Part I:

Multivariate Bayesian Analysis of Atmosphere-Ocean General Circulation Models

GSP-NCAR, May 2nd 2005

Bayes Model for Climate Projections

Synthesizing temperature and precipitation climate projections from the outputs of several AOGCMs’ with a hierarchical Bayesian approach.

In collaboration with: Stephan Sain - CU at Denver
Claudia Tebaldi - NCAR
Jerry Meehl - NCAR
Linda Mearns - NCAR
Tom Wigley - NCAR
Doug Nychka - NCAR

Study Climate with AOGCMs

AOGCM: Atmosphere-Ocean General Circulation Models

Numerical models that calculate the detailed large-scale motions of the atmosphere and the ocean explicitly from hydrodynamical equations.

Models Do Not Agree

Example: Atmospheric Model

Input

- External forcings (radiation, volcanos, ...)
- Anthropogenic forcings (GHG, aerosols, ...)
- Initial conditions

- Flow dynamics, PDEs
- Large scale discretization and simplifications
- Parametrization

Output

- “Mean” temperatures and precipitation
- Pressure, wind, ...
Models Do Not Agree

- Variability of global temperature increase across 16 models. MAGICC/SCENGEN program (Wigley, 2003).
- Probabilistic description of regional climate changes. (Tebaldi et al. 2005).

Statistical Model

For models \(i = 1, \ldots, N\), stack the gridded output into vectors:

- \(X_i = \) simulated present climate
- \(Y_i = \) simulated future climate

Objective:

Probabilistic description of modeled climate change

Data

Data provided for the Fourth Assessment Report of IPCC:

- Several models (CNRM, CSIRO, GFDL, GISSR, HADCM, INMCM, IPSL, MIROC, MPI, MRI, PCM, \ldots)
- 2.8° x 2.8° resolution (8192 data points)
- A2 scenario ("business as usual")
- Temperature, \ldots, monthly and seasonal averages over years 1961–1990 and 2070–2099
- NCEP reanalysis as "observations"

Statistical Model

Data level:

\[
D_i = Y_i - X_i = \text{simulated climate change} = Y_i - X_i = \text{large scale structure } + \text{small scale structure} = Y_i - X_i = \mathbf{M} \theta_i + \epsilon_i, \quad \text{with } \epsilon_i \sim \mathcal{N}(0, \phi, \Sigma)
\]

Process level:

\[
\theta_i \sim \mathcal{N}(\mu, \psi_1 I)
\]

Prior level:

\[
\begin{align*}
\mu & \sim \mathcal{N}(0, \sigma^2 I) \\
\phi & \sim \mathcal{I}(\xi_1, \xi_2) \\
\psi & \sim \mathcal{I}(\xi_3, \xi_4)
\end{align*}
\]

Basis Functions

We need a "rich" truncated basis set.

Current candidate:

- Harmonic functions on the sphere
- Indicators for continents, sea ice, \ldots
- Patterns of current climate from NCEP reanalysis

Covariance Matrices

Examples of positive definite functions on the sphere:

1. representation with an infinite series of Legendre polynomials

\[
c(h; \sigma, \tau) = \sigma (1 - 2\tau \cos(h) + \tau^2)^{-3/2}
\]

2. restriction of a positive definite function on \(\mathbb{R}^3\) to the sphere

\[
c(h; \sigma, \tau) = \sigma \exp(-\tau \sin(h/2))
\]

We only parameterize the scale \(\sigma\) of the covariance matrices.

The "range" \(\tau\) is choosen according an "empirical Bayes" approach.
Gibbs Sampler

- Full conditionals for the parameters are available
- Gibbs sampler programmed in R
- Run 10000 iterations (5000 burn-in, keep every 10th, takes ≈ 2 hours)
- Assessing convergence with: trace plots, different starting values, ...

Posterior Climate Change

Posterior 20% quantile of temperature change

Posterior Climate Change

Probability that we observe at least a 2°K temperature increase

Posterior Model Realisations

Discussion and Further Work

- Promising approach (statistically and climatologically)
- Improve model for precipitation
- Generalize covariance parameterization
- Use “current” climate for better priors
- Extend to multivariate setting
- Implement ensemble runs
Part II:

**KriSp:**

a Package for Interpolation of Large Datasets Using Covariance Tapering

May 2nd, GSP NCAR

---

**Tapering and Kriging**

Introduce a sparseness structure in the covariance via tapering to gain computational advantages in large kriging problems constraint to maintaining asymptotic optimality.

In collaboration with Marc Genton and Doug Nychka.

---

**Motivation**

Precipitation anomaly in April 1948

![Map of precipitation anomaly in April 1948](image)

**Best Linear Unbiased Predictor**

Suppose a spatial process of the form

\[ Z(x), \quad x \in D, \quad \mathbb{E}(Z) = 0, \quad \text{Cov}(Z) = \Sigma \]

with observations \( Z = (Z(x_1), \ldots, Z(x_n))^T, \; x_i \in D \subset \mathbb{R}^d. \)

The kriging estimator (BLUP) is

\[ \hat{Z}(x_0) = \mathbf{c}^T \Sigma^{-1} Z \]

with \( \mathbf{c}_i = \text{Cov}(Z(x_0), Z(x_i)) \).

---

**Motivation**

Prediction at a large number of points involves either

- solving one large linear system,
- solving many tiny linear systems

or, being smarter and

- “approximating” the covariance and solving “efficiently” one large but sparse linear system.

---

**Synopsis**

Completed:
- Submitted paper
- Beta-version of package with comprehensive tutorial

In progress:
- Apply to other statistical problems, like microarrays

In future:
- Nonstationarity, MLE, . . .