

# RESEARCH STATEMENT

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## 1. BACKGROUND

My work lies in an area at the intersection of topology, higher category theory and mathematical physics, which more precisely concerns the study of **topological quantum field theories** (TQFT). From the mathematical point of view, a TQFT allows to relate a symmetric monoidal category of topological or geometrical nature, namely a **cobordism category**, to a symmetric monoidal category of algebraic nature, e.g. vector spaces over a field or modules over a commutative algebra. This allows on one side to obtain topological invariants of manifolds, and on the other to prove results in algebra using topological or geometric arguments. More precisely, the Atiyah-Segal axioms [Ati89, Seg88] formulate a  $n$ -dimensional TQFT as a symmetric monoidal functor from  $\text{Cob}_n$ , the cobordism category with objects closed  $n - 1$ -manifolds and morphisms given by  $n$ -bordisms, to  $\text{Vect}_{\mathbb{C}}$ , the category of vector spaces over the field  $\mathbb{C}$  of complex numbers. As a consequence of these axioms, a TQFT assigns a numerical invariant to a closed  $n$ -manifold. Particularly important examples of topological quantum field theories include the 3-dimensional Reshetikhin-Turaev TQFTs [RT91, Tu94] associated to a modular tensor category and Turaev-Viro TQFTs [TV92] associated to spherical fusion categories. Recently [Mor13], Dijkgraaf-Witten theory for a finite group  $G$  has been constructed as a TQFT via a linearisation of the stack of  $G$ -bundles over manifolds, which can be seen as a quantisation procedure of a classical theory, as suggested in [FHLT09]. All the above are furthermore examples of **extended** topological quantum field theories.

An extended topological quantum field theory attaches data not only to  $n$ -manifolds and  $n - 1$ -manifolds, but also to lower dimensional manifolds which appear as *corners*. To properly formulate this requires the introduction of **higher** categories, which are a generalisation of the notion of a category, where one considers not just objects and morphisms, but also morphisms between morphisms, and so on. Examples of higher categories include the 2-categories of 2-vector spaces [KV94], of algebras, bimodules and intertwiners, and of bundle gerbes over a manifold [W07]. A TQFT which assigns data to all lower dimensional corners up to the point is said to be **fully extended**. Fully extended TQFTs have been completely classified in [Lu09] using the language of  $\infty$ -categories, where it is shown that a fully extended  $n$ -dimensional framed TQFT is determined by the object assigned to the point, and that any *fully dualisable* object in the target  $(\infty, n)$ -category gives rise to such a TQFT. This is a remarkable result which proves what is known as the *cobordism hypothesis*, a conjecture formulated by John Baez and James Dolan [BD95] concerning the algebraic structure of the higher category of  $n$ -dimensional cobordisms, namely that the  $n$ -category of completely extended cobordism  $\text{Cob}_n$  is the free symmetric monoidal  $n$ -category with duals generated by a single object.

My interest in this field lies in the investigation of **boundary conditions** and **defects** in TQFTs. In recent developments [FFRS07, Kap10, KS11], it has become clear that the study of boundary conditions and defects will provide the tools for a deeper understanding of the mathematical structures required to describe symmetries and dualities of quantum field theories. On the other hand, many possible applications ranging from quantum computing to impurity problems in condensed matter physics require to consider quantum field theories on manifolds with *physical boundaries*, or on manifolds which are separated by *domain walls* into different parts on which different phases of the quantum field theory are realized. Furthermore, recent progress has been made concerning the relation between the study of boundary conditions and anomalies in quantum field theories [FT12].

I am also interested in applications of  $\infty$ -category theory to questions in geometry and topology motivated by Topological Quantum Field Theory, and in the study of nonperturbative aspects of **factorization algebras**, as developed by Costello and Gwilliam.

## 2. WORK TO DATE

In a recent paper with my collaborators Christoph Schweigert and Jürgen Fuchs [FSV12], I have investigated the 2-category of boundary conditions and surface defects for 3d TQFTs of Reshetikhin-Turaev type. Given a modular tensor category  $\mathcal{C}$ , consistent boundary conditions for the associated TQFT can be imposed only if  $\mathcal{C}$  is braided equivalent to the Drinfeld center of a fusion category  $\mathcal{A}^b$ . Similarly, we show that given two modular tensor categories  $\mathcal{C}_1$  and  $\mathcal{C}_2$ , surface defects between the associated TQFTs exist only if  $\mathcal{C}_1^{rev} \boxtimes \mathcal{C}_2$  is braided equivalent to the Drinfeld center of a fusion category  $\mathcal{A}^d$ . When boundary conditions can be imposed, their 2-category is equivalent to the 2-category of **module categories** over the fusion category  $\mathcal{A}^b$ . In the same way, when surface defects between two TQFTs do exist, their 2-category is equivalent to the 2-category of **module categories** over the fusion category  $\mathcal{A}^d$ . We then observe that the obstruction for a TQFT to admit boundary conditions lies in the *Witt group* of fusion categories, which has been subject to investigation in recent works [DNO13].

