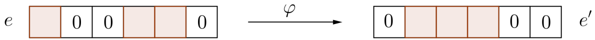

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$\boxed{\text{How to choose } \mathbb{E}\text{?}}$

Restricted Errors

M. Baldi, S. Bitzer, A. Pavoni, P. Santini, A. Wachter-Zeh, **V.W.** "Zero knowledge protocols and signatures from the restricted syndrome decoding problem ", Preprint, 2023

$$\boxed{(\mathbb{E}, \cdot) < (\mathbb{F}_q^\star, \cdot) \ \rightarrow \ g \in \mathbb{F}_q^\star \text{ of prime order } z \ \rightarrow \ \mathbb{E} = \{g^i \mid i \in \{1, \dots, z\}\}}$$

$$q = 13 \quad \rightarrow \quad g = 3 \text{ order } z = 3 \quad \rightarrow \quad \mathbb{E} = \{1, 3, 9\}$$

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$(\mathbb{E}^n, \star)$ $\xrightarrow{\ell}$ $(\mathbb{F}_z^n, +)$

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$(\mathbb{E}^n, \star)$

- $e = (1,9,3,3) \in \{1,3,9\}^4$

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new size:	before: $n\log_2((q-1)n)$	new: $n\log_2(z)$
fast arithmetic:	before: $(\mathbb{F}_q^n,\cdot)$	new: $(\mathbb{F}_z^n,+)$

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Can do even better

$(\mathbb{F}_z^n,+)$

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Restricted-G SDP

Restricted Syndrome Decoding Problem

Given $H \in \mathbb{F}_q^{(n-k)\times n}$, $s \in \mathbb{F}_q^{n-k}$, $\mathbb{E} \subseteq \mathbb{F}_q^\star$, find $e \in \mathbb{E}^n$ s.t. $s = eH^\top$.

- $(\mathbb{E}^n, \star) \cong (\mathbb{F}_z^n, +)$
- $e = (1, 9, 3, 3) \in \mathbb{E}^4 = \{1, 3, 9\}^4$

Restricted-G SDP

Restricted-G Syndrome Decoding Problem

Given $H \in \mathbb{F}_q^{(n-k)\times n}$, $s \in \mathbb{F}_q^{n-k}$, $\mathbb{E} \subseteq \mathbb{F}_q^\star$, $G = \langle x_1, \dots, x_m \rangle \leq \mathbb{E}^n$ find $e \in G$ s.t. $s = eH^\top$.

- $(\mathbb{E}^n, \star) \cong (\mathbb{F}_z^n, +)$

$\rightarrow$ Subgroup $(G, \star) \leq (\mathbb{E}^n, \star)$
$G = \langle x_1, \dots, x_m \rangle$

$\rightarrow$ $e' = \prod_{i=1}^m x_i^{u_i} \in G$

- $e = (1, 9, 3, 3) \notin G$

- $x_1 = (9, 1, 9, 1), x_2 = (9, 9, 1, 9), x_3 = (1, 9, 9, 3)$
- $e' = x_1^2 \star x_2^1 \star x_3^0 = (1, 9, 3, 9) \in G$

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- $M_G = [\ell(x_i)] \in \mathbb{F}_z^{m\times n}$
- $\ell(e') = yM_G, \; y \in \mathbb{F}_z^m$

$\rightarrow$ fast arithmetic

- $e = (1, 9, 3, 3) \; \notin G$
- $x_1 = (9, 1, 9, 1), x_2 = (9, 9, 1, 9), x_3 = (1, 9, 9, 3)$
- $e' = x_1^2 \star x_2^1 \star x_3^0 = (1, 9, 3, 9) \in G$
- $M_G = \begin{pmatrix} 2 & 0 & 2 & 0 \\ 2 & 2 & 0 & 2 \\ 0 & 2 & 2 & 1 \end{pmatrix}$
- $\ell(e') = (0, 2, 1, 2) = (2, 1, 0)M_G$

Restricted-G SDP

Restricted-G Syndrome Decoding Problem

Given $H \in \mathbb{F}_q^{(n-k)\times n}$, $s \in \mathbb{F}_q^{n-k}$, $\mathbb{E} \subseteq \mathbb{F}_q^\star$, $G = \langle x_1, \dots, x_m \rangle \leq \mathbb{E}^n$ find $e \in G$ s.t. $s = eH^\top$.

