Spatial Hierarchical Bayes Model for AOGCM Climate Projections

JSM, Aug. 2004

Bayes Model for Climate Projections

Synthesizing temperature and precipitation climate projections from the outputs of several AOGCMs' weighted according to model bias and convergence.

In collaboration with: Stephan Sain - CU at Denver
Tom Wigley - NCAR
Doug Nychka - NCAR

As Beautiful. . .

. . . So Dangerous

lostate.edu/flood97.html

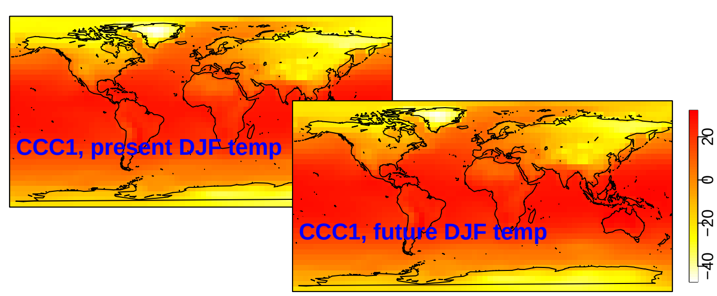
Study Climate with AOGCMs

AOGCM: Atmospheric-Ocean General Circulation Models

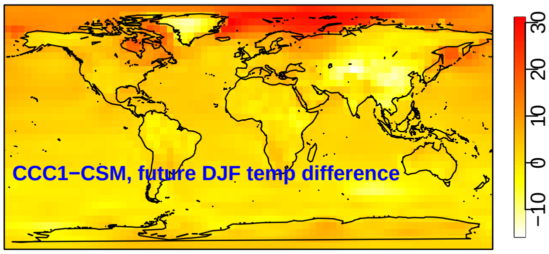
Numerical models that calculate the detailed large-scale motions of the atmosphere or the ocean explicitly from hydrodynamical equations.

Study Climate with AOGCMs

AOGCM: Atmospheric-Ocean General Circulation Models



Models Do Not Agree



Example: Atmospheric Model

Input
- External forcings (radiation, volcanos,. . .)
- Anthropogenic forcings (GHG, aerosols,. . .)
- Initial conditions

↓

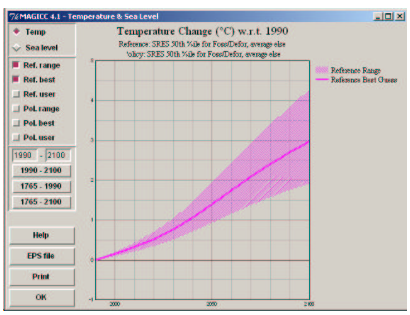
- Flow dynamics, PDEs
- Large scale discretization and simplifications
- Parametrization

↓

Output
- "Mean" temperatures and precipitation
- Pressure, wind, . . .

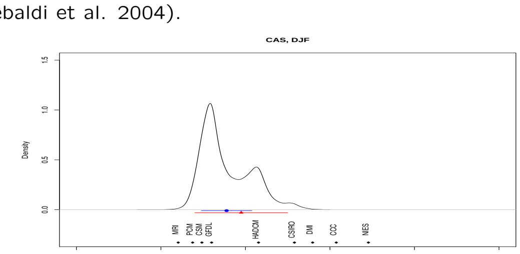
Models Do Not Agree

- Variability of global temperature increase across 16 models. MAGICC/SCENGEN program (Wigley, 2003).



Models Do Not Agree

- Variability of global temperature increase across 16 models. MAGICC/SCENGEN program (Wigley, 2003).
- Probabilistic description of regional climate changes. (Tebaldi et al. 2004).



Data

Data from MAGICC/SCENGEN 4.1 (Wigley, 2003):

- 16 models (CCC1, CSM, ECHx, HADx, GFDL, . . .)
- $5^\circ \times 5^\circ$ resolution (2592 data points)
- Perturbation experiment with 1% CO_2 increase per year
- Monthly averages over 20 years, 1-20 and 61-80
- . . .