Finally, we also construct (noncanonically) from a surface defect  $S$  a special symmetric commutative Frobenius algebra in  $\mathcal{C}$ , which corresponds to a 2d RCFT, as predicted in [FFRS07]. The results above agree with those arising in more specific and concrete situations, for instance in the study of toroidal Chern-Simons theory [KS11]: in this case the gauge theory is governed by a metric group  $(D, q)$ , and its associated modular tensor category  $\mathcal{C}(D, q)$ . From the mathematical point of view, the results described above provide a useful bridge between algebraic results in terms of module categories and their classification, and topological results in terms of TQFTs.

In [FSV13], via an educated guess motivated by the classification of module categories over  $\text{Vect}^\omega(G)$ , where  $\omega \in Z_{grp}^3(G, \mathbb{C}^*)$ , we give a functorial construction of Dijkgraaf-Witten theory based on  $(G, \omega)$  including boundary conditions and surface defects in terms of **relative  $G$ -bundles**. More specifically, in the simplest case, for  $H \subset G$ , a relative  $(G, H)$ -bundle on a relative manifold  $(M, Y)$  consists of a  $G$ -bundle over  $M$  which admits a restriction of its structure group to  $H \subset G$  when restricted over  $Y$ . When appropriate morphisms are introduced, relative bundles form a groupoid for each relative manifold, and give rise to a stack over the site of *decorated* manifolds. Using a linearisation procedure [Mor13, FHLT09], we present the categories associated to one-dimensional manifolds with boundaries and defects points, and we show that the results agree with the prediction of the general formalism developed in [FSV12], taking into account the results of [Ost03]. In particular, we generalise the “parmesan” combinatoric techniques from [Will08] concerning the trasgression of  $n$ -cocycles for the cohomology of the classifying space, or rather stack  $BG$  to be able to transgress along 1-dimensional manifolds with possible boundaries and codimension 1 defects, i.e. marked points.

In [FPSV14], we investigated symmetries of extended Dijkgraaf-Witten theory in 3d via invertible defects. In particular, in the case of an abelian group  $A$ , we show that the group of autoequivalences of  $\mathcal{C} := D(A) - \text{mod}$  is generated by **transmission functors**, which are constructed via gauge theoretic techniques. Equivalently, the above results can be regarded as a geometric realisation, within Dijkgraaf-Witten theory, of the isomorphism between the Brauer-Picard group of invertible bimodule categories over  $\text{Vect}(G)$  and  $\text{Aut}(D(G) - \text{mod})$ , established in [ENO10].

Together with my collaborator Domenico Fiorenza, I have studied TQFTs with boundary conditions and the description of **anomalies** [FV14]<sup>1</sup>. It is well known that a 3d TQFT gives rise to a 2d modular functor, which can be seen as a representation of the groupoid with objects

<sup>1</sup>Some of our observations have been studied and expanded in [Nui13].

given by oriented 2-manifolds without boundaries, and morphisms given by isotopy classes of orientation preserving homeomorphisms. Moreover, for any surface  $\Sigma$ , a modular functor provides a representation of  $MCG(\Sigma)$ , the mapping class group of  $\Sigma$ . Theories with anomalies will give rise only to *projective* representation of the mapping class group of surfaces. This is usually encoded in the fact that a 3d TQFT with anomalies is a sort of *lax natural transformation* from a trivial TQFT to an invertible one. Generalizing ideas from [FT12], in [FV14] we have developed a construction of 2d (projective) modular functors arising from invertible extended 4d TQFTs with boundaries, using the higher categorical techniques developed in [Lu09]. In this picture, the anomalous 3d TQFT is seen as a boundary theory for a 4d topological field theory, and its behaviour is governed precisely by the boundary conditions. Using completely different techniques, this circle of ideas has been exploited in [Wal91], and we are working towards a detailed comparison of these results.

The circle of ideas surrounding the project above can be seen as a manifestation of a *holographic principle*, which can be expressed as the fact that a  $n$ -dimensional field theory presenting anomalies can be understood as a boundary theory for a well behaved  $n + 1$ -dimensional field theory. We expect the ideas and techniques developed in this project to have concrete applications, for instance to the study of boundary conditions in 4-dimensional quantum field theories, as those studied in [KW07], where the category of boundary conditions, or *D-branes*, is related to a category arising in the Langland program.