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$\rightarrow$ Subgroup $(G, \star) \leq (\mathbb{E}^n, \star)$
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- $M_G = [\ell(x_i)] \in \mathbb{F}_z^{m \times n}$
- $\ell(e') = y M_G, \ y \in \mathbb{F}_z^m$

$\rightarrow$ fast arithmetic

- $e = (1, 9, 3, 3) \ \notin G$
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smaller sizes: $n \log_2((q-1)n)$ $\rightarrow$ rest.: $n \log_2(z)$ $\rightarrow$ rest.-G: $m \log_2(z)$

Is this Safe?

Restricted Syndrome Decoding Problem

Given $H \in \mathbb{F}_q^{(n-k)\times n}$, $s \in \mathbb{F}_q^{n-k}$, $\mathbb{E} \subseteq \mathbb{F}_q^{\star}$, find $e \in \mathbb{E}^n$ s.t. $s = eH^\top$.

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Information set decoding?

- Restricted errors first introduced: $g = -1 \rightarrow z = 2$
- several proposals for small z e.g. $z = 4, 6$

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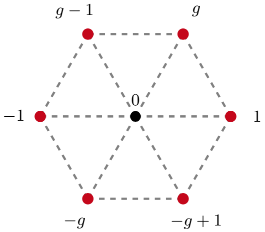

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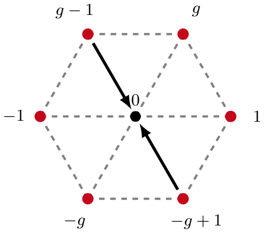

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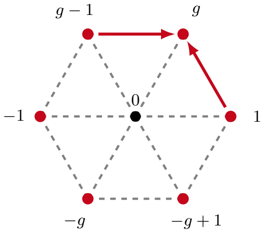

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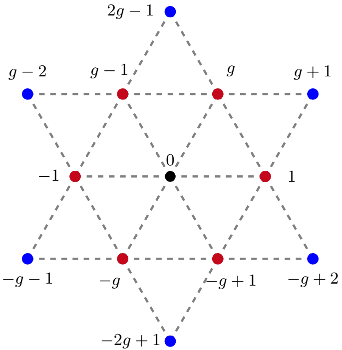

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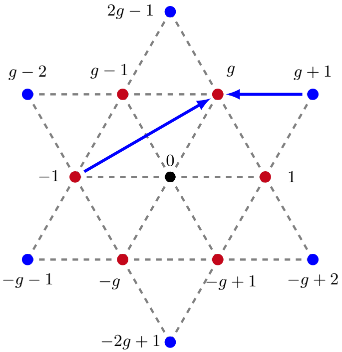

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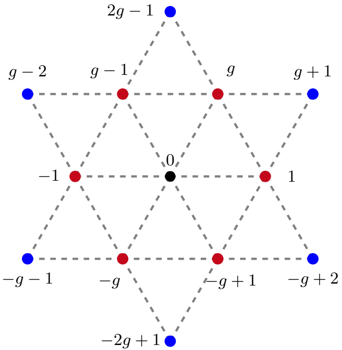

$\rightarrow$ our $\mathbb{E}$ has no additive structure

Performance of Restricted-G Signatures

Restricted CVE

- classical: $q = 31, n = 256, k = 204$ $\rightarrow$ signature size: 43 kB

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- rest.: $q = 127, z = 7, n = 2k = 127$ $\rightarrow$ signature size: 10 kB
- rest.-G: $q = 509, z = 127, m = 24, n = 2k = 42$ $\rightarrow$ signature size: 7 kB

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- classical: $q = 31, n = 256, k = 204$ $\rightarrow$ signature size: 43 kB
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Conclusion

- Can replace classical SDP with Restricted SDP/ Restricted-G SDP in any code-based ZK protocol.
- Achieve smaller signature sizes, smaller running times

Questions?

