# Algorithmique de base

Master 1, Université de Rennes

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**Exercise 1.**

1. Use a recursion tree to determine a good asymptotic upper bound on the recurrence $C(n) = C(\frac{n}{2}) + n^2$. Use the substitution method to verify your answer.
2. Use a recursion tree to determine a good asymptotic upper bound on the recurrence $C(n) = 2C(n-1) + 1$. Use the substitution method to verify your answer.
3. Use a recursion tree to determine a good asymptotic upper bound on the recurrence $C(n) = 2C(\lfloor \frac{n}{2} \rfloor) + n$. Use the substitution method to verify your answer.
4. Use a recursion tree to determine a good asymptotic upper bound on the recurrence $C(n) = 4C(\frac{n}{2} + 2) + n$. Use the substitution method to verify your answer.
5. Use a recursion tree to determine a good asymptotic upper bound on the recurrence $C(n) = C(n-1) + C(\frac{n}{2}) + n$. Use the substitution method to verify your answer.

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**Exercise 2.** Let $G = (V, E)$ be a directed graph with $n$ vertices and $m$ edges.

1. Show that $\sum_{v \in V} \deg^-(v) = \sum_{v \in V} \deg^+(v)$.
2. Show that $\sum_{v \in V} \deg^-(v) = m$.
3. What can we say if $G$ is undirected without loops?
4. Is there a simple graph with 5 vertices with the following degrees?

   (a) $3, 3, 3, 3, 4$

   (b) $1, 2, 3, 4, 5$

   If so, draw it.

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**Exercise 3.** Let $G$ be an undirected connected graph. An Eulerian circuit of $G$ is a circuit which passes once and only once through each edge of $G$. We say that $G$ is an Eulerian graph if it admits an Eulerian circuit.

1. Show that if $G$ is Eulerian, then for every vertex $x$ of $G$, $\deg(x)$ is even.
2. Show the converse.

**Exercise 4.** 1. Determine the undirected graph with adjacency matrix

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

2. Is this graph connected? Is it Eulerian? Is it Hamiltonian?
3. Compute the vector of distances to vertex 1 as well as the vector of predecessors using the algorithm of Dijkstra.

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**Exercise 5.** Show that an undirected graph of which all vertices have degrees greater than or equal to 2 has one cycle.

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**Exercise 6.** For a graph $G = (V, E)$ on $n$ vertices, the following statements are equivalent, and they all characterize a tree on $n$ vertices.

1. $G$ is connected and has no cycles.
2. $G$ is connected and has $n - 1$ edges.
3. $G$ has $n - 1$ edges and no cycles.
4. For all $u, v \in V$ there is a unique path between $u$ and $v$.

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