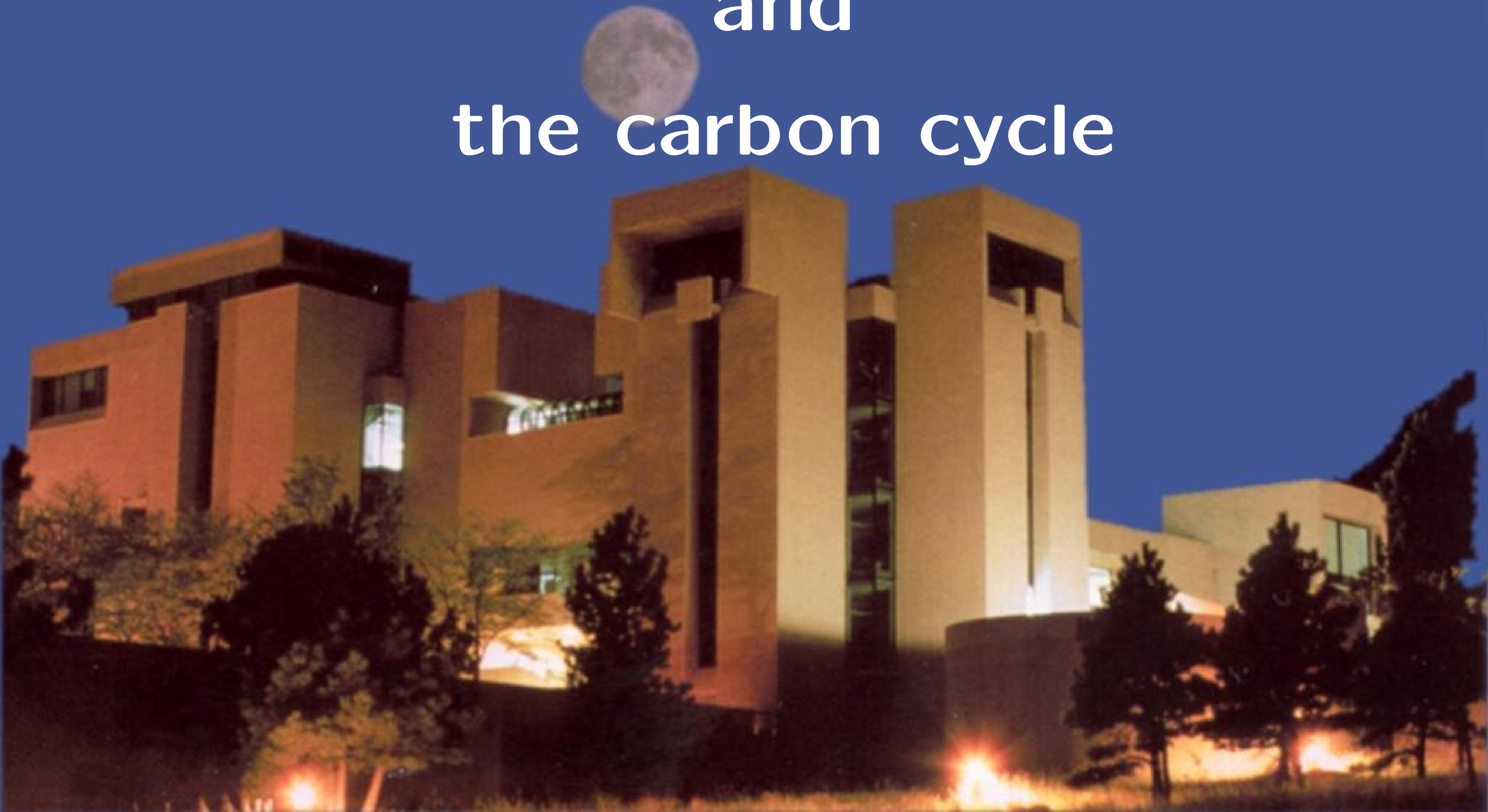
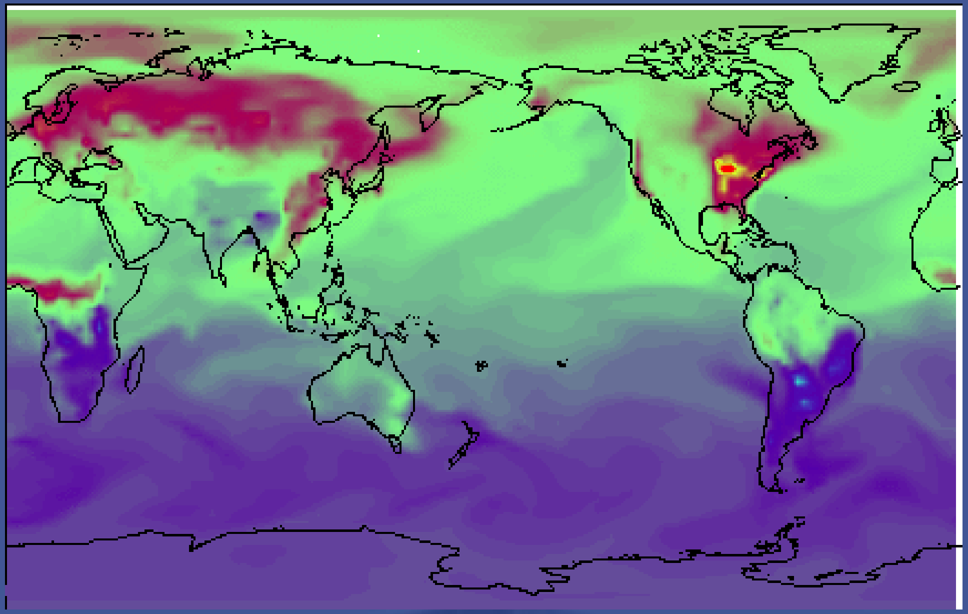
Offline transport models and the carbon cycle