CROSS

Codes & Restricted Objects Signature Scheme

`http://cross-crypto.com/`

Thank you!

Running times

Running time given in kCycles, CROSS has only PoC, no optimization, parallelization

Scheme	**Key gen.**	**Signature gen.**	**Verification**
SPHINCS	1794	5802	6506
Dilitihium	49	140	61
CROSS	19	187	184

Is this Safe?

$G = \langle x_1, \ldots, x_m \rangle$: use generators?

No: $\prod_{i=1}^{m} x_i^{u_i} H^\top = s$

$\rightarrow$ not compatible unlike $\sum_{i=1}^{m} \lambda_i x_i H^\top = s$

Solving Restricted SDP in subgroup G

- we want q, z such that $\mathbb{E}$ has no additive structure
- Publicly known: $x_1, \ldots, x_m$ generators of multiplicative group G
- $x_\ell = (g^{i_{1,\ell}}, \ldots, g^{i_{n,\ell}})$
- define $M_G \in \mathbb{F}_z^{m \times n}$ having rows $(i_{1,\ell}, \ldots, i_{n,\ell})$

$$\begin{array}{cc} & \begin{array}{ccc} \phantom{i_{1,\ell}} & J & \phantom{i_{n,\ell}} \end{array} \\ M_G = & \begin{bmatrix} i_{1,\ell} & \boxed{\cdots} & i_{n,\ell} \end{bmatrix} \\ & \begin{array}{ccc} \phantom{i_{1,\ell}} & \hookrightarrow \text{ rank } m' & \end{array} \end{array}$$

$$m' \geq \min \left\{ \mid J \mid, \frac{\lambda}{\log_2(z)} \right\} \to \text{no improvement over enumerating all possible errors in these positions}$$

Comparison

Scheme	Public Key size	Signature size	Total size	Variant
SPHINCS$^+$	<0.1	16.7	16.7	Fast
	<0.1	7.7	7.7	Short
Falcon	0.9	0.6	1.5	-
Dilitihium	1.3	2.4	3.7	-
CROSS	0.1	7.7	7.8	Fast
	0.1	7.2	7.3	Short
GPS	0.1	24.0	24.1	Fast
	0.1	19.8	19.9	Short
FJR	0.1	22.6	22.7	Fast
	0.1	16.0	16.1	Short
SDItH	0.1	11.5	11.6	Fast
	0.1	8.3	8.4	Short
Ret. of SDitH	0.1	12.1	12.1	Fast, V3
	0.1	5.7	5.8	Shortest, V3

Comparison

Scheme	Public Key size	Signature size	Total size	Variant
WAVE	3200	2.1	3202	-
Durandal	15.2	4.1	19.3	-
Ideal Rank BG	0.5	8.4	8.9	Fast
	0.5	6.1	6.6	Short
MinRank Fen	18.2	9.3	27.5	Fast
	18.2	7.1	25.3	Short
Rank SDP Fen	0.9	7.4	8.3	Fast
	0.9	5.9	6.8	Short
Beu	0.1	18.4	18.5	Fast
	0.1	12.1	12.2	Short
PKP BG	0.1	9.8	9.9	Fast
	0.1	8.8	8.9	Short
FuLeeca	1.3	1.1	2.4	-

Hash-and-Sign: CFS

PROVER	VERIFIER
KEY GENERATION	
$\mathcal{S} = H$ parity-check matrix	
$\mathcal{P} = (t, HP)$ permuted H	
SIGNING	
Choose message m	
$s = \text{Hash}(m)$ Find e: $s = eH^\top = eP(HP)^\top$, and $\text{wt}(e) \leq t$	
$\xrightarrow{m, eP}$	
	VERIFICATION
	Check if $\text{wt}(eP) \leq t$ and $eP(HP)^\top = \text{Hash}(m)$