Statistical Model

For models $i = 1, \dots, N$, stack the gridded output into vectors:

$\mathbf{X}_0$ = observed present climate = present climate + meas. error
$\mathbf{X}_i$ = simulated present climate$_i$ = present climate + model bias$_i$
$\mathbf{Y}_i$ = simulated future climate$_i$ = future climate + model bias$_i$

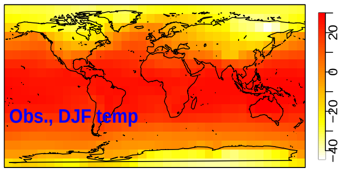
Objective:

Probabilistic description of modeled climate change

Statistical Model

$$\begin{aligned} \mathbf{X}_0 &= \boldsymbol{\mu}_x + \boldsymbol{\varepsilon} && \text{(observed present climate)} \\ \mathbf{X}_i &= \boldsymbol{\mu}_x + \mathbf{u}_i + \boldsymbol{\sigma}_i && \text{(simulated present climate)} \\ \mathbf{Y}_i &= \boldsymbol{\mu}_y + \mathbf{v}_i + \boldsymbol{\nu}_i && \text{(simulated future climate)} \end{aligned}$$

$$\boldsymbol{\mu}_x = \mathbf{M}_c\boldsymbol{\theta} \qquad \boldsymbol{\varepsilon} \sim \mathcal{N}_n(\mathbf{0}, \mathbf{S})$$



Statistical Model

$$\begin{aligned} \mathbf{X}_0 &= \boldsymbol{\mu}_x + \boldsymbol{\varepsilon} && \text{(observed present climate)} \\ \mathbf{X}_i &= \boldsymbol{\mu}_x + \mathbf{u}_i + \boldsymbol{\sigma}_i && \text{(simulated present climate)} \\ \mathbf{Y}_i &= \boldsymbol{\mu}_y + \mathbf{v}_i + \boldsymbol{\nu}_i && \text{(simulated future climate)} \end{aligned}$$

$$\begin{aligned} \boldsymbol{\mu}_x &= \mathbf{M}_c\boldsymbol{\theta} \qquad \boldsymbol{\varepsilon} \sim \mathcal{N}_n(\mathbf{0}, \mathbf{S}) \\ \boldsymbol{\mu}_x &= \mathbf{M}_c\boldsymbol{\theta} \qquad \mathbf{u}_i = \mathbf{M}_b\boldsymbol{\beta}_i \qquad \boldsymbol{\sigma}_i \sim \mathcal{N}_n(\mathbf{0}, \boldsymbol{\Sigma}_i) \end{aligned}$$

Statistical Model

$$\begin{aligned} \mathbf{X}_0 &= \boldsymbol{\mu}_x + \boldsymbol{\varepsilon} && \text{(observed present climate)} \\ \mathbf{X}_i &= \boldsymbol{\mu}_x + \mathbf{u}_i + \boldsymbol{\sigma}_i && \text{(simulated present climate)} \\ \mathbf{Y}_i &= \boldsymbol{\mu}_y + \mathbf{v}_i + \boldsymbol{\nu}_i && \text{(simulated future climate)} \end{aligned}$$

$$\begin{aligned} \boldsymbol{\mu}_x &= \mathbf{M}_c\boldsymbol{\theta} \qquad \boldsymbol{\varepsilon} \sim \mathcal{N}_n(\mathbf{0}, \mathbf{S}) \\ \boldsymbol{\mu}_x &= \mathbf{M}_c\boldsymbol{\theta} \qquad \mathbf{u}_i = \mathbf{M}_b\boldsymbol{\beta}_i \qquad \boldsymbol{\sigma}_i \sim \mathcal{N}_n(\mathbf{0}, \boldsymbol{\Sigma}_i) \\ \boldsymbol{\mu}_y &= \mathbf{M}_c\boldsymbol{\eta} \qquad \mathbf{v}_i = \mathbf{M}_b\boldsymbol{\gamma}_i \qquad \boldsymbol{\nu}_i = \boldsymbol{\omega}_i + \rho_i\boldsymbol{\sigma}_i \\ & \qquad \qquad \boldsymbol{\omega}_i \sim \mathcal{N}_n(\mathbf{0}, \boldsymbol{\Omega}_i) \quad \boldsymbol{\omega}_i \perp \boldsymbol{\sigma}_i \end{aligned}$$

Covariance Matrices

The covariance matrices $\mathbf{S}$, $\boldsymbol{\Sigma}_i$, $\boldsymbol{\Omega}_i$ are positive definite.