Together with Domenico Fiorenza and Urs Schreiber, we have investigated extensions of the mapping class group of closed surfaces which arise from automorphisms of higher topological structures on manifolds [FSV15]. We introduce the notion of a  $\rho$ -**structure**, namely an object in the slice  $(\infty, 1)$ -category over the homotopy type of  $GL(n; \mathbb{R})$ , regarded as an  $\infty$ -stack. This is quite a natural generalisation of **tangential structures**, which are at the heart of the cobordism hypothesis in [Lu09]. The  $\infty$ -group of diffeomorphism of a manifold  $M$  equipped with a  $\rho$ -structure can then be seen to extend, in a suitable higher categorical sense, the (homotopy type of)  $\text{Diff}(M)$ , and gives rise, in particular, to central extensions of the mapping class group.

In [HSV16], we have studied **homotopy actions** of topological groups on bicategories. Loosely speaking, a group  $G$  acts *homotopically* on a space  $X$  when the action is required to satisfy the various structural properties only up to homotopy. Similarly, one can consider the notion of an **homotopy fixed point**, and its generalisation to the realm of higher category theory. We provide a precise and detailed formulation of a homotopy action of a topological group on a bicategory, and its homotopy fixed points, and show some general results. In particular, we apply the above results to the study of the trivial homotopy action of  $SO(2)$  on the bicategory of finite dimensional semi-simple algebras, and to 2-Vector spaces, i.e. abelian finitely semi-simple linear categories. We show that homotopy fixed points for such trivial actions are far from trivial: indeed, they provide an algebra with the structure of a symmetric Frobenius algebra, and a 2-vector space with the structure of a Calabi-Yau category, respectively. The motivation to study homotopy  $SO(2)$ -actions arises from the cobordism hypothesis [Lu09, S-P]: indeed,  $SO(2)$ -homotopy fixed points are intimately related to fully extended 2d TQFTs which are furthermore oriented. Exploiting this relation between  $SO(2)$ -homotopy fixed points and oriented fully extended 2d TQFTs, in [HV17] we have shown in detail that the Serre automorphism for any bicategory has a geometric origin, and given conditions for its trivializability.

### 3. RESEARCH PLAN

**Extended TQFTs and Symmetries.** The relationship between symmetries and topological defects in topological quantum field theories is a very interesting one, and yet not completely explored. In particular a complete definition of a symmetry for a 3d extended TQFT in the functorial sense should involve natural transformations of 2-functors and their modifications on a side, and the category of automorphisms of the object the theory assigns to the circle.

Moreover, there are results in 2 dimensions [FFRS07] which suggest that all symmetries should be realisable in terms of invertible defects.

Using the constructions in the previous section, in future work I plan to investigate symmetries of Dijkgraaf-Witten theories and their relation to invertible surface defects, in particular to relate the automorphisms of the stack of  $G$ -bundles with the category of braided equivalences of the Drinfeld center of the category  $G$ -vector spaces. In particular, one expects the 2-group describing the symmetries of Dijkgraaf-Witten as an extended 3d TQFT to be strictly “bigger” than just the automorphism 2-group of the stack of  $G$ -bundles, which corresponds to the *classical* symmetries: from gauge theoretic intuition, indeed, I expect that nongeometric symmetries, as for instance the analog of electric-magnetic duality, should arise only at the quantum level, but should be nevertheless captured by a-per-faberinvertible surface defects. As a start, in the work [FPSV14], I have concentrated on verifying these expectations in the simplified case when  $G$  is an Abelian group, and the 3-cocycle  $\omega$  vanishes.

As pointed out in [DW90], a given 3-cocycle  $\omega \in Z_{grp}^3(G, U(1))$  gives rise to a Chern-Simons 2-gerbe over  $\text{Bun}(G)$ , the stack of  $G$ -bundles. In this sense, a Dijkgraaf-Witten theory based on  $(G, \omega)$  can be regarded as a nonlinear sigma model on the background given by  $\text{Bun}(G)$  and the Chern-Simons 2-gerbe  $\omega$ . It is important then to first formulate the correct notion of symmetry, or automorphism of such a background, since we are dealing with higher categorical objects. I plan to study in which sense an automorphism of  $\text{Bun}(G)$  must be compatible with the Chern-Simons 2-gerbe  $\omega$ , and to show that such automorphisms indeed produce invertible module categories over the category of  $\omega$ -twisted  $G$ -graded vector space  $\text{Vect}^\omega(G)$ , as expected by [FSV12, FSV13, FPSV14].