CO_2 Concentrations at the Surface

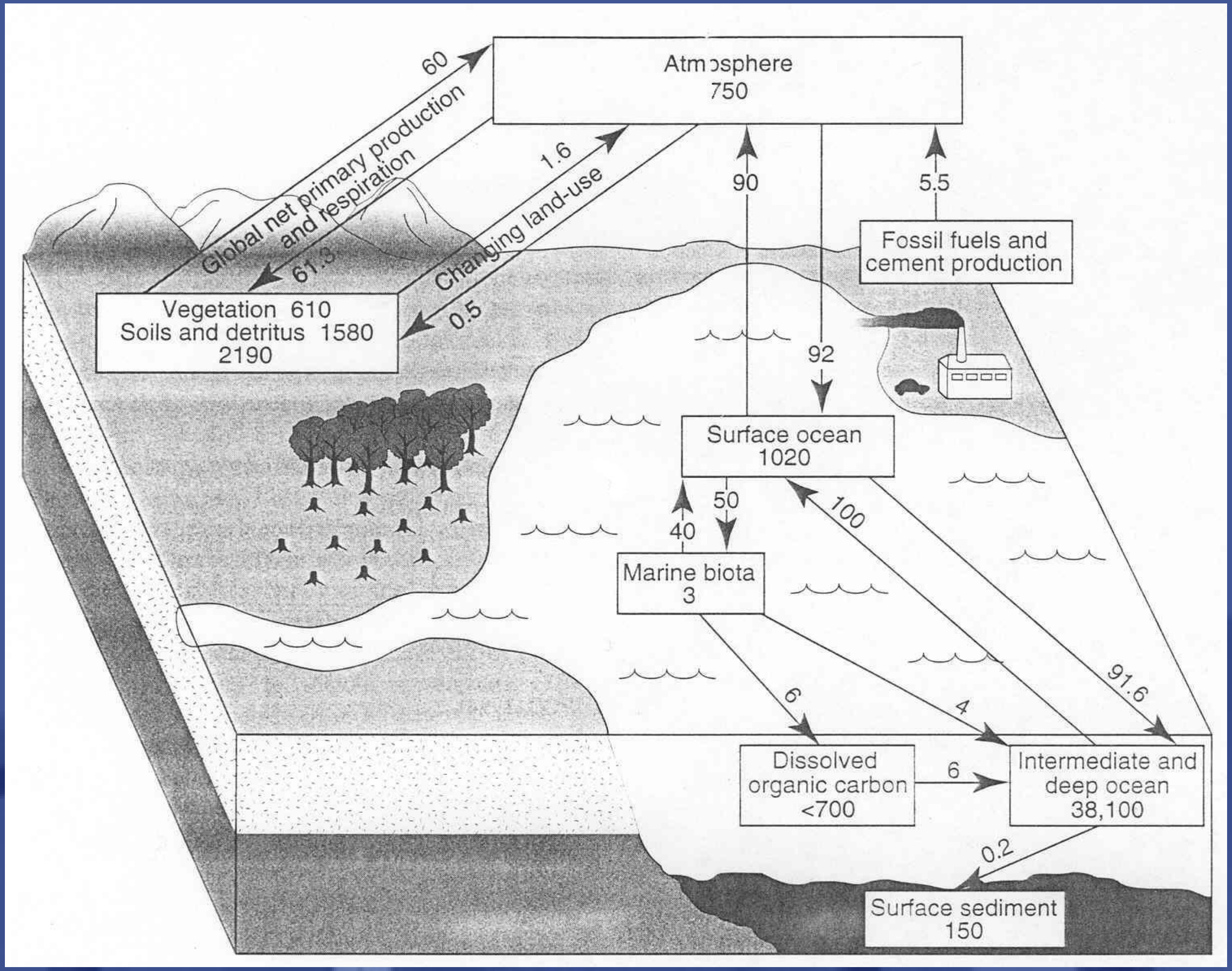


Scientific Questions

- Examine how atmospheric carbon concentrations have changed due to anthropogenic activities.

- Understand what and how the carbon budget affects the climate and thus the environment.

