Inverse problems for sources and sinks of CO_2: Sensitivity analysis of the transport model

Inverse problems for the carbon cycle

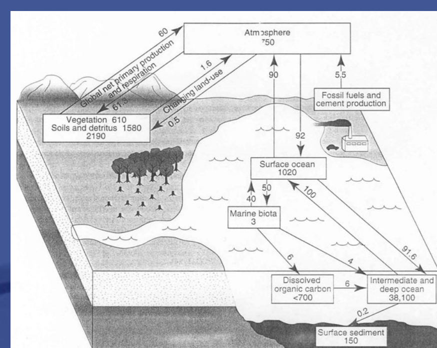
- Examine how atmospheric carbon dioxide concentrations have changed due to anthropogenic activities.
- Understand what and how the carbon budget affects the environment.
- Identify the 'missing sink' for carbon dioxide.

In collaboration with D. Baker and P. Hess.

Outline of the Talk

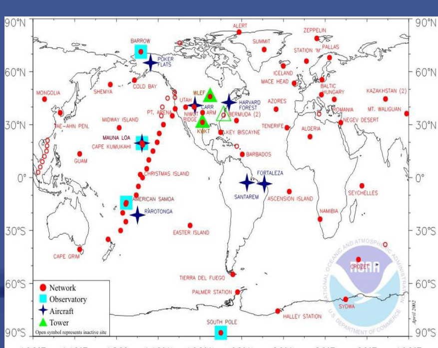
1. Illustrate the carbon cycle
2. Sketch existing estimation techniques
3. Motivate a statistical approach
4. Discuss a first sensitivity analysis
5. Outline further research

Schematic CO_2 Cycle



Observation network
Concentrations
Fluxes

Schematic CO_2 Cycle



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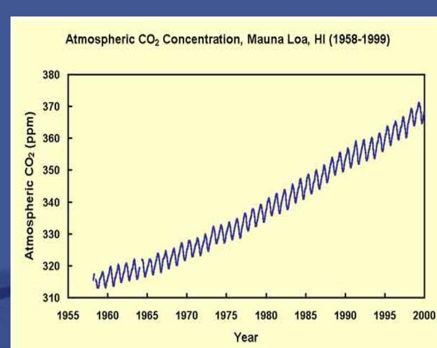

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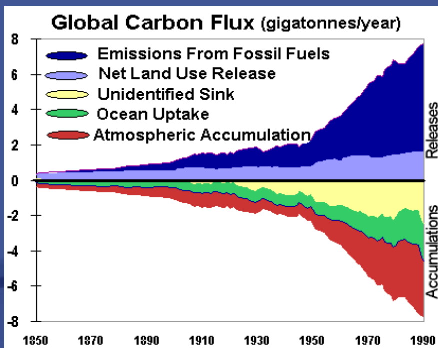

Observation network

Concentrations

Fluxes

Modeling the CO_2 Cycle

We model the CO_2 cycle with:

$\mathbf{z}_t$ measured concentrations at a given network,

$\mathbf{x}_t$ actual concentrations (mixing ratios),
are dynamically constrained to $\mathbf{x}_{t-\delta}$ and fluxes,

$\mathbf{u}_t$ surface CO_2 fluxes,
represent the sources and sinks in the model.

We solve for the unknown fluxes $\{\mathbf{u}_t\}$

Inverse Method 1 2

The CO_2 surface flux problem is formulated as an optimality problem (simplified vector notation)

$$\min_{\{\mathbf{u}\}} \sum_t (h(\mathbf{x}_t) - \mathbf{z}_t)^\mathsf{T} \mathbf{W}_1 (h(\mathbf{x}_t) - \mathbf{z}_t)$$
$$+ \sum_t (\mathbf{u}_t - \mathbf{u}_t^{\text{con}})^\mathsf{T} \mathbf{W}_2 (\mathbf{u}_t - \mathbf{u}_t^{\text{con}})$$
$$+ (\mathbf{x}_0 - \mathbf{x}_0^{\text{con}})^\mathsf{T} \mathbf{W}_3 (\mathbf{x}_0 - \mathbf{x}_0^{\text{con}})$$

subject to dynamical constraints

where:

$h(\cdot)$ is the measurement function,
$\mathbf{W}_i$ are weight matrices,
superscript 'con' are constraints,
subscript '0' are initial conditions.

Inverse Method 1 2

Run model forward:

with initial concentrations and forced by a priori fluxes
⇝ obtain modeled measurement history.

Run (adjoint) model backward:

forced by the weighted measurement differences
⇝ get the adjoint to the concentrations,
⇝ get new estimates for the fluxes and initial concentrations.

Repeat until convergence is achieved.

Statistical Approach to CO_2 Cycle

Questions of Analysis:

- How does a observation network influence the space-time resolution of the uncertainty?
- Can we improve the knowledge of the state with inclusion of spatial covariates?
- ...