Examples of positive definite functions on the sphere:

1. representation with an infinite series of Legendre polynoms

$$c(h; \sigma, \tau) = \sigma\Big(1 - 2\tau\cos(h) + \tau^2\Big)^{-3/2}$$

2. restriction of a positive definite function on $\mathbb{R}^3$ to the sphere

$$c(h; \sigma, \tau) = \sigma\exp\Big(-\tau\sin(h/2)\Big)$$

We only parameterize the scale σ of the covariance matrices.

The "range" τ is choosen according an "empirical Bayes" approach.

Priors

Let $\mathbf{C}$ be a positive definite matrix defined by $c(\cdot; 1, \tau)$ and put

$$\boldsymbol{\Sigma}_i = \phi_i\mathbf{C} \qquad \boldsymbol{\Omega}_i = \psi_i\mathbf{C}$$

We use vague (dispersed) priors

$$\begin{aligned} \boldsymbol{\theta} &\sim \mathcal{N}_p(\mathbf{0}, \xi_1^2\mathbf{I}) & \boldsymbol{\eta} &\sim \mathcal{N}_p(\mathbf{0}, \xi_2^2\mathbf{I}) \\ \boldsymbol{\beta}_i &\sim \mathcal{N}_q(\mathbf{0}, \xi_3^2\mathbf{I}) & \boldsymbol{\gamma}_i &\sim \mathcal{N}_q(\mathbf{0}, \xi_4^2\mathbf{I}) \\ \phi_i &\sim \mathrm{I}\Gamma(\xi_5, \xi_6) & \psi_i &\sim \mathrm{I}\Gamma(\xi_7, \xi_8) \\ \rho_i &\sim \mathcal{N}(0, \xi_9^2) \end{aligned}$$

Posteriors

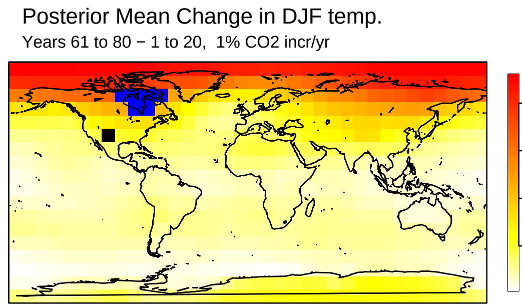
Full conditionals for the parameters are available

$$\begin{aligned} \boldsymbol{\theta}, \boldsymbol{\eta} \mid \ldots &\sim \mathcal{N}_p(\cdot, \cdot) \\ \boldsymbol{\beta}_i, \boldsymbol{\gamma}_i \mid \ldots &\sim \mathcal{N}_q(\cdot, \cdot) \\ \phi_i, \psi_i \mid \ldots &\sim \mathrm{I}\Gamma(\cdot, \cdot) \\ \rho_i \mid \ldots &\sim \mathcal{N}(\cdot, \cdot) \end{aligned}$$

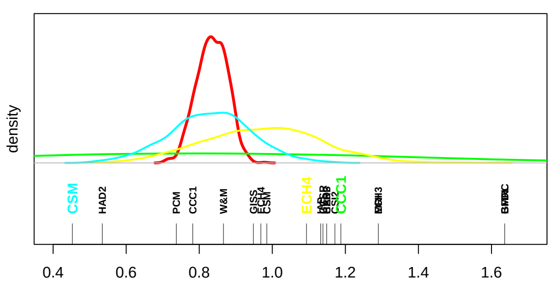
Gibbs Sampler

- Gibbs sampler programmed in R
- Run 50000 iterations
  (40000 burn-in, keep every 10th, takes ≈ 12 hours)
- Visual inspection of convergence

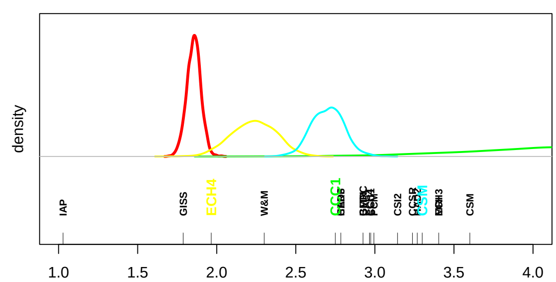
Global Posterior Climate Change



Local Posterior Climate Change



Regional Posterior Climate Change



Discussion and Further Work

- A first "running horse"
- Promising approach (statistically and climatologically)

- Evaluate and up/down-scale first results
- Generalize covariance parameterization
- Use "current" climate for better priors
- Extend to multivariate setting