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Problem: Distinguishability

Hash-and-Sign: CFS

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KEY GENERATION	
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$\mathcal{P} = (t, HP)$ permuted H	
SIGNING	
Choose message m	
$s = \text{Hash}(m)$	
Find e: $s = eH^\top = eP(HP)^\top$,	
and $\text{wt}(e) \leq t$	
$\xrightarrow{m, eP}$	
	VERIFICATION
	Check if $\text{wt}(eP) \leq t$
	and $eP(HP)^\top = \text{Hash}(m)$

Not any s is syndrome of low weight e

ZKID

PROVER		VERIFIER
		VERIFICATION
commitments c_0, c_1	$\xrightarrow{c_0, c_1}$	
	$\xleftarrow{b}$	$b \in \{0, 1\}$
response r_b	$\xrightarrow{r_b}$	
		Verify c_b using $r_b, \mathcal{P}$

SIGNING		
Choose message m		
Construct signature s from $\mathcal{S}$, m		
	$\xrightarrow{m,s}$	
		VERIFICATION
		Verify signature s using $\mathcal{P}$, m

Signature Scheme

ZKID

PROVER		VERIFIER
		VERIFICATION
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Fiat-Shamir

SIGNING		
Choose message m		
Construct signature s from $\mathcal{S}$, m		
	$\xrightarrow{m, s}$	
		VERIFICATION
		Verify signature s using $\mathcal{P}$, m

Signature Scheme

Fiat-Shamir

PROVER		VERIFIER
KEY GENERATION		
Given $\mathcal{P}, \mathcal{S}$ of some ZKID and message m		
SIGNING		
Choose commitment c $b = \text{Hash}(m, c)$ Compute response r_b Signature $s = (b, r_b)$	$\xrightarrow{m,s}$	
		VERIFICATION
		Using $r_b, \mathcal{P}$ construct c check if $b = \text{Hash}(m, c)$

CVE

PROVER		VERIFIER
KEY GENERATION		
Choose e with $\mathrm{wt}(e) \leq t$		
H parity-check matrix		
Compute $s = eH^\top$	$\xrightarrow{\mathcal{P}=(H,s,t)}$	
		VERIFICATION
Choose $u \in \mathbb{F}_q^n$, $\sigma \in \mathcal{S}_n$		
Set $c_1 = \mathrm{Hash}(\sigma, uH^\top)$		
Set $c_2 = \mathrm{Hash}(\sigma(u), \sigma(e))$	$\xrightarrow{c_1, c_2}$	
	$\xleftarrow{z}$	Choose $z \in \mathbb{F}_q^\times$
Set $y = \sigma(u + ze)$	$\xrightarrow{y}$	
$r_1 = \sigma$	$\xleftarrow{b}$	Choose $b \in \{1, 2\}$
$r_2 = \sigma(e)$	$\xrightarrow{r_b}$	$b = 1$: $c_1 = \mathrm{Hash}(\sigma, \sigma^{-1}(y)H^\top - zs)$
		$b = 2$: $\mathrm{wt}(\sigma(e)) = t$
		and $c_2 = \mathrm{Hash}(y - z\sigma(e), \sigma(e))$

CVE

PROVER		VERIFIER
KEY GENERATION		Recall SDP: (1) $s = eH^\top$ (2) $\mathrm{wt}(e) \leq t$
Choose e with $\mathrm{wt}(e) \leq t$		
H parity-check matrix		
Compute $s = eH^\top$	$\xrightarrow{\mathcal{P}=(H,s,t)}$	
		VERIFICATION
Choose $u \in \mathbb{F}_q^n$, $\sigma \in \mathcal{S}_n$		
Set $c_1 = \mathrm{Hash}(\sigma, uH^\top)$		
Set $c_2 = \mathrm{Hash}(\sigma(u), \sigma(e))$	$\xrightarrow{c_1, c_2}$	
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CVE