As argued in [FSV12], module categories play a relevant role in general Reshetikhin-Turaev and Turaev-Viro theories. In recent work [ENO10], the **Brauer-Picard groupoid**  $\text{BrPic}(\mathcal{A})$  of bimodule categories over a fusion category  $\mathcal{A}$  has been introduced: this is a 2-group with objects given by invertible bimodule categories over  $\mathcal{A}$  and with morphisms given by equivalent classes of invertible bimodule functors. One of the main results in [ENO10] is to provide an equivalence between  $\text{BrPic}(\mathcal{A})$  and the 2-group of braided equivalences  $\text{EqBr}(Z(\mathcal{A}))$  of the Drinfeld center  $Z(\mathcal{A})$  of  $\mathcal{A}$ . I plan to provide a TQFT description of these results, motivated by the following argument, and results in [FPSV14]. Very recently it has been shown [DSPS13] that fusion categories provide fully dualizable objects in the 3-category of finite tensor categories  $TC$ : by the cobordism hypothesis, to any fusion category  $\mathcal{A}$  there corresponds a 3-dimensional (framed) TQFT with target  $TC$ , which assigns the Drinfeld center  $Z(\mathcal{A})$  to the framed circle. Moreover, in [Lu09] it is shown that any choice of a  $\mathcal{A}$ -bimodule  $\mathcal{M}$  will produce a TQFT  $F$  defined on a cobordism category *with* defects. Consider then a cylinder with an embedded defect circle “in the middle”: the theory  $F$  must then assign to such a cobordism a functor between  $Z(\mathcal{A})$  and itself. If  $\mathcal{M}$  is invertible, by the general properties of TQFTs we expect such a functor to be an equivalence and, moreover, to be braided. Motivated by this, I conjecture that this assignment of an invertible bimodule category to a braided equivalence *coincides* with the equivalence constructed in [ENO10], providing a novel topological interpretation of such an algebraic result, and a sign of a fruitful interaction among different fields of mathematics.

**Equivariant factorization algebras from abelian Chern-Simons theory.** A **factorization algebra** over a manifold  $M$  is an algebra over an operad that looks similar to the little disk operad, but with the difference that the “open ball” are subsets of a fixed manifold  $M$ . Denoting with  $\text{Fact}_M$  such an operad, a factorization algebra on  $M$  taking values in a symmetric monoidal category  $\mathcal{C}$  can be defined as a morphism  $\text{Fact}_M \rightarrow \text{End}(\mathcal{C})$ , where  $\text{End}(\mathcal{C})$  denotes the endomorphism operad associated to  $\mathcal{C}$ . A particular interesting target category  $\mathcal{C}$  to consider is the category of chain complexes of  $k$ -modules. In [CG16], Costello and Gwilliam show how to produce factorization algebras from perturbative Quantum Field Theories, and interpret such factorization algebras as algebras of local observables, both classical and quantum. In particular, they study in detail the case of Chern-Simons theory with abelian structure group. They show how the classical factorization algebra associated to  $U(1)$ -Chern-Simons theory on a manifold  $M$  can be obtained from the shifted de Rham complex  $\Omega^*(M)[1]$ . Nevertheless,

their approach is entirely perturbative, hence it does not distinguish between  $\mathbb{R}$ -Chern-Simons and  $U(1)$ -Chern-Simons. Together with Corina Keller, a master student of mine, we equip the factorization algebra  $\text{Obs}^{cl}$  of classical observables constructed by Costello-Gwilliam with a homotopy  $\mathcal{G}$ -equivariant structure, where  $\mathcal{G} := \text{Maps}(M, U(1))$ , the gauge group of  $U(1)$ -Chern-Simons. We show that such a homotopy action does detect nonperturbative phenomena in abelian Chern-Simons theory via its fixed points, and that such an approach opens in general the possibility of applying the framework of factorization algebras beyond perturbation theory. Notice that the above results generalize immediately to Chern-Simons theories with structure group the  $n$ -torus  $T^n$ .

We plan to apply the results above to the perturbative quantization of abelian Chern-Simons theory. This in particular requires studying the equivariant properties of the shifted symplectic form which the classical factorization algebra of observables carries, and the compatibility with taking homotopy fixed points. We expect moreover that the above homotopy action survives the deformation quantization of  $\text{Obs}^{cl}$ . In [CG16], the authors argue that the quantization of  $\text{Obs}^{cl}$  is related to quantum groups: namely, by studying its behaviour on stratified manifolds carrying line defects, they construct a braided monoidal category  $\mathcal{B}$ , which should classify the decoration of Wilson lines. We expect that the compatibility with the homotopy action of the gauge group enforces additional properties on  $\mathcal{B}$ , namely modularity. We plan finally to relate this approach to abelian Chern-Simons theory to the results obtained in [FSV12, KS11].

**Boundary conditions, defects and homotopy actions.** In [Lu09], Lurie discusses a version of the cobordism hypothesis for manifolds with singularities, and in particular how to describe boundary conditions and defects for fully extended TQFTs. He first argues that given a symmetric monoidal  $(