What is going on in the on ramp call?

Violetta Weger

Young Cryptographers in Genova 2024

November 28, 2024

Post-quantum Cryptography

Asymmetric

Public-key

Post-quantum Cryptography

Asymmetric

- RSA signature, encryption
- DH, DSA
- ECDH, ECDSA

Public-key

→ Integer factorization

→ Discrete logarithm over $\mathbb{F}_p$

→ Discrete logarithm over ell. curves

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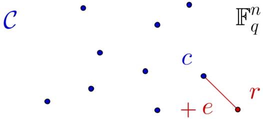

- $\mathcal{C} = \langle G \rangle \subseteq \mathbb{F}_q^n$ linear subspace
- Decode: $r = mG + e$ find closest $c = mG$
- $\mathrm{wt}_H(e) = |\{i : e_i \neq 0\}|$

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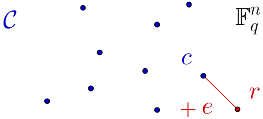

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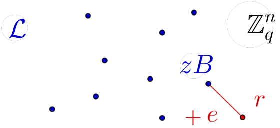
Lattice-based



- $\mathcal{L} = \{\sum z_i b_i \mid z_i \in \mathbb{Z}\} = \langle B \rangle \subseteq \mathbb{Z}_q^n$
- SVP: $r = zB + e$ find closest zB
- $||e||_2 = \sqrt{\sum e_i^2}, ||e||_\infty = \max\{|e_i|\}$

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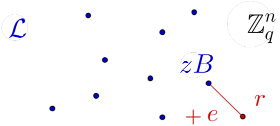

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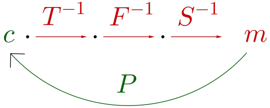
Multivariate



- $P = (p_1, \dots, p_m) \in \mathbb{F}_q[x_1, \dots, x_n]$
- Given $P(m) = c$ find m
- $P = S \circ F \circ T$, F quadr., S, T affine

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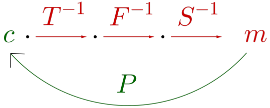

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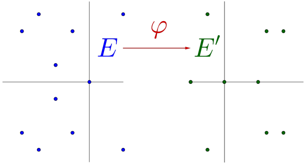
Isogeny-based



- E, E' ell. curves over $\mathbb{F}_q$
- find isogeny $\varphi : E \to E'$

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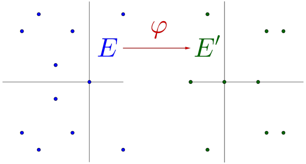

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Post-quantum crypto

- Code-based
- Multivariate
- Hash-based
- Lattice-based
- Isogeny-based

Timeline

2016	NIST standardization call	for post-quantum PKE/KEM and signatures

Timeline

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	Standardized KEM:	KYBER
	4th round:	BIKE, Classic McEliece, HQC
2022	Standardized signatures:	DILITHIUM, FALCON, SPHINCS+

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necessary: EUF-CMA, attackers $\geq 2^{64}$ signatures, security levels $\sim$ breaking AES

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On ramp announcement — only signatures

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Example: Level 1: AES-128: 2^{157} quantum / 2^{143} classical gates

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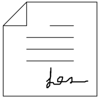
nice to haves: side-channel resistant, BUFF, multi-key attacks, well-understood math

Idea of Signature Schemes

Signer

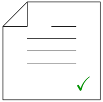


- Key Generation: $\mathcal{P}$ public, $\mathcal{S}$ secret
- Signing: use $\mathcal{S}$ and message m to generate signature σ

$\mathcal{P}$ →

m, σ →

Verifier



- Verification: use $\mathcal{P}$ and message m to verify signature σ

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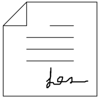

- Key Generation: $\mathcal{P}$ public, $\mathcal{S}$ secret
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EUF-CMA

small $\mathcal{P}$

small σ

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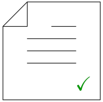

fast verification

- Verification: use $\mathcal{P}$ and message m to verify signature σ

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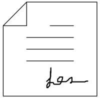

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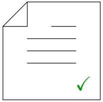

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Approaches for signatures:

- Hash-and-Sign
- ZK Protocol
- ZK + MPC

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	On ramp announcement	
2023	1st round candidates:	40 submissions

1st round Candidates

Code-based: 6

- CROSS
- Enhan. pqsigRM
- FuLeeca
- LESS
- MEDS
- Wave

Lattice-based: 7

- EagleSign
- EHT
- HAETAE
- Hawk
- HuFu
- Raccoon
- Squirrels

MPCitH: 7

- Biscuit
- MIRA
- MiRitH
- MQOM
- PERK
- RYDE
- SDitH

Other: 5

- ALTEQ
- eMLE-Sig
- KAZ-SIGN
- Preon
- Xifrat1-Sign.I

Isogeny: 1

- SQISign

Multivariate: 10

- 3wise
- DME-Sign
- HPPC
- MAYO
- PROV
- QRUOV
- SNOVA
- TUOV
- UOV
- VOX

Symmetric: 4

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- Ascon-Sign
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- RYDE
- SDitH
- Wave

Other: 1

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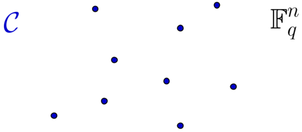
Basics



- Code $\mathcal{C} \subseteq \mathbb{F}_q^n$ linear subspace
- G generator matrix $\quad \rightarrow \quad c = mG$

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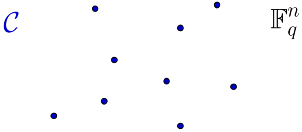

- Code $\mathcal{C} \subseteq \mathbb{F}_q^n$ linear subspace
- H parity-check matrix $\rightarrow$ $cH^\top = 0$

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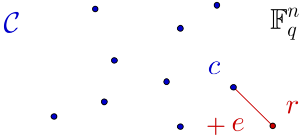

- Code $\mathcal{C} \subseteq \mathbb{F}_q^n$ linear subspace
- H parity-check matrix $\rightarrow$ $rH^\top = eH^\top = s$
- Hamming weight: $\mathrm{wt}_H(e) = |\{i \mid e_i \neq 0\}|$

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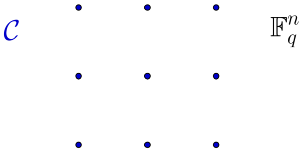

- algebraic structure
- e.g. RS, Goppa codes

$\rightarrow$ efficient decoders

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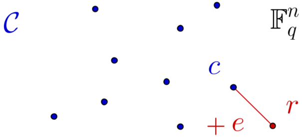


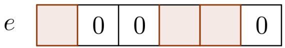
- random code
- decoding is NP-hard

→ Information set decoding

Syndrome Decoding Problem (SDP)

Given H, s, weight t, find e s.t.