Schematic CO_2 Cycle

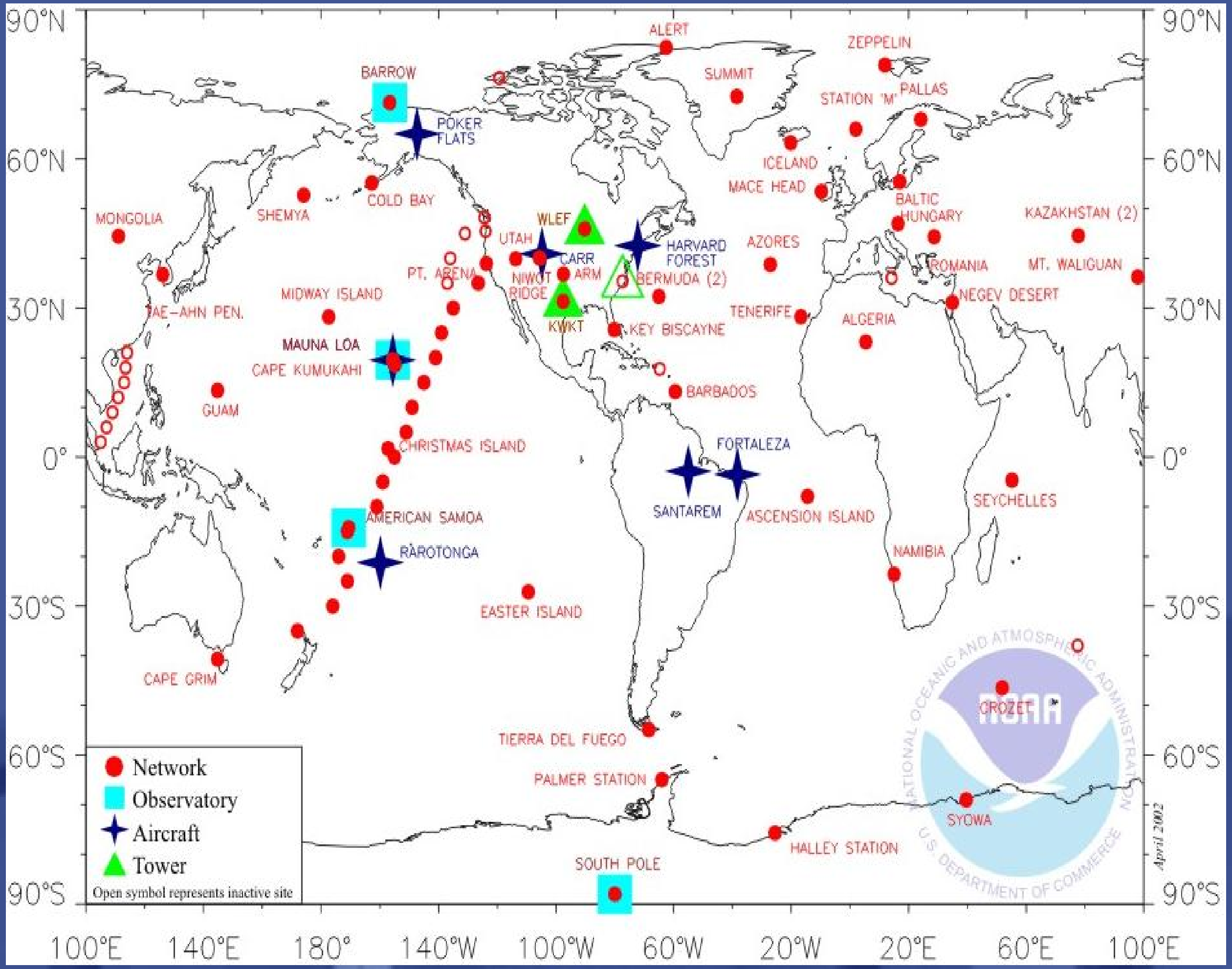


Observation network

Concentrations

Fluxes

Schematic CO_2 Cycle



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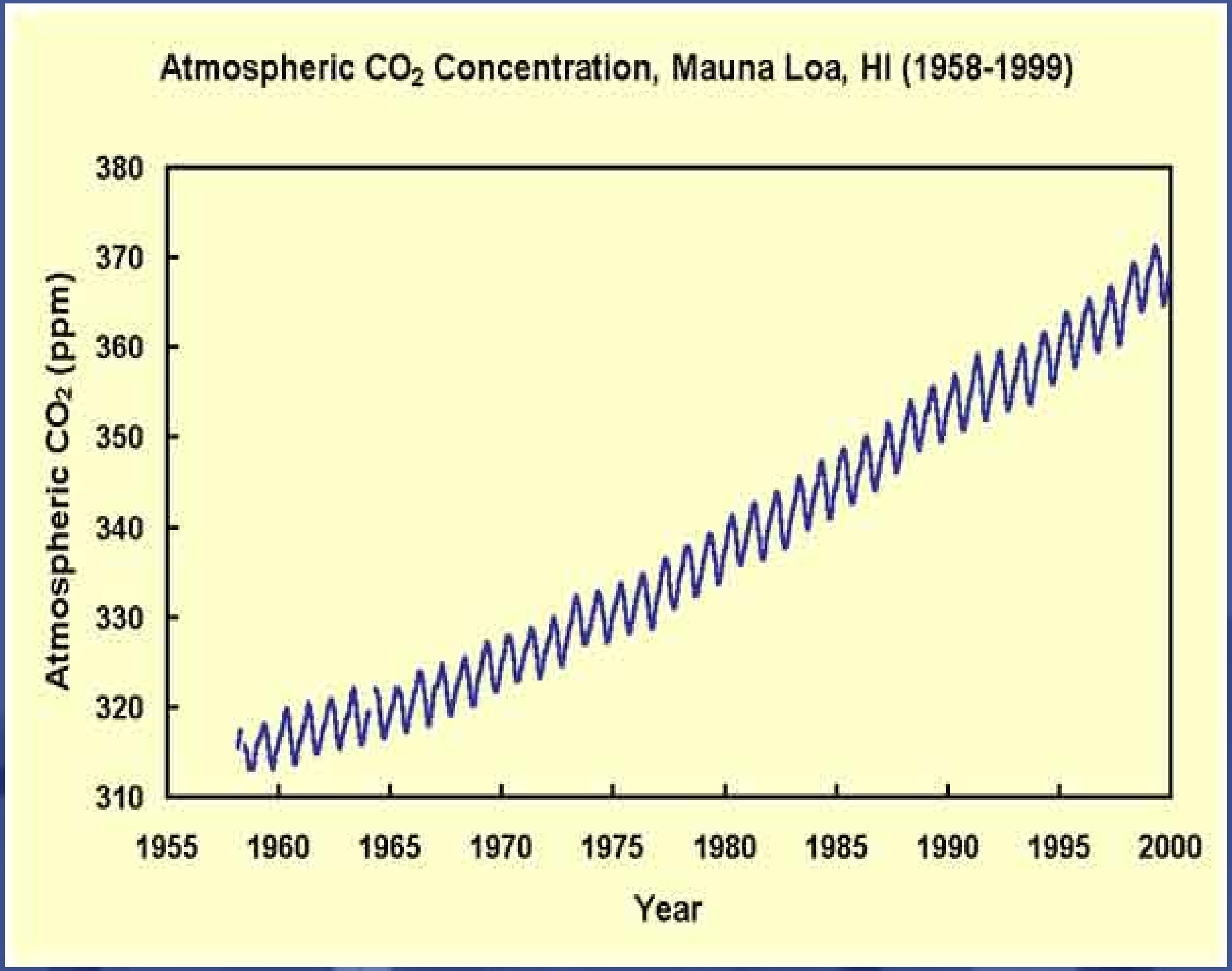

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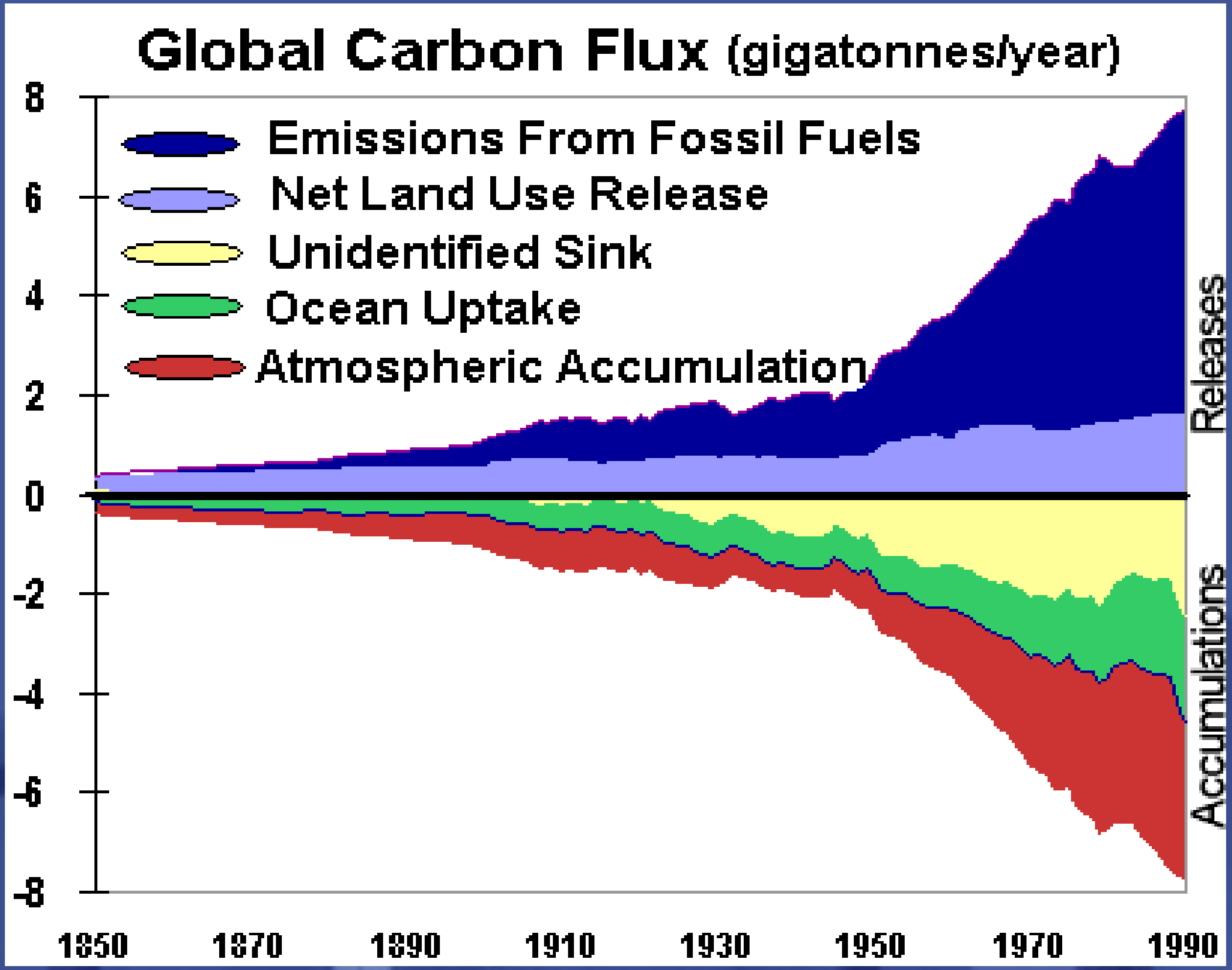