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Choose e with $\mathrm{wt}(e) \leq t$		
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Compute $s = eH^\top$	$\xrightarrow{\mathcal{P}=(H,s,t)}$	
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Problem: big signature sizes

Cheating Probability

- Cheating probability = Probability of impersonator getting accepted
- For security level 2^λ want cheating probability $2^{-\lambda}$
- If cheating probability δ, with N rounds $\to$ cheating probability δ^N

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- If cheating probability δ, with N rounds $\rightarrow$ cheating probability δ^N
- might need many rounds: large communication cost
- solution: compression technique
- do not send c_0^i, c_1^i in each round i
- before 1. round send $c = \text{Hash}(c_0^1, c_1^1, \ldots, c_0^N, c_1^N)$
- ith round: receiving challenge b prover sends r_b^i, c_{1-b}^i
- end: verifier checks $c = \text{Hash}(c_0^1, c_1^1, \ldots, c_0^N, c_1^N)$

C. Aguilar, P. Gaborit, J. Schrek. "A new zero-knowledge code based identification scheme with reduced communication", IEEE Information Theory Workshop, 2011.

Cheating Probability

- Cheating probability = Probability of impersonator getting accepted
- For security level 2^λ want cheating probability $2^{-\lambda}$
- If cheating probability δ, with N rounds $\rightarrow$ cheating probability δ^N
- might need many rounds: large communication cost
- other solution: MPC in the head
- third party: trusted helper sends commitments $\rightarrow \delta = 0$
- instead prover sends seeds of commitment: not ZK $\rightarrow$ cut and choose
- $x < N$ times send response, $N - x$ times send the seed of commitment
- to compress: use Merkle root or seed tree

T. Feneuil, A. Joux, M. Rivain. "Syndrome decoding in the head: Shorter signatures from zero-knowledge proofs", 2022.

Comparison

	ZKID	Hash-and-Sign
reduction to NP-hard		
low public key size		
low signature size		
fast verification		

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reduction to NP-hard	$\checkmark$	$\times$	
low public key size	CVE: 70 B	WAVE: 3 MB	NIST: 3 KB
low signature size			
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reduction to NP-hard	$\checkmark$	$\times$	
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low signature size	$\sim$	$\checkmark$	
fast verification			

Comparison

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low public key size	CVE: 70 B	WAVE: 3 MB	NIST: 3 KB
low signature size	CVE: 43 KB	WAVE: 1 KB	NIST: 2 KB
fast verification			

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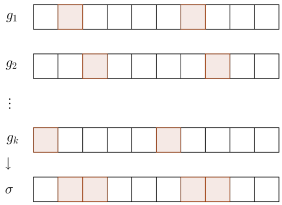
fast verification	$\sim$	✓	

Statistical Attacks



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- Low Hamming weight generators will produce low Hamming weight signatures
- Observing many signatures reveals the support of the secret low Hamming weight generators

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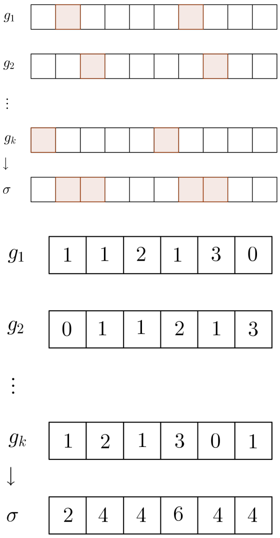

Set up

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- Observing many signatures reveals the support of the secret low Hamming weight generators

- Low Lee weight generators: $\mathrm{supp}_L(x) = (\mathrm{wt}_L(x_1), \dots, \mathrm{wt}_L(x_n))$
- Signatures have low Lee weight
- Recovering Lee support of secret generators: much harder