1. $s = eH^\top$
2. $\mathrm{wt}_H(e) = t$



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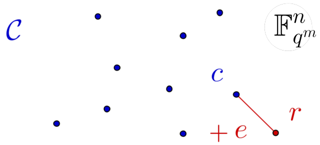

- Code $\mathcal{C} \subseteq \mathbb{F}_{q^m}^n$ linear subspace
- H parity-check matrix $\rightarrow$ $rH^\top = eH^\top = s$
- Rank weight: $\mathrm{wt}_R(e) = \dim(\langle e_1, \dots, e_n \rangle_{\mathbb{F}_q})$

Rank SDP

Given H, s, weight t, find e s.t.

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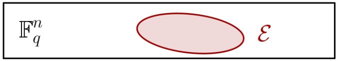
$$\mathrm{wt}_R(e) = \dim_{\mathbb{F}_q}(\mathcal{E})$$



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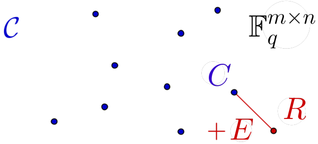


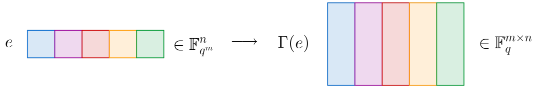
- Code $\mathcal{C} \subseteq \mathbb{F}_q^{m\times n}$ linear subspace
- $G_1, \dots, G_k \quad \rightarrow \quad C = \sum \lambda_i G_i$,
- Rank weight: $\mathrm{wt}_R(E) = \mathrm{rk}(E)$

MinRank

Given $\mathcal{C} \subseteq \mathbb{F}_q^{m\times n}$, R, t, find E s.t.

1. $R - E \in \mathcal{C}$
2. $\mathrm{rk}(E) = t$

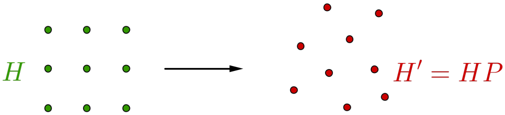
Γ basis of $\mathbb{F}_{q^m}/\mathbb{F}_q$: $\mathrm{wt}_R(e) = \mathrm{rk}(\Gamma(e))$ basis



Classical Approach: Hash and Sign

structured code

efficient decoding

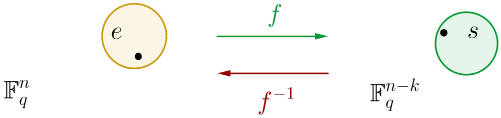


random code

hard to decode

Idea McEliece: use Goppa code as secret code

trapdoor



$\mathrm{wt}_H(e) = t$

$s = HPe^\top$

encryption

messages

ciphertexts

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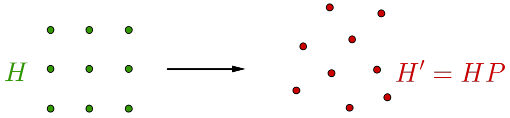

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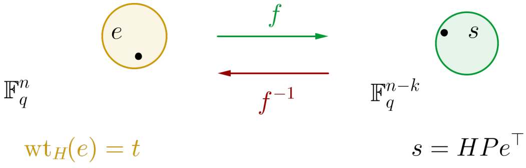

signature

signatures

messages

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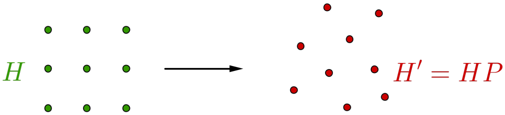

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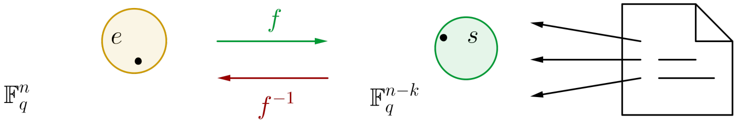

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Hash

signature

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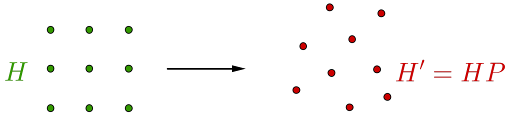

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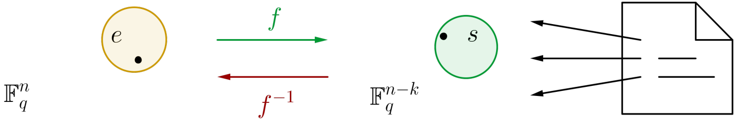

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Hash

signature

signatures

messages

repeat

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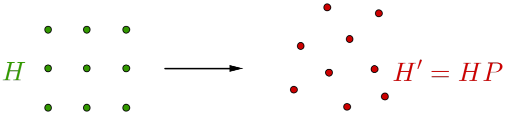

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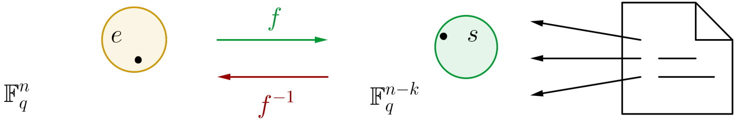

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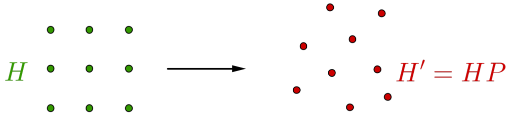
Disadvantage: slow signing, large public key

Advantage: small signatures, fast verification

Classical Approach: Hash and Sign

structured code

efficient decoding

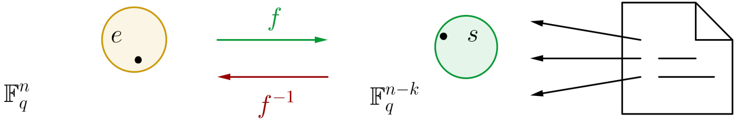


random code

hard to decode

Idea McEliece: use Goppa code as secret code

trapdoor



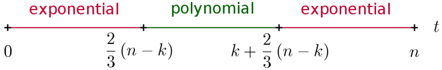
$\mathrm{wt}_H(e) = t$

$s = HPe^\top$

Disadvantage: slow signing, large public key

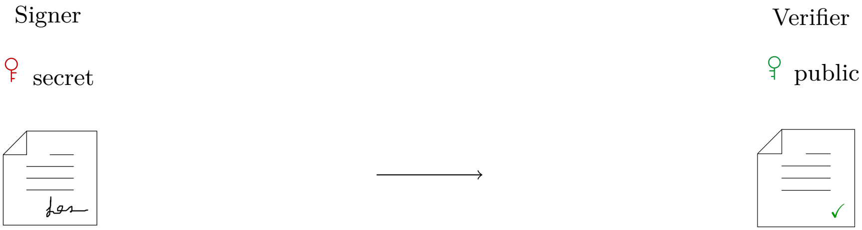
Advantage: small signatures, fast verification

Wave: $(u, u+v)$ ternary code and t large



Zero-Knowledge Protocol

Signature Scheme



Zero-Knowledge Protocol

ZK Protocol

Prover		Verifier
secret		public
	⟵	
Commitments c	Interaction	
	⟶	Challenge
		recover c ✓
Response r		

Zero-Knowledge Protocol

Signature Scheme

Signer

🔑 secret

Commitments c

Challenge= $\mathsf{Hash}(m, c)$

Response r

Fiat-Shamir

$\longrightarrow$

Verifier

🔑 public

✓

Zero-Knowledge Protocol

Signature Scheme

Impersonator

secret

cheating prob.