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Transport Model

A (parametrized chemistry) transport model numerically solves the constituent continuity equation over the entire globe.

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Constituent

- Carbons
- Ozone
- Aerosols
- Liquid water

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Continuity equation

$$\frac{\partial x}{\partial t} = -\mathbf{v}\nabla x + P(x) - L(x)$$

x: constituent

$\mathbf{v}$: velocity

$P(x), L(x)$: production and loss

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$\rightsquigarrow$ where do we get $\mathbf{v}$ from?

$\rightsquigarrow$ how to we solve the equation?

General Circulation Model

General circulation models are a numerical representation of the atmosphere and its phenomena over the entire Earth.

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Numerical representation

- Physical laws of flow
- Simplification
- Discretization

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- Conservation of momentum
- Conservation of mass
- Equation of state for ideal gases
- Conservation of energy
- Conservation equation for water mass

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$$\frac{d\mathbf{v}}{dt} = -\alpha\nabla p - \nabla\phi + \mathbf{F} - 2\Omega \times \mathbf{v}$$

$$\frac{\partial\rho}{\partial t} = -\nabla \cdot (\rho\mathbf{v})$$

$$p\alpha = RT$$

$$Q = C_p\frac{dT}{dt} - \alpha\frac{dp}{dt}$$

$$\frac{\partial\rho q}{\partial t} = -\nabla \cdot (\rho\mathbf{v}q) + \rho(E - C)$$

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Numerical representation

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  – Spherical coordinates
  – Traditional approximation
  – Hydrostatic model
  – Quasi-geostropic model
  – Barotropic model

General Circulation Model

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Numerical representation

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  - Leapfrog
  - Implicit (semi/fully)
  - Euler (forward/backward)

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- Surface (towers, ships)
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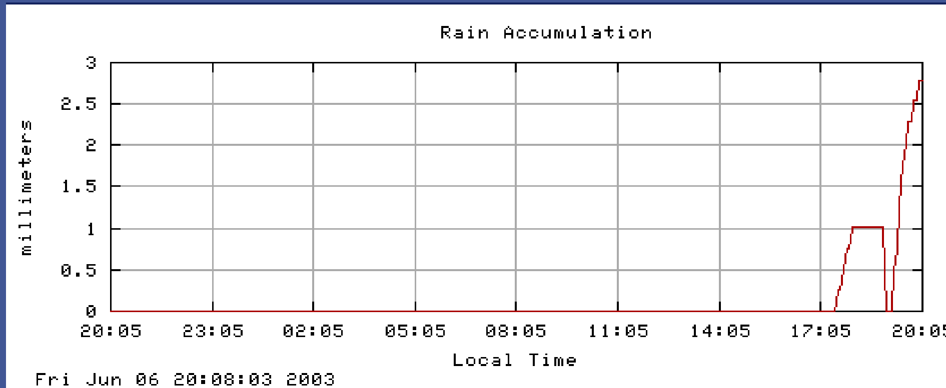

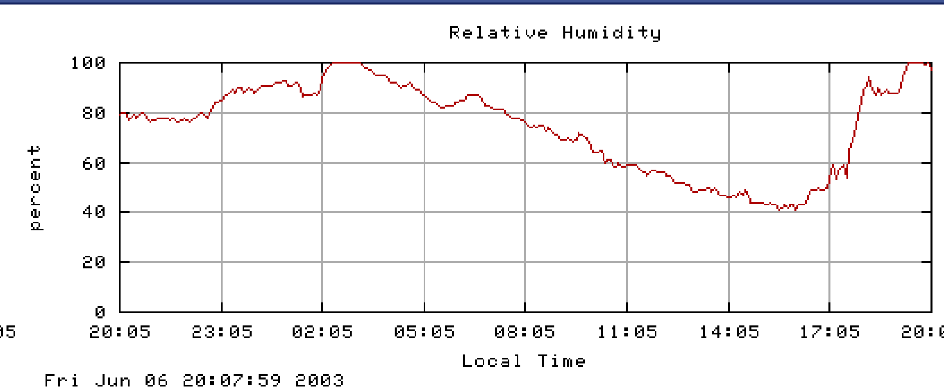

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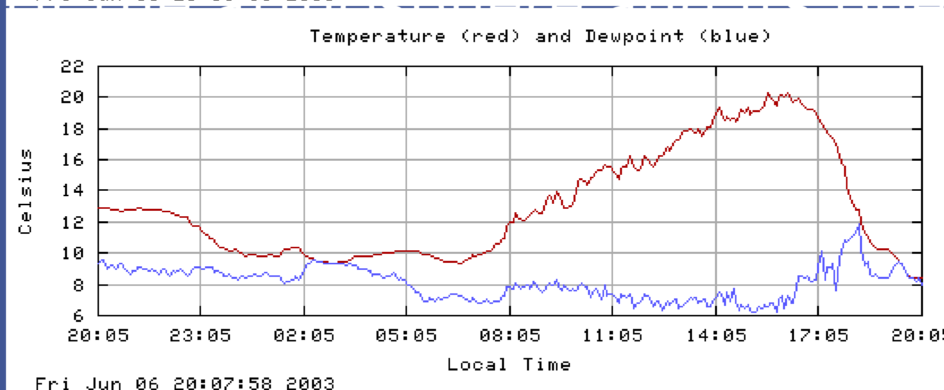