State-Space Decomposition 1 2

We denote

$Z(\mathbf{s}_i, t)$ observed CO_2 concentrations at time t at location $\mathbf{s}_i$,
(obtained with different measurement techniques),

$X(\mathbf{s}, t)$ actual concentrations (mixing ratios),
are dynamically constrained to $X(\mathbf{s}, t-\delta)$ and fluxes,

$U(\mathbf{s}, t)$ surface CO_2 fluxes,
represent the sources and sinks in the model.

We solve for the unknown fluxes $U(\cdot, \cdot)$

State-Space Decomposition 1 2

We decompose the observed mixing ratios $Z(\cdot,\cdot)$ as

$$Z(\mathbf{s}_i,t) = h(X(\mathbf{s},t),\mathbf{s}_i,t) + \varepsilon(\mathbf{s}_i,t)$$

$$X(\mathbf{s},t+\delta) = \sum_{\mathbf{u}\in\mathcal{U}_\mathbf{s}} K(\mathbf{u},\mathbf{s},t)X(\mathbf{u},t) + U(\mathbf{s},t)$$

$$U(\mathbf{s},t) = \sum_k \beta_k v_k(\mathbf{s},t) + \kappa(\mathbf{s},t)U(\mathbf{s},t-\delta) + e(\mathbf{s},t)$$

⇝ Space equation:
Linking observations with concentrations

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⇝ State equation:
Linking sources and sinks with concentrations

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where
$K(\cdot,\cdot,\cdot)$ is the (random) transport function
$\mathcal{U}_\mathbf{s}$ contains $\mathbf{s}$ and its first order neighbors

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where
$v_k(\cdot,\cdot)$ are covariates
$\kappa(\cdot,t)U(\cdot,t-\delta)$ is an autoregressive element
$e(\cdot,\cdot)$ is a Gaussian spatial process

Markov Chain Monte Carlo

The concentrations and fluxes can be simulated with a Gibbs sampler.

Write $\mathbf{U} = \big(U(\mathbf{s}_i,t_j)\big)$, $\mathbf{X} = \big(X(\mathbf{s}_i,t_j)\big)$ and
$\mathbf{X}_k$, $\mathbf{U}_k$ the respective kth column.

Then
$\mathbf{U}_k \big| \mathbf{U}_{k-1}$ is Gaussian,

$\mathbf{X}_k \big| \{\mathbf{X}_{j\neq k}, \mathbf{U}_k\}$ is Gaussian.

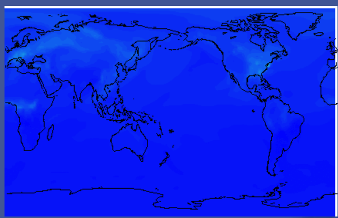
Apply a Gibbs sampler to get a sequence $\mathbf{X}_1,\ldots,\mathbf{X}_T$.

Sensitivity Analysis: Goal

Acquaint with

- transport model
- model input/output
- first quantitative result

Sensitivity Analysis: Simulation Setup



Transport model from NASA, Goddard Space Flight Center.

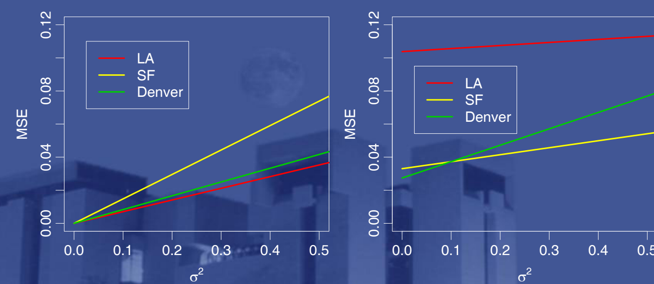
PCTM, Lin/Rood advection code w/ simple vertical mixing schemes.

- Eliminate sources and sinks
- Inject CO_2 "Dirac" point sources at LA, SF and Denver
- Run model forward with measurement and/or transport errors
- Try to recover the intensity with regression techniques after 4 days

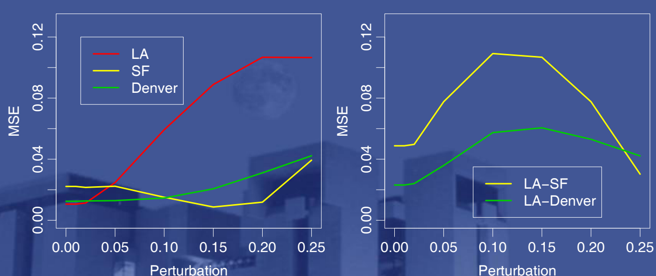
Sensitivity Analysis: Results

Measurement error

Measurement error and wind perturbation



Sensitivity Analysis: Results



Work in Progress

- Change to NCAR model (Mozart)
- Incorporate adjoint step
- Setup Gibbs sampler
- Answer scientific questions, for example:

  Quantify the uncertainty of sources and sinks on a regional scale.