Fiat-Shamir

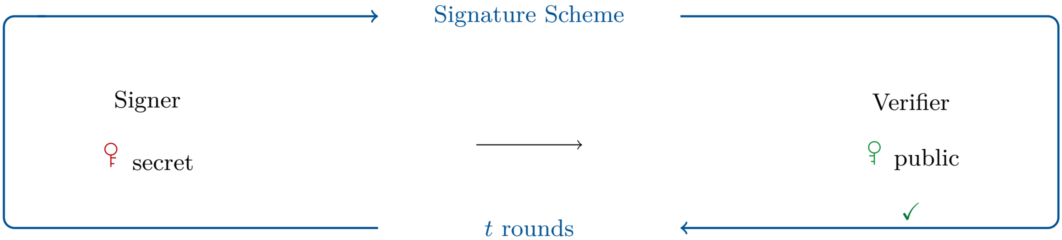
$\longrightarrow$

Verifier

public

✓

Zero-Knowledge Protocol



Zero-Knowledge Protocol

ZK Protocol

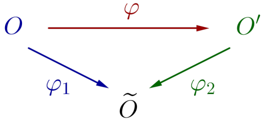
Prover: secret

← Interaction →

Verifier: public ✓

Isomorphism Problems Given O, O', find φ s.t. $\varphi(O) = O'$

Prover: φ



Verifier: O, O'

1. $\varphi_1(O) = \tilde{O}$ ✓ /
2. $\varphi_2(O') = \tilde{O}$ ✓

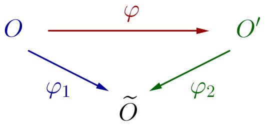
Zero-Knowledge Protocol

ZK Protocol

Prover		Verifier
secret	$\longleftarrow$ Interaction $\longrightarrow$	public ✓

Isomorphism Problems Given O, O', find φ s.t. $\varphi(O) = O'$

φ

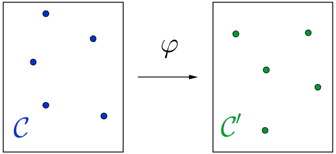


O, O'

1. $\varphi_1(O) = \tilde{O}$ ✓ /
2. $\varphi_2(O') = \tilde{O}$ ✓

$\rightarrow$ MEDS, LESS

Code Equivalence



Code equivalence

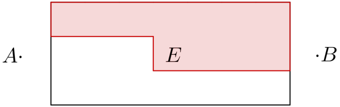
Given $G, G' \in \mathbb{F}_q^{k \times n}$ find isometry φ s.t.

$$\varphi(\langle G \rangle) = \langle G' \rangle$$

Hamming isometries $\varphi \in (\mathbb{F}_q^\star)^n \rtimes S_n$

$\rightarrow$ LESS

Rank isometries $\varphi \in \mathrm{GL}_m(\mathbb{F}_q) \times \mathrm{GL}_n(\mathbb{F}_q)$



$\rightarrow$ MEDS

Disadvantages: medium/large public keys

Advantages: medium/small signatures

Zero-Knowledge Protocol

ZK Protocol

Prover		Verifier
	$\longleftarrow$	
♀ secret	Interaction	♀ public
	$\longrightarrow$	
		✓

SDP Given H, s, t, find e s.t. 1. $s = eH^\top$, 2. $\mathrm{wt}_H(e) = t$

Zero-Knowledge Protocol

ZK Protocol

Prover $\longleftarrow$ Verifier

secret — Interaction — public

$\longrightarrow$

✓

SDP Given H, s, t, find e s.t. 1. $s = eH^\top$, 2. $\mathrm{wt}_H(e) = t$

e of $\mathrm{wt}_H(e) = t$

H, s, t

1. ✓ / 2. ✓

Zero-Knowledge Protocol

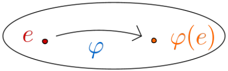
ZK Protocol

Prover $\longleftarrow$ Interaction $\longrightarrow$ Verifier

secret | public ✓

SDP: Given H, s, t, find e s.t. 1. $s = eH^\top$, 2. $\mathrm{wt}_H(e) = t$

e of $\mathrm{wt}_H(e) = t$



H, s, t

φ: 1. ✓ / $\varphi(e)$: 2. ✓

Zero-Knowledge Protocol

ZK Protocol

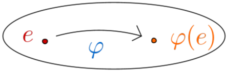
Prover ← Interaction → Verifier

secret — public

✓

SDP Given H, s, t, find e s.t. 1. $s = eH^\top$, 2. $\mathrm{wt}_H(e) = t$

e of $\mathrm{wt}_H(e) = t$



H, s, t

φ: 1. ✓ / $\varphi(e)$: 2. ✓

1. Problem

cheating prob. $\sim \frac{1}{2}$

→ many rounds

Zero-Knowledge Protocol

ZK Protocol

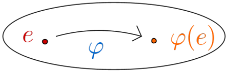
Prover | $\longleftarrow$ Interaction $\longrightarrow$ | Verifier

secret | | public

✓

SDP Given H, s, t, find e s.t. 1. $s = eH^\top$, 2. $\mathrm{wt}_H(e) = t$

e of $\mathrm{wt}_H(e) = t$



H, s, t

φ: 1. ✓ / $\varphi(e)$: 2. ✓

1. Problem — cheating prob. $\sim \frac{1}{2}$ — $\rightarrow$ many rounds

$\rightarrow$ Solution — MPCitH: change protocol

MPC in-the-head

ZK Protocol

Prover		Verifier
🔑 secret $\mathcal{S}$		🔑 public
$(N-1)$-private MPC:	$\xrightarrow{c_i, \alpha_i}$	
Split $\mathcal{S}$ into N shares: s_i	$\xleftarrow{\ell}$	Challenge $\ell \in \{1, \dots, N\}$
Commitments c_i for s_i	$\xrightarrow{s_i \text{ for } i \neq \ell}$	
Broadcasts $\alpha_i = f(s_i)$		Check c_i, α_i for $i \neq \ell$ ✓

$(N-1)$-private MPC — Secret $\mathcal{S}$ split into N shares s_i

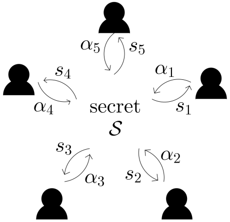
$\leq N-1$ many $s_i \to$ no info. on $\mathcal{S}$ — broadcasts α_i to check validity of $\mathcal{S}$

Example $e = \sum_{i=1}^{N} e^{(i)}$, $f(e^{(i)}) = e^{(i)} H^\top = s^{(i)} \to$ can check $\sum_{i=1}^{N} s^{(i)} = s$