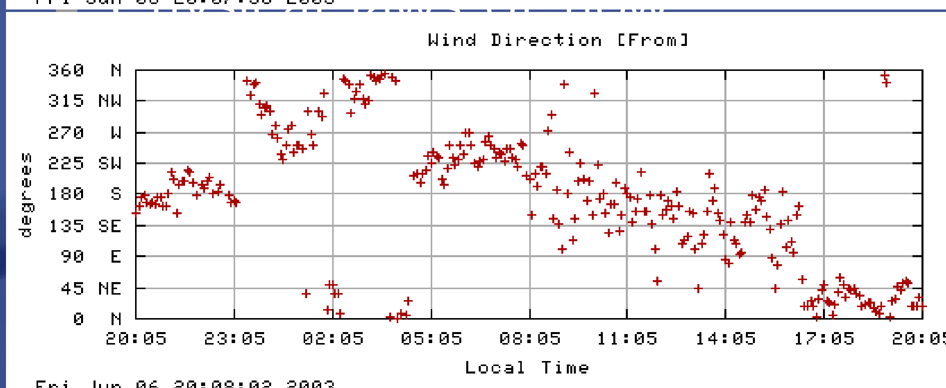

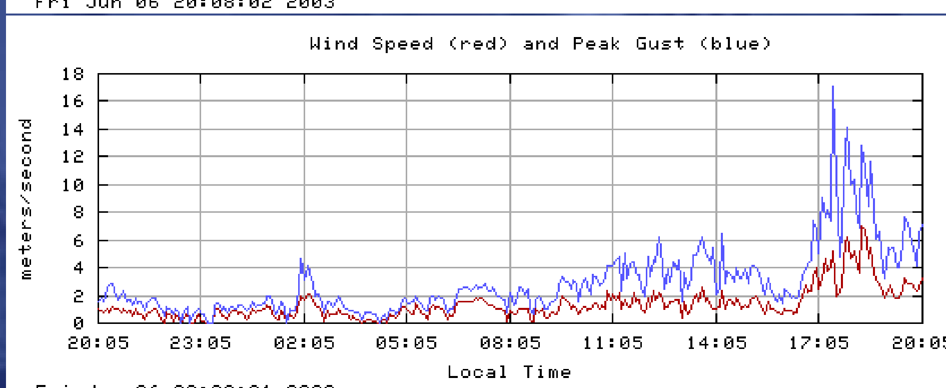

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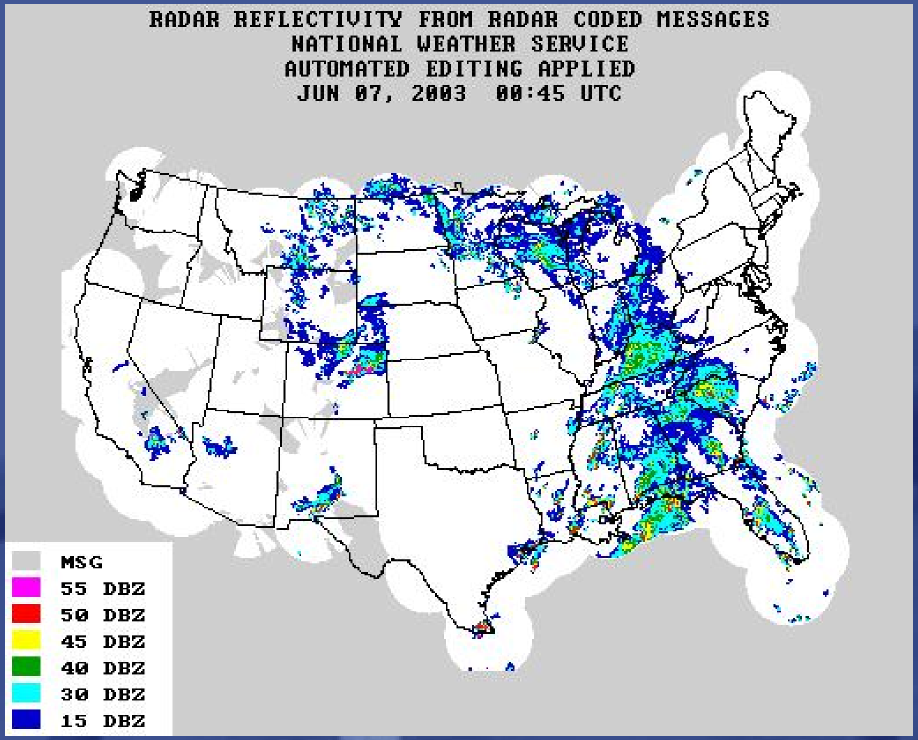

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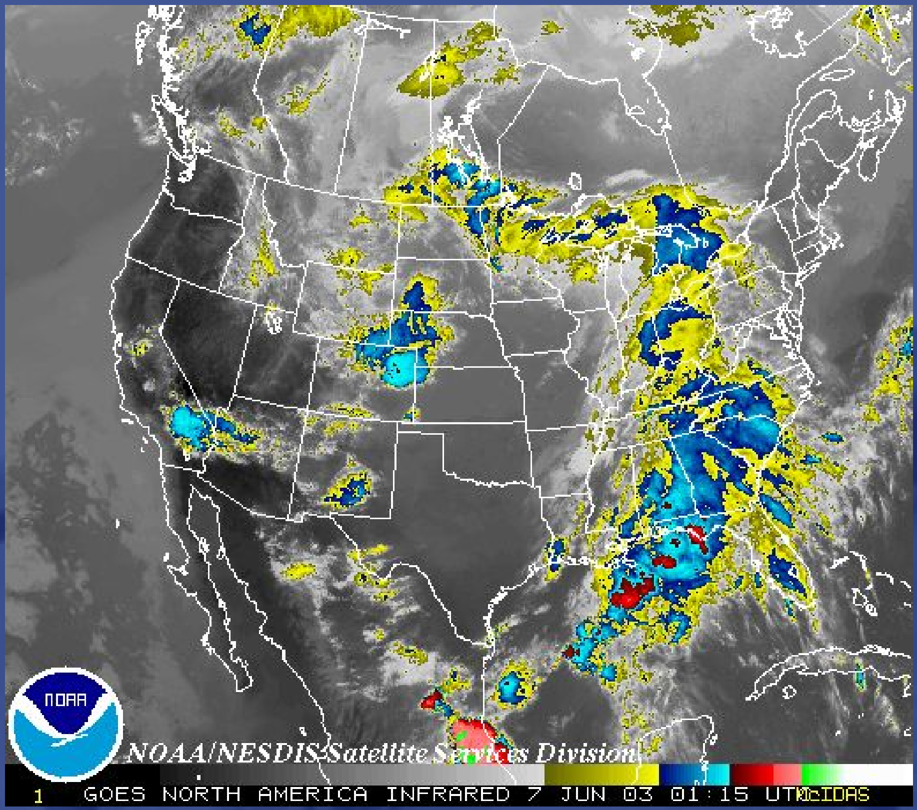

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Model output

Wind, humidity, temperature, pressure and specific density (at a "regular" grid).

