Numerik 1 Übungstunde, Gruppe 1 (Donnerstag)

Woche 10

Übungstunden werden registriert. Bitte informieren Sie den Assistenten, falls Sie nicht in der Aufzeichnung erscheinen möchten.

Streaming (öffentlich) und Aufzeichnungen (Anmeldung nötig):

https://seminarlive.mnf.uzh.ch/semlive/index.php?id=event_detail&module=mat801&semester=fs23&group=1

Notizen (diese Datei) und Python scripts:

https://user.math.uzh.ch/florian/stable/page/2023Mat801/notes.php

Horner:

Input p (polynomial) -> coefficients in the monomial basis

x (point in $\mathbb{R}$ (usually $[-1,1]$))

Output $p(x)$

Input: $(a_i)_{i=0}^{\deg p}$ $\qquad p(x) = \sum_{i=0}^{\deg p} a_i x^i$

Ex $\quad p(x) = 1 + 2x + x^2 + 3x^3$

$= 1 + x(2 + x(1 + 3(x)))$

evaluation $= a_{\deg p}$ $\qquad (=3)$

For idx in $\deg p - 1$ down to 0

evaluation $*= x$ $\qquad (3x) \qquad ((3x+1)x)$

evaluation $+= a_{ind}$ $\qquad (3x+1) \qquad ((3x+1)x+2)$

return evaluation

division: $\quad \dfrac{p(x)}{x-\alpha} = q(x)\ (+ \boxed{r(x)}^{=0})$

$p(x) = \sum_{0}^{n+1} a_i x^i$

$q(x) = \sum_{i=0}^{n} b_i x^i$

$p(x) = q(x)\ (x - \alpha)$

$$\sum_{i=0}^{n+1} a_i x^i = \sum_{i=0}^{n} b_i (x-\alpha) x^i$$

$$= \sum_{i=0}^{n-1} \left(b_i x^{i+1} - \alpha b_{i+1} x^{i+1} \right) + \underline{b_n x^{n+1}} + \underline{\alpha b_0}$$

n equations for n coefficients

$$\begin{cases} a_{n+1} = \boxed{b_n} \\ a_{j+1} = \boxed{b_j} - \alpha \boxed{b_{j+1}} \end{cases} \qquad 0 \le j \le n-1$$

unknowns! → previous step

$\boxed{a_0} = \boxed{\alpha}\boxed{b_0}$ → need to fit this one as well!

given just computed

is it true? Yes!

$r(x) = a_0 - \alpha b_0 \overset{\text{assumption}}{=} 0$

Programming

- we deal with polynomials, what is a good way to store them in the computer?
- We choose for this exercise the monomial basis, so $p(x) = \sum_{i=0}^{n} a_i x^i$ is represented as $(a_i)_{i=0}^{n=\deg p}$
- stored as a list [a_i for i in range (deg p + 1)]

$a_{\deg p - i}$