MPC in-the-head

Prover

secret $\mathcal{S}$



ZK Protocol

$\xrightarrow{c_i, \alpha_i}$

$\xleftarrow{\ell}$

$\xrightarrow{s_i \text{ for } i \neq \ell}$

Verifier

public

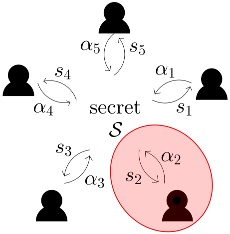
Challenge $\ell \in \{1, \dots, N\}$

Check c_i, α_i for $i \neq \ell$ ✓

MPC in-the-head

Prover

secret $\mathcal{S}$



ZK Protocol

$\xrightarrow{c_i, \alpha_i}$

$\xleftarrow{\ell}$

$\xrightarrow{s_i \text{ for } i \neq \ell}$

Verifier

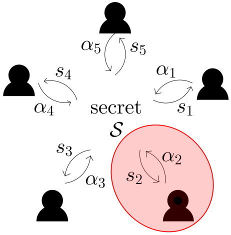
public

Challenge $\ell \in \{1, \ldots, N\}$

Check c_i, α_i for $i \neq \ell$ ✓

MPC in-the-head

Prover	ZK Protocol	Verifier
secret $\mathcal{S}$		public
	$\xrightarrow{c_i, \alpha_i}$	
	$\xleftarrow{\ell}$	Challenge $\ell \in \{1, \ldots, N\}$
	$\xrightarrow{s_i \text{ for } i \neq \ell}$	Check c_i, α_i for $i \neq \ell$ ✓



→ New cheating probability: $\sim 1/N$

MPC in-the-head

ZK Protocol

Prover		Verifier
secret $\mathcal{S}$		public
m-private MPC:	$\xrightarrow{c_i, \alpha_i}$	
Split $\mathcal{S}$ into N shares: s_i	$\xleftarrow{I}$	Challenge $\|I\| = m$
Commitments c_i for s_i	$\xrightarrow{s_i \text{ for } i \in I}$	Check c_i, α_i for $i \in I$ ✓
Broadcasts $\alpha_i = f(s_i)$		

→ New cheating probability: $\sim 1/\binom{N}{m}$

MPC in-the-head

ZK Protocol

Prover		Verifier
secret $\mathcal{S}$		public
$(N-1)$-private MPC:	$\xrightarrow{c_i, \alpha_i}$	
Split $\mathcal{S}$ into N shares: s_i	$\xleftarrow{\ell}$	Challenge $\ell \in \{1, \ldots, N\}$
Commitments c_i for s_i	$\xrightarrow{s_i \text{ for } i \neq \ell}$	Check c_i, α_i for $i \neq \ell$ ✓
Broadcasts $\alpha_i = f(s_i)$		

→ New cheating probability: $\sim 1/N$

$\sim t/N$ rounds, but more computations

MPC in-the-head

ZK Protocol

Prover		Verifier
secret $\mathcal{S}$		public
$(N-1)$-private MPC:	$\xrightarrow{c_i, \alpha_i}$	
Split $\mathcal{S}$ into N shares: s_i	$\xleftarrow{\ell}$	Challenge $\ell \in \{1, \dots, N\}$
Commitments c_i for s_i	$\xrightarrow{s_i \text{ for } i \neq \ell}$	Check c_i, α_i for $i \neq \ell$ ✓
Broadcasts $\alpha_i = f(s_i)$		

$\rightarrow$ New cheating probability: $\sim 1/N$

$\sim t/N$ rounds, but more computations

Disadvantages: slow

Advantages: small sizes

MPC in-the-head

ZK Protocol

Prover		Verifier
secret $\mathcal{S}$		public
$(N-1)$-private MPC:	$\xrightarrow{c_i, \alpha_i}$	
Split $\mathcal{S}$ into N shares: s_i	$\xleftarrow{\ell}$	Challenge $\ell \in \{1, \dots, N\}$
Commitments c_i for s_i	$\xrightarrow{s_i \text{ for } i \neq \ell}$	Check c_i, α_i for $i \neq \ell$ ✓
Broadcasts $\alpha_i = f(s_i)$		

$\rightarrow$ New cheating probability: $\sim 1/N$

$\sim t/N$ rounds, but more computations

Disadvantages: slow

Using rank SDP $\rightarrow$ RYDE

Advantages: small sizes

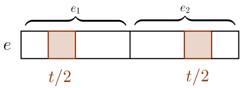
Using MinRank $\rightarrow$ MIRA, MiRitH

More novel problems

d-split SDP

Given H, s, t, find (e_1, e_2) s.t.

1. $s = eH^\top$
2. $\mathrm{wt}_H(e_i) = t/2$

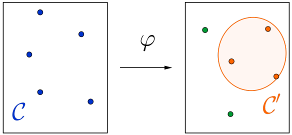


$\rightarrow$ SDitH

Subcode equivalence

Given $G \in \mathbb{F}_q^{k \times n}$, $G' \in \mathbb{F}_q^{k' \times n}$ find P s.t.

$$\langle GP \rangle \subset \langle G' \rangle$$



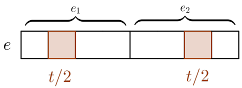
$\rightarrow$ PERK

More novel problems

d-split SDP

Given H, s, t, find (e_1, e_2) s.t.

1. $s = eH^\top$ 2. $\mathrm{wt}_H(e_i) = t/2$

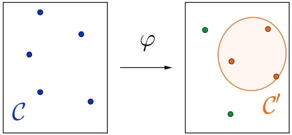


$\to$ SDitH

Permuted Kernel

Given $G \in \mathbb{F}_q^{k\times n}$, $H' \in \mathbb{F}_q^{n-k'\times n}$ find P s.t.

$$H'(GP)^\top = 0$$



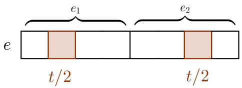
$\to$ PERK

More novel problems

d-split SDP

Given H, s, t, find (e_1, e_2) s.t.

1. $s = eH^\top$
2. $\mathrm{wt}_H(e_i) = t/2$

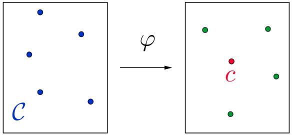


$\rightarrow$ SDitH

Relaxed permuted kernel problem

Given $G \in \mathbb{F}_q^{k \times n}, H' \in \mathbb{F}_q^{n-k' \times n}$ find x, P:

$$H'(xGP)^\top = 0$$

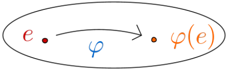


$\rightarrow$ PERK

Zero-Knowledge Protocol

SDP Given H, s, t, find e s.t. 1. $s = eH^\top$, 2. $\mathrm{wt}_H(e) = t$

e of $\mathrm{wt}_H(e) = t$



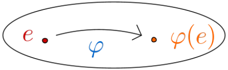
H, s, t

φ: 1. ✓ / $\varphi(e)$: 2. ✓

Zero-Knowledge Protocol

SDP Given H, s, t, find e s.t. 1. $s = eH^\top$, 2. $\mathrm{wt}_H(e) = t$

e of $\mathrm{wt}_H(e) = t$



H, s, t

φ: 1. ✓ / $\varphi(e)$: 2. ✓

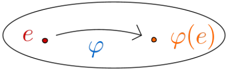
2. Problem

1 round: large commun. cost

Zero-Knowledge Protocol

SDP Given H, s, t, find e s.t. 1. $s = eH^\top$, 2. $\mathrm{wt}_H(e) = t$

e of $\mathrm{wt}_H(e) = t$



H, s, t

φ: 1. ✓ / $\varphi(e)$: 2. ✓

2. Problem

1 round: large commun. cost

$S = \{\mathrm{wt}_H(e) = t\}$

$\varphi : S \to S$ linear, transitive

$\to |\varphi|$ large

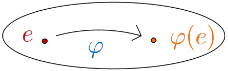
$\varphi \in (\mathbb{F}_q^\star)^n \rtimes S_n$

$|\varphi| \sim t \log_2(n(q-1))$

Zero-Knowledge Protocol

SDP Given H, s, t, find e s.t. 1. $s = eH^\top$, 2. $\mathrm{wt}_H(e) = t$

♀ e of $\mathrm{wt}_H(e) = t$



♀ H, s, t

φ: 1. ✓ / $\varphi(e)$: 2. ✓

2. Problem 1 round: large commun. cost

$S = \{\mathrm{wt}_H(e) = t\}$ $\varphi : S \to S$ linear, transitive $\to |\varphi|$ large

$\varphi \in (\mathbb{F}_q^\star)^n \rtimes S_n$ $|\varphi| \sim t \log_2(n(q-1))$

$\to$ Solution change underlying problem $\to$ CROSS

Hard Problems

Syndrome Decoding Problem Given p.c. matrix H, syndrome s, weight t, find e s.t.

lin. constraint 1. $s = eH^\top$ 2. $\mathrm{wt}_H(e) = t$ non-lin. constraint

Hard Problems

Restricted SDP (R-SDP) Given p.c. matrix H, syndrome s, restriction $\mathbb{E}$, find e s.t.

lin. constraint 1. $s = eH^\top$ 2. $e \in \mathbb{E}^n$ non-lin. constraint

$\mathbb{E} = \{g^i \mid i \in \{1, \ldots, z\}\} < \mathbb{F}_q^\star$ $g \in \mathbb{F}_q^\star$ of prime order z

Hard Problems

Restricted SDP (R-SDP) Given p.c. matrix H, syndrome s, restriction $\mathbb{E}$, find e s.t.

lin. constraint 1. $s = eH^\top$ 2. $e \in \mathbb{E}^n$ non-lin. constraint

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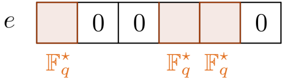
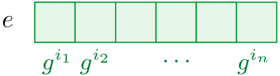


$\rightarrow$



- NP-hard
- adaption of ISD: exponential cost

R-SDP

Benefits

restriction $\mathbb{E} = \{g^i \mid i \in \{1, \dots, z\}\}$

rest. vectors $e = (g^{i_1}, \dots, g^{i_n}) \in \mathbb{F}_q^n$

R-SDP

Benefits

restriction $\mathbb{E} = \{g^i \mid i \in \{1, \dots, z\}\}$	$\xrightarrow{\ell}$	exponents $\mathbb{F}_z^n$
rest. vectors $e = (g^{i_1}, \dots, g^{i_n}) \in \mathbb{F}_q^n$		$\ell(e) = (i_1, \dots, i_n)$

R-SDP

Benefits

restriction $\mathbb{E} = \{g^i \mid i \in \{1, \ldots, z\}\}$	$\xrightarrow{\ell}$	exponents $\mathbb{F}_z^n$
rest. vectors $e = (g^{i_1}, \ldots, g^{i_n}) \in \mathbb{F}_q^n$		$\ell(e) = (i_1, \ldots, i_n)$
secret space $S = \mathbb{E}^n, \varphi : S \to S$	$\xrightarrow{\ell}$	$\lvert e\rvert = \lvert\varphi\rvert = n \log_2(z)$
$\varphi(e) = e' \star e, e' = (g^{j_1}, \ldots, g^{j_n})$		

R-SDP

Benefits

restriction $\mathbb{E} = \{g^i \mid i \in \{1, \dots, z\}\}$	$\xrightarrow{\ell}$	exponents $\mathbb{F}_z^n$
rest. vectors $e = (g^{i_1}, \dots, g^{i_n}) \in \mathbb{F}_q^n$		$\ell(e) = (i_1, \dots, i_n)$
secret space $S = \mathbb{E}^n, \varphi : S \to S$	$\xrightarrow{\ell}$	$\lvert e \rvert = \lvert \varphi \rvert = n \log_2(z)$
$\varphi(e) = e' \star e, e' = (g^{j_1}, \dots, g^{j_n})$		$\ell(\varphi(e)) = \ell(e) + \ell(e')$

R-SDP

Benefits

restriction $\mathbb{E} = \{g^i \mid i \in \{1, \dots, z\}\}$	$\xrightarrow{\ell}$	exponents $\mathbb{F}_z^n$
rest. vectors $e = (g^{i_1}, \dots, g^{i_n}) \in \mathbb{F}_q^n$		$\ell(e) = (i_1, \dots, i_n)$
secret space $S = \mathbb{E}^n, \varphi : S \to S$	$\xrightarrow{\ell}$	$\lvert e \rvert = \lvert \varphi \rvert = n \log_2(z)$
$\varphi(e) = e' \star e, e' = (g^{j_1}, \dots, g^{j_n})$		$\ell(\varphi(e)) = \ell(e) + \ell(e')$

Example

$\mathbb{E} = \{1, 3, 9\} \subset \mathbb{F}_{13}$	$\xrightarrow{\ell}$	exponents in $\mathbb{F}_3^4$
$e = (1, 9, 3, 3)$		$\ell(e) = (0, 2, 1, 1)$
$\downarrow \ \star(3, 3, 9, 1)$		$\downarrow \ +(1, 1, 2, 0)$
$\tilde{e} = (3, 1, 1, 3)$		$\ell(\tilde{e}) = (1, 0, 0, 1)$

R-SDP(G)

R-SDP   Given H, s, $\mathbb{E}$, find e s.t.   1. $s = eH^\top$ 2. $e \in \mathbb{E}^n$ $(\mathbb{E}^n, \star) \simeq (\mathbb{F}_z^n, +)$

R-SDP(G)

R-SDP(G) Given H, s, G, find e s.t. 1. $s = eH^\top$ 2. $e \in G$ $(G, \star) < (\mathbb{E}^n, \star)$

Benefits

$$x_1 = (g^{i_1}, \ldots, g^{i_n})$$
$$\vdots$$
$$x_m = (g^{j_1}, \ldots, g^{j_n})$$

R-SDP(G)

R-SDP(G) Given H, s, G, find e s.t. 1. $s = eH^\top$ 2. $e \in G$ $(G, \star) < (\mathbb{E}^n, \star)$

Benefits

$$x_1 = (g^{i_1}, \dots, g^{i_n}) \\ \vdots \\ x_m = (g^{j_1}, \dots, g^{j_n}) \quad \xrightarrow{\ \ell\ } \quad M = \begin{pmatrix} i_1 & \cdots & i_n \\ \vdots & & \vdots \\ j_1 & \cdots & j_n \end{pmatrix} \in \mathbb{F}_z^{m \times n}$$

R-SDP(G)

R-SDP(G) Given H, s, G, find e s.t. 1. $s = eH^\top$ 2. $e \in G$ $G \simeq \mathcal{C} \subset \mathbb{F}_z^n$

Benefits

$$\begin{array}{c} x_1 = (g^{i_1}, \dots, g^{i_n}) \\ \vdots \\ x_m = (g^{j_1}, \dots, g^{j_n}) \end{array} \xrightarrow{\ \ell\ } M = \begin{pmatrix} i_1 & \cdots & i_n \\ \vdots & & \vdots \\ j_1 & \cdots & j_n \end{pmatrix} \in \mathbb{F}_z^{m \times n}$$

$$e = {x_1}^{u_1} \star \cdots \star {x_m}^{u_m} \qquad \ell(e) = (u_1, \dots, u_m)M$$

$$\varphi : G \to G, \varphi(e) = e' \star e \xrightarrow{\ \ell\ } |e| = |\varphi| = m \log_2(z) < 1.5\lambda$$

R-SDP(G)

> **R-SDP(G)** Given H, s, G, find e s.t. 1. $s = eH^\top$ 2. $e \in G$ $G \simeq \mathcal{C} \subset \mathbb{F}_z^n$

Benefits

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$$\varphi : G \to G, \varphi(e) = e' \star e \quad \xrightarrow{\ell} \quad |e| = |\varphi| = m \log_2(z) < 1.5\lambda$$

Example

$$\mathbb{E} = \{1, 3, 9\} \subset \mathbb{F}_{13} \quad \xrightarrow{\ell} \quad \text{exponents in } \mathbb{F}_3^4$$

$$x_1 = (3, 1, 1, 3) \qquad x_2 = (1, 3, 9, 1) \qquad M = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 \end{pmatrix}$$

$$e = {x_1}^{②} \star {x_2}^{①} = (9, 3, 9, 9) \qquad \ell(e) = (2, 1, 2, 2) = (2, 1)M$$

Summary

Hash & Sign

Large weight SDP	$\longrightarrow$	WAVE	large public key

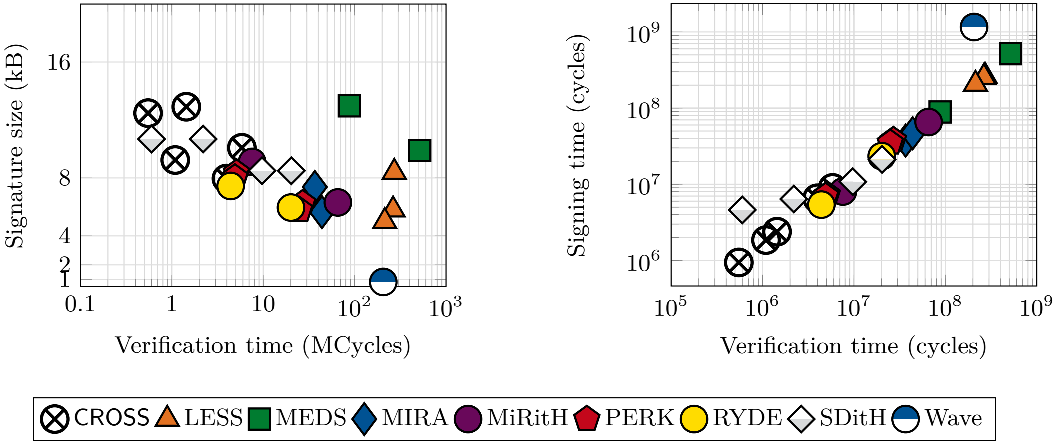
ZK Protocol

Restricted SDP	$\longrightarrow$	CROSS	
CEP	$\longrightarrow$	LESS	
Matrix CEP	$\longrightarrow$	MEDS	large signature

ZK + MPC

d-split SDP	$\longrightarrow$	SDitH	
Rank SDP	$\longrightarrow$	RYDE	
MinRank	$\longrightarrow$	MIRA/MiRitH	
PKP	$\longrightarrow$	PERK	slow

Comparison



Timings taken from `https://pqshield.github.io/nist-sigs-zoo/`

Timeline

2016	NIST standardization call	for post-quantum PKE/KEM and signatures	
	Standardized KEM:	KYBER	
	4th round:	BIKE, Classic McEliece, HQC	
2022	Standardized signatures:	DILITHIUM, FALCON, SPHINCS+	
2023	On ramp announcement		
	1st round candidates:	29 survivors	9 code-based
2024			

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	1st round candidates:	29 survivors	9 code-based
2024	2nd round announced	14 schemes	6 code-based

2nd Round Candidates

Code-based: 9

- CROSS
- LESS
- MEDS
- MIRA
- MiRitH
- PERK
- RYDE
- SDitH
- Wave

Other: 1

- Preon

Lattice-based: 5

- HAETAE
- Hawk
- HuFu
- Raccoon
- Squirrels

Symmetric: 4

- AIMer
- Ascon-Sign
- FAEST
- SPHINCSα

Multivariate: 9

- Biscuit
- MAYO
- MQOM
- PROV
- QRUOV
- SNOVA
- TUOV
- UOV
- VOX

Isogeny: 1

- SQISign

2nd Round Candidates

Code-based: 6

- CROSS
- LESS
- MEDS
- MiRatH
- PERK
- RYDE
- SDitH
- Wave

Other: 0

- Preon

Lattice-based: 1

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- Hawk
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Symmetric: 1

- AIMer
- Ascon-Sign
- FAEST
- SHPINCSα

Multivariate: 5

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Isogeny: 1

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2nd Round Candidates

NIST.IR.8528 Status report

1) security 2) cost and performance 3) implementation

Code-based: 6

- CROSS
- LESS
- MiRatH
- PERK
- RYDE
- SDitH

Lattice-based: 1

- Hawk

Symmetric: 1

- FAEST

Isogeny: 1

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2nd Round Candidates

NIST.IR.8528 Status report

1) security 2) cost and performance 3) implementation

a) simplicity b) uniqueness c) elegance

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- CROSS
- LESS
- MiRatH
- PERK
- RYDE
- SDitH

Lattice-based: 1

- Hawk

Symmetric: 1

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non-lattice, better performance than SPHINCS

new, improve performance

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NIST.IR.8528 Status report

1) security   2) cost and performance   3) implementation

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Symmetric: 1

- FAEST

Isogeny: 1

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Multivariate: 5

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- MQOM
- QRUOV
- SNOVA
- UOV

non-lattice, better performance than SPHINCS

new, improve performance: threshold, VOLE

2nd Round Candidates

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- RYDE
- SDitH

Lattice-based: 1

- Hawk

Symmetric: 1

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Isogeny: 1

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Multivariate: 5

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non-lattice, better performance than SPHINCS

complex, technical

2nd Round Candidates

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Lattice-based: 1

- Hawk

Symmetric: 1

- FAEST

Isogeny: 1

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Multivariate: 5

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no floating points

new

2nd Round Candidates

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Lattice-based: 1

- Hawk

Symmetric: 1

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new, recent attacks

How will the 2nd round go?

Timeline

- Submission deadline: Jan. 17
- 3rd round decision?
- How many schemes?

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- Cost of restricted SDP
- Cost of rank SDP
- Cost of q-ary SDP

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- How hard is code equivalence?

Abhi's talk!

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Open Problems

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- Cost of restricted SDP
- Cost of rank SDP
- Cost of q-ary SDP
- How hard is code equivalence?

Slides

Stay tuned!

Thank you

VOLE

Vector Oblivious Linear Transfer

ZK Protocol

Prover	Verifier
secret s	public
v random	Δ eval. point
$f(x) = sx + v$	
	$q = f(\Delta)$

VOLE

Vector Oblivious Linear Transfer

ZK Protocol

Prover

secret s

v random

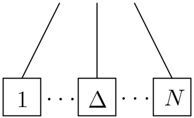
$f(x) = sx + v$

$q = f(\Delta)$

$\leftarrow s, v$

$\leftarrow \Delta$

GGM Tree



$\leftarrow \Delta$

$\mathsf{seed}_i \rightarrow$

Verifier

public

Δ eval. point

seed_i for $i \neq \Delta$

$q = f(\Delta)$

VOLE correlation $q = s\Delta + v = f(\Delta)$

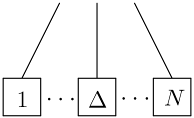
dishonest prover needs to guess Δ before committing to GGM tree: $\mathbb{P} \sim 1/p$

VOLE

Vector Oblivious Linear Transfer

ZK Protocol

Prover		GGM Tree		Verifier
secret s	$\overset{s, v}{\longleftarrow}$		$\overset{\Delta}{\longleftarrow}$	public
v random	$\overset{\Delta}{\longleftarrow}$		$\overset{\mathsf{seed}_i}{\longrightarrow}$	Δ eval. point
$f(x) = sx + v$				seed_i for $i \neq \Delta$
$q = f(\Delta)$				$q = f(\Delta)$



MPC

$s = \sum s_i$ MPC $\overset{\ell}{\longleftarrow}$ $N - 1$ views

VOLE

$s = \sum s_i$ GGM $\overset{\Delta}{\longleftarrow}$ $N - 1$ seeds

$v = \sum i s_i$ $q = \sum s_i(\Delta - i) = s\Delta + v$

VOLE

Vector Oblivious Linear Transfer

ZK Protocol

Prover

secret s

v random

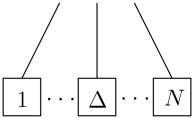
$f(x) = sx + v$

$q = f(\Delta)$

$s, v \leftarrow$

$\Delta \leftarrow$

GGM Tree



$\Delta \leftarrow$

$\mathsf{seed}_i \rightarrow$

Verifier

public

Δ eval. point

seed_i for $i \neq \Delta$

$q = f(\Delta)$

$f(x) = \sum_{i=0}^{d} f_i x^i,$

$s = f_d$

$f_1(x), f_2(x)$

$f_1(\Delta) + f_2(\Delta) = (f_1 + f_2)(\Delta)$

$f_1(\Delta) f_2(\Delta) = (f_1 f_2)(\Delta)$

VOLE

Vector Oblivious Linear Transfer

ZK Protocol

Prover

secret s

v random

$f(x) = sx + v$

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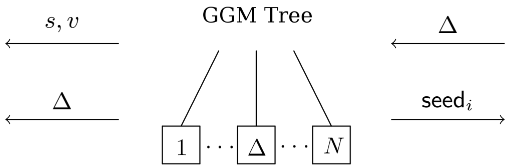

Verifier

public

Δ eval. point

seed_i for $i \neq \Delta$

$q = f(\Delta)$

Disadvantages: slow

Advantages: small sizes

Main Features

Implementation

- optimized AVX2 — **fast** < 1 MCycle (NIST cat. I)
- memory-optimized — fits on Cortex-M4 microcontroller
- constant worst-case runtime — no signature rejection
- available on lib open quantum safe

Ingredients

- Restricted Syndrome Decoding
  - → compact objects & efficient arithmetic
  - → NP-hard problem
- Zero-Knowledge protocol
  - → simple and well-studied
  - → EUF-CMA security
  - → BUFF security
  - → standard optimizations

Future of CROSS

What's next?

- Hardware implementation
- Side-channel protection
- Worst-case to average-case reduction
- Smaller signatures: VOLE

Website

CROSS

Codes & Restricted Objects Signature Scheme
http://cross-crypto.com/

Attacks

- $\mathbb{E}, G$ have multiplicative structure
  $e = (g^{i_1}, \ldots, g^{i_n})$
- $s = eH^\top$ has additive structure
  $s_j = \sum_{\ell=1}^{n} h_{j,\ell} g^{i_\ell}$ for $j \in \{1, \ldots, n-k\}$

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- Take $\mathbb{E}$ with no additive structure
- good: $q = 13, g = 3, \mathbb{E} = \{1, 3, 9\}$
- bad: $q = 13, g = 5, \mathbb{E} = \{1, 5, -1, -5\}$

Attacks

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- combinatorial:

  ISD algorithms

S. Bitzer, A. Pavoni, V. Weger, P. Santini, M. Baldi, and A. Wachter-Zeh. "Generic Decoding of Restricted Errors", ISIT, 2023.

M. Baldi, S. Bitzer, A. Pavoni, P. Santini, A. Wachter-Zeh, and V. Weger. "Zero knowledge protocols and signatures from the restricted syndrome decoding problem", PKC, 2024.

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- combinatorial:

  ISD algorithms

- algebraic attacks:

  $e_i^z = 1$ Gröbner basis

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M. Baldi, et al. "CROSS", NIST PQC round 1, 2023.

W. Beullens, P. Briaud, M. Øygarden. "A Security Analysis of Restricted Syndrome Decoding Problems", 2024.

Performance

NIST cat. I

Problem	q, z	Type	(n, k, m)	rounds	|Sign.| (kB)	Sign (MCycles)	Verify (MCycles)
R-SDP	$(127, 7)$	fast	$(127, 76, -)$	163	19.1	1.28	0.78
		balanced		252	12.9	2.38	1.44
		short		960	10.1	8.96	5.84
R-SDP(G)	$(509, 127)$	fast	$(55, 36, 25)$	153	12.5	0.94	0.55
		balanced		243	9.2	1.85	1.09
		short		871	7.9	6.54	3.96

private and public keys < 0.1 kB

key gen. < 0.1 MCycle

Measurements collected on an AMD Ryzen 5 Pro 3500U, clocked at 2.1GHz. The computer was running Debian GNU/Linux 12

CVE

PROVER		VERIFIER
KEY GENERATION		
Choose e with $\mathrm{wt}_H(e) = t$		
H parity-check matrix		
Compute $s = eH^\top$	$\xrightarrow{\mathcal{P}=(H,s,t)}$	
		VERIFICATION
Choose $u \in \mathbb{F}_q^n$, $\varphi \in \mathcal{M}_n$		
Set $c_1 = \mathrm{Hash}(\varphi, uH^\top)$		
Set $c_2 = \mathrm{Hash}(\varphi(u), \varphi(e))$	$\xrightarrow{c_1, c_2}$	
	$\xleftarrow{z}$	Choose $z \in \mathbb{F}_q^\times$
Set $y = \varphi(u + ze)$	$\xrightarrow{y}$	
$r_1 = \varphi$	$\xleftarrow{b}$	Choose $b \in \{1, 2\}$
$r_2 = \varphi(e)$	$\xrightarrow{r_b}$	$b = 1$: $c_1 = \mathrm{Hash}(\varphi, \varphi^{-1}(y)H^\top - zs)$
		$b = 2$: $\mathrm{wt}_H(\varphi(e)) = t$
		and $c_2 = \mathrm{Hash}(y - z\varphi(e), \varphi(e))$

CVE

Recall SDP: (1) $s = eH^\top$ (2) $\mathrm{wt}_H(e) = t$

PROVER		VERIFIER
KEY GENERATION		
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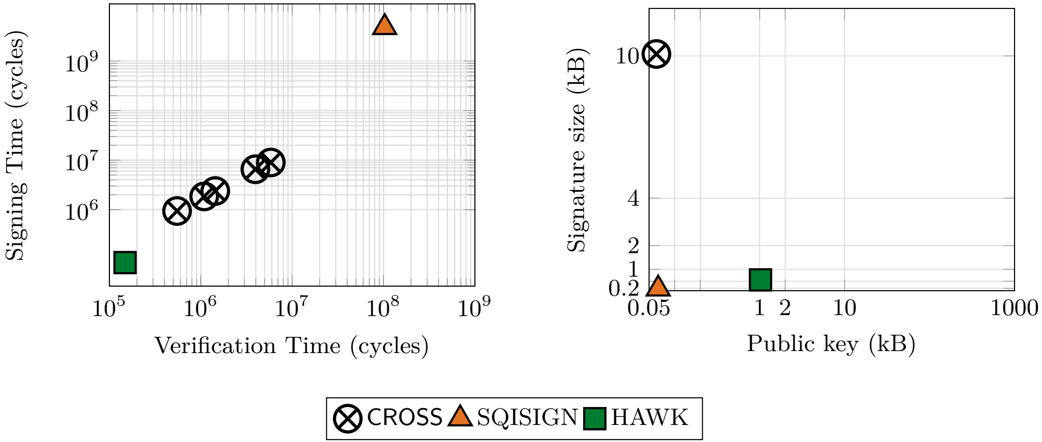
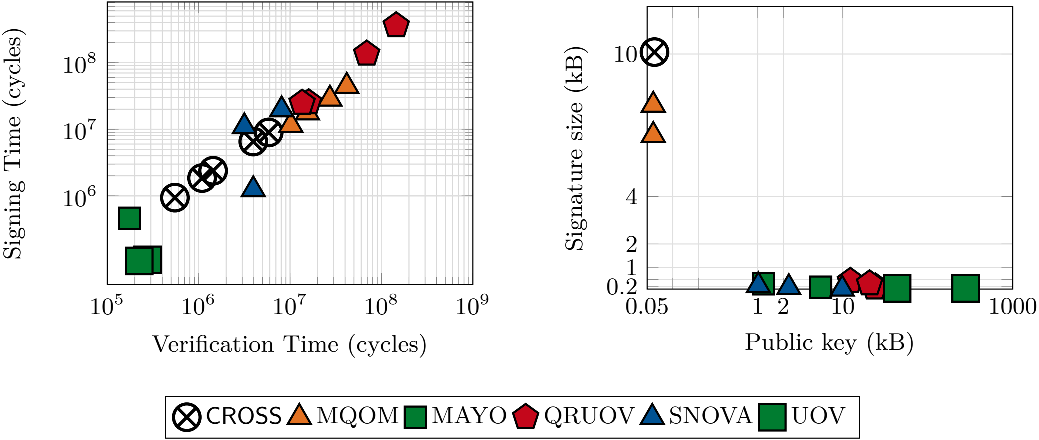
		and $c_2 = \mathrm{Hash}(y - z\varphi(e), \varphi(e))$

Problem: big signature sizes

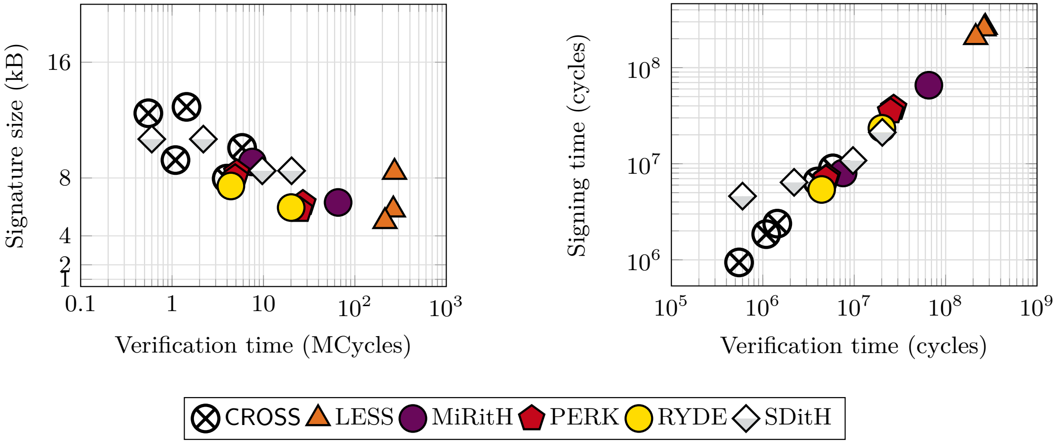
vs: Isogenies and lattices



vs: Multivariate



Comparison



Signature size (kB)
16
8
4
2
1
0.1
1
10
10^2
10^3
Verification time (MCycles)
Signing time (cycles)
10^8
10^7
10^6
10^5
10^6
10^7
10^8
10^9
Verification time (cycles)
CROSS LESS MiRitH PERK RYDE SDitH