Offline Transport Model

In a offline transport model, the dynamics of the model are supplied by archived meteorological datasets.

Dynamics = winds, temperatures, surface pressures
and cloud mass fluxes.

Modeling the CO_2 Cycle

We model the CO_2 cycle with:

$\mathbf{z}_t$ measured concentrations at a given network,

$\mathbf{x}_t$ actual concentrations (mixing ratios),

are dynamically constrained to $\mathbf{x}_{t-\delta}$ and fluxes,

$\mathbf{u}_t$ surface CO_2 fluxes,

represent the sources and sinks in the model.

> We solve for the unknown fluxes $\{\mathbf{u}_t\}$

Inverse Method 1 2

The CO_2 surface flux problem is formulated as an optimality problem (simplified vector notation)

$$\begin{aligned}
\min_{\{\mathbf{u}\}} \; & \sum_t (h(\mathbf{x}_t) - \mathbf{z}_t)^{\mathsf{T}} \mathbf{W}_1 (h(\mathbf{x}_t) - \mathbf{z}_t) \\
& + \sum_t (\mathbf{u}_t - \mathbf{u}_t^{\text{con}})^{\mathsf{T}} \mathbf{W}_2 (\mathbf{u}_t - \mathbf{u}_t^{\text{con}}) \\
& + (\mathbf{x}_0 - \mathbf{x}_0^{\text{con}})^{\mathsf{T}} \mathbf{W}_3 (\mathbf{x}_0 - \mathbf{x}_0^{\text{con}})
\end{aligned}$$

subject to dynamical constraints

where:

$h(\cdot)$ is the measurement function,
$\mathbf{W}_i$ are weight matrices,
superscript 'con' are constraints,
subscript '0' are initial conditions.

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Inverse Method 1 2

Run model forward:

with initial concentrations and forced by a priori fluxes
$\leadsto$ obtain modeled measurement history.

Run (adjoint) model backward:

forced by the weighted measurement differences
$\leadsto$ get the adjoint to the concentrations,
$\leadsto$ get new estimates for the fluxes and
initial concentrations.

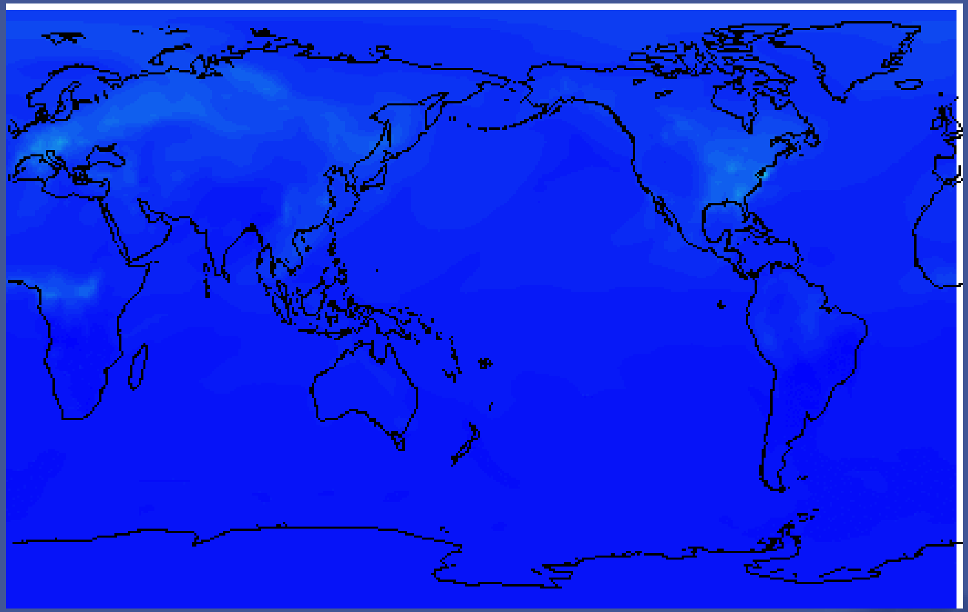
Repeat until convergence is achieved.

Statistical Work

Quantify the uncertainty of carbon sources and sinks on a regional scale.

- Acquaint with transport model
- First quantitative result of a sensitivity analysis
- Setup Gibbs sampler
- . . .

Sensitivity Analysis: Simulation Setup



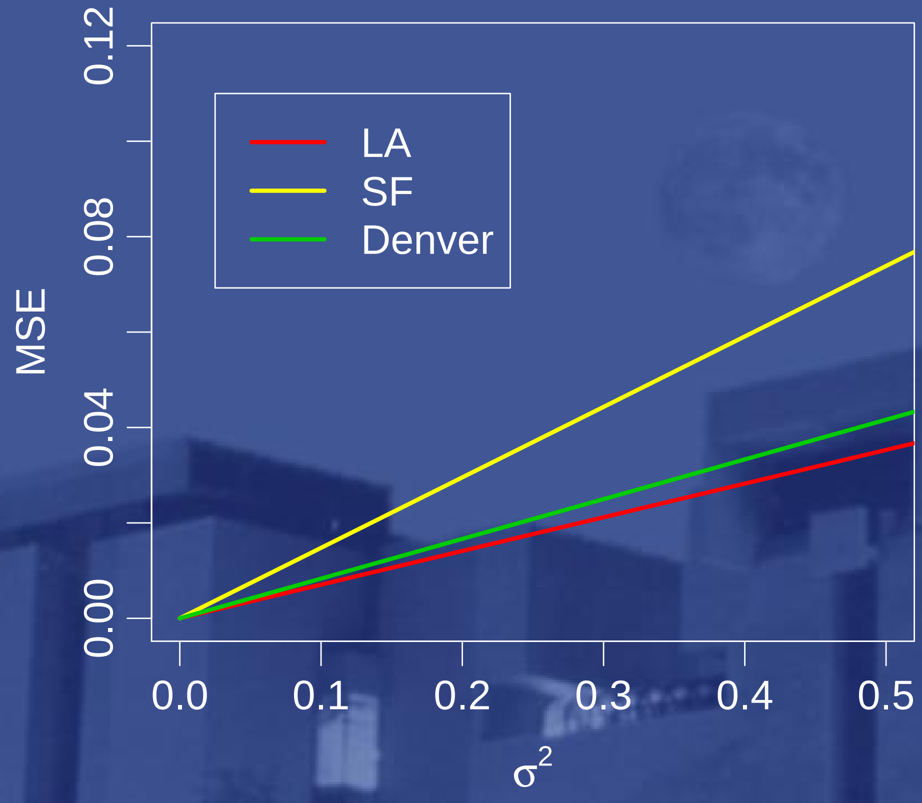
Transport model from NASA, Goddard Space Flight Center.

PCTM, Lin/Rood advection code w/ simple vertical mixing schemes.

- Eliminate sources and sinks
- Inject CO_2 “Dirac” point sources at LA, SF and Denver
- Run model forward with measurement and/or transport errors
- Try to recover the intensity with regression techniques after 4 days

Sensitivity Analysis: Results

Measurement error



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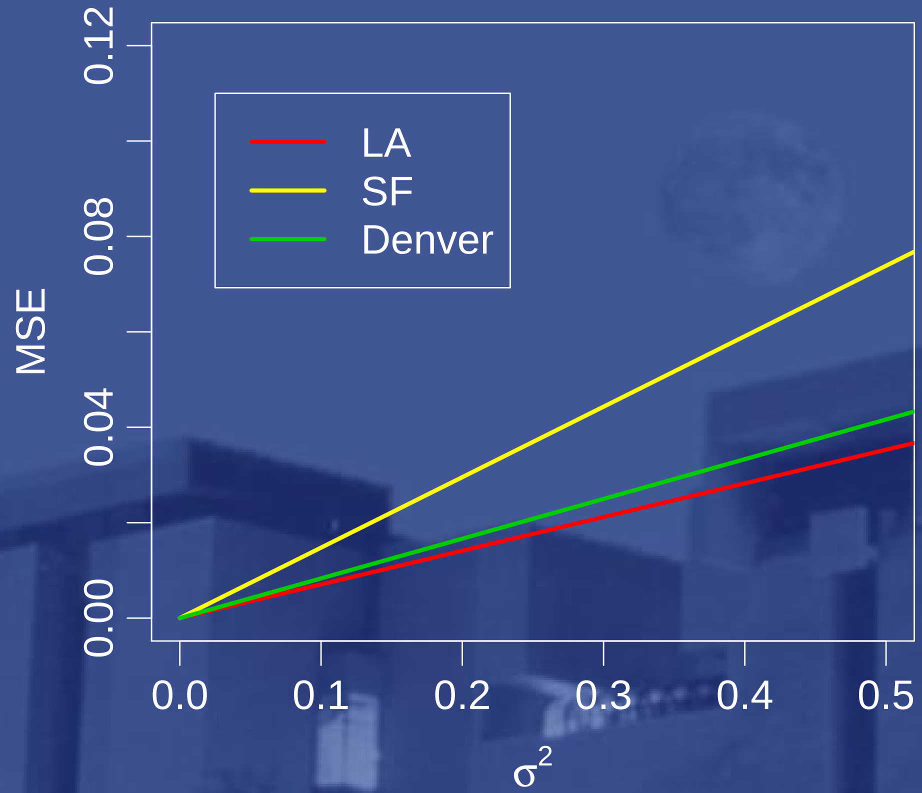
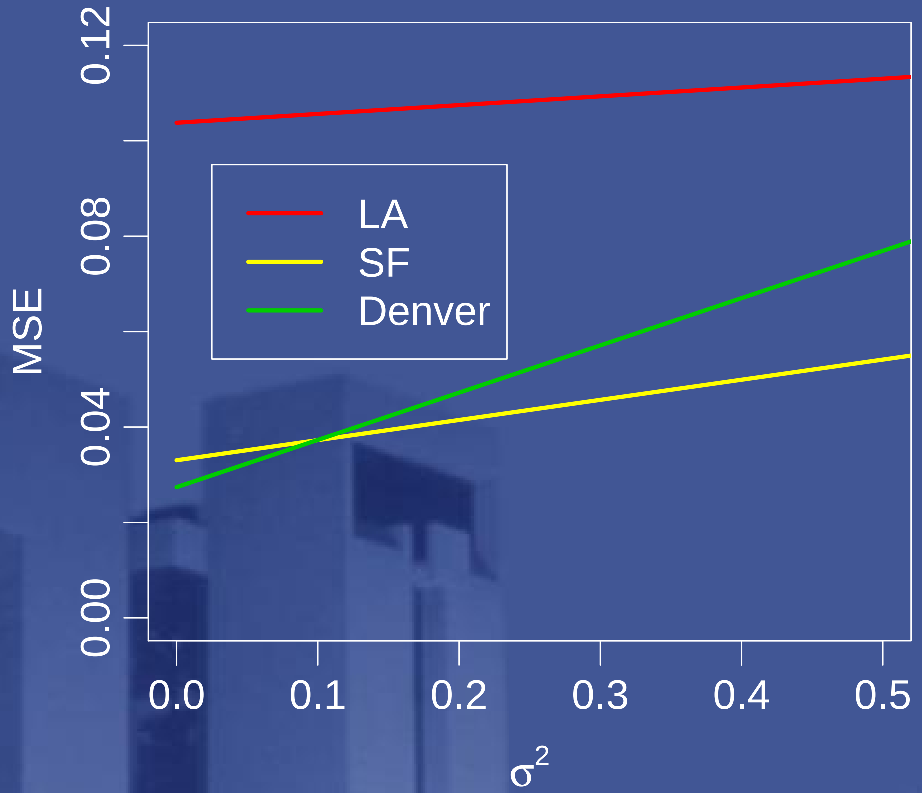


Measurement error
and wind perturbation



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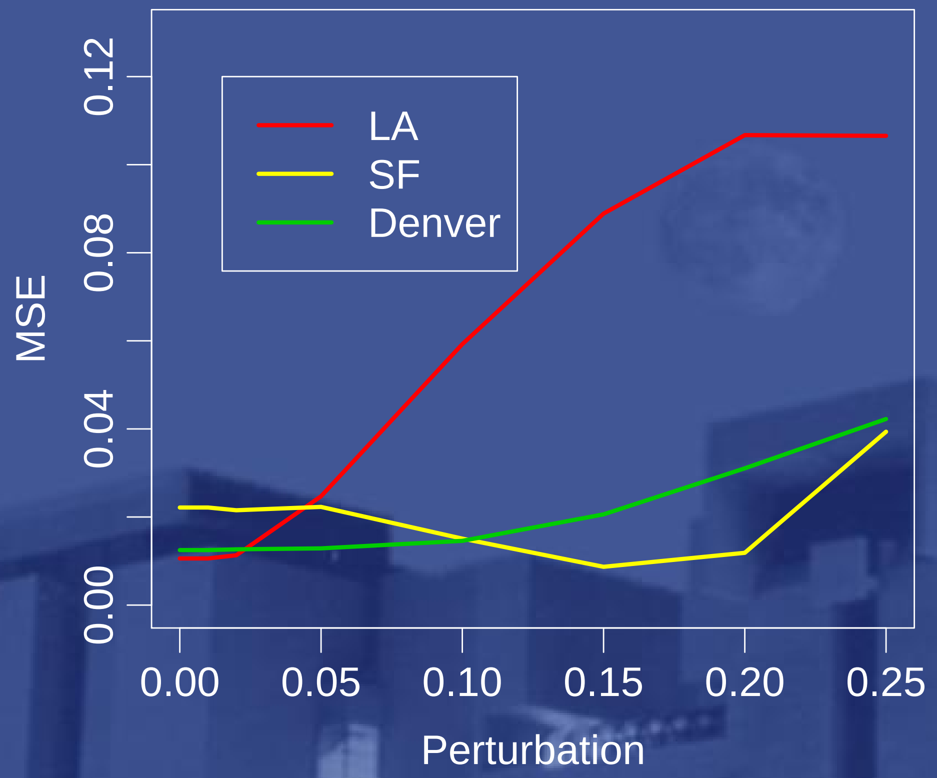

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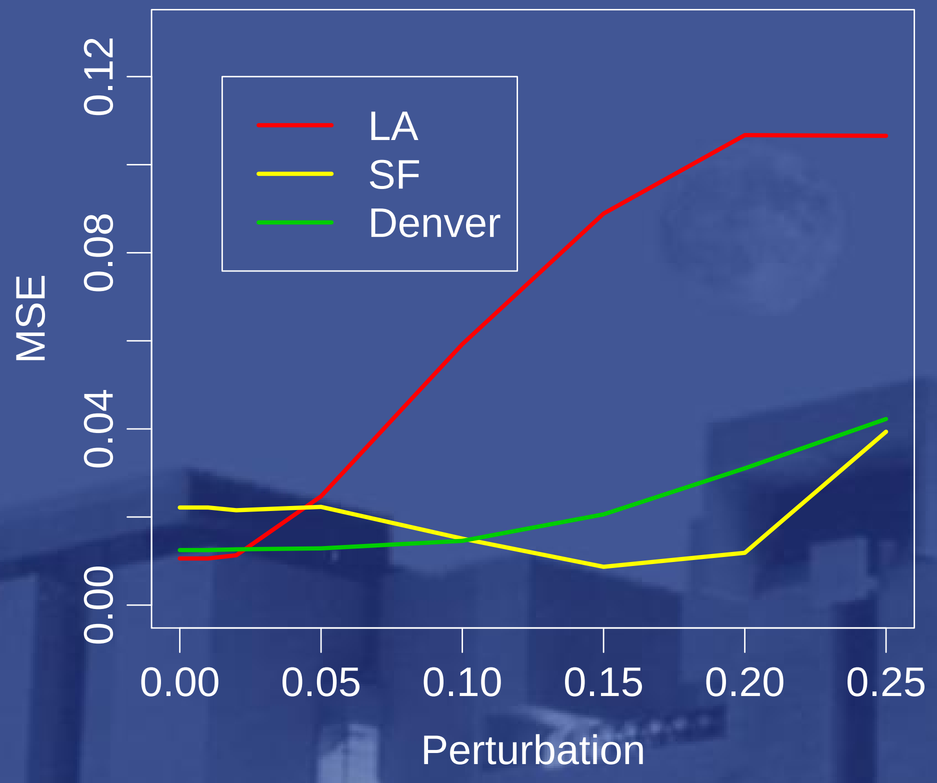

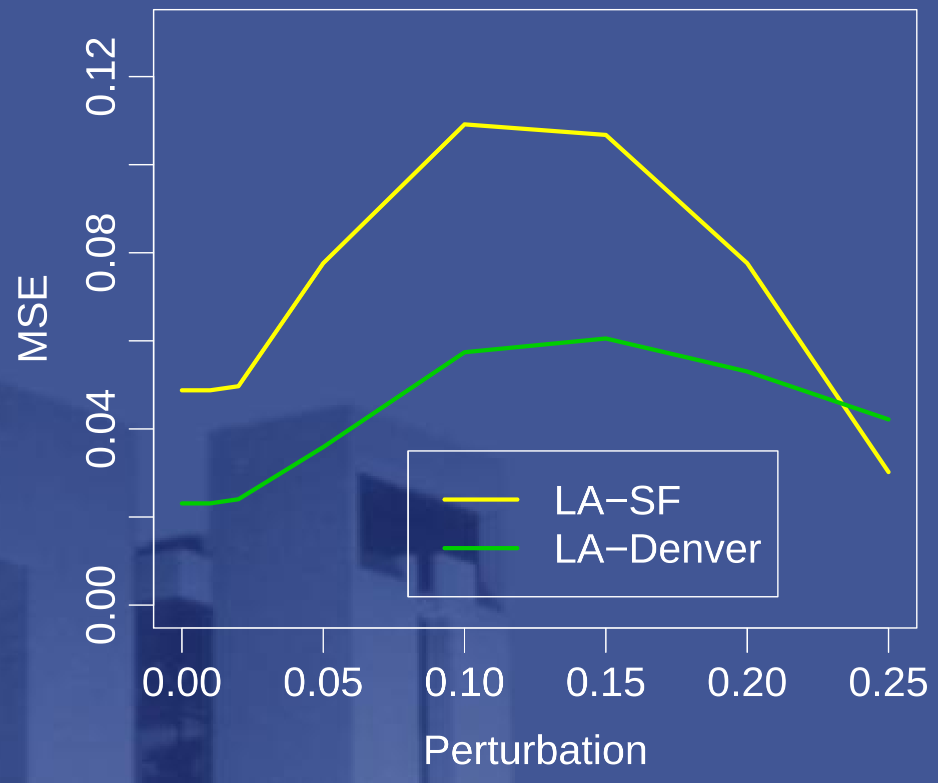

Sensitivity Analysis

Regression model

$$\mathbf{y} = \mathbf{K}\beta + \varepsilon \qquad\qquad \mathbf{y} = \mathbf{K}_{\text{pert}}\beta_{\text{pert}} + \varepsilon$$

where

- $\mathbf{y}$ concentration anomaly after 4 days
- $\mathbf{K}$, $\mathbf{K}_{\text{pert}}$ true and perturbed transport operator
- ε measerment error

The mean squared error is

$$\text{MSE} = \sigma^2(\mathbf{K}^{\mathsf{T}}\mathbf{K})^{-1}$$

$$\text{MSE} = \sigma^2(\mathbf{K}_{\text{pert}}^{\mathsf{T}}\mathbf{K}_{\text{pert}})^{-1} + (\mathbf{M} - \mathbf{I})\beta\beta^{\mathsf{T}}(\mathbf{I} - \mathbf{M}^{\mathsf{T}})$$

with $\mathbf{M} = (\mathbf{K}_{\text{pert}}^{\mathsf{T}}\mathbf{K}_{\text{pert}})^{-1}\mathbf{K}_{\text{pert}}^{\mathsf{T}}\mathbf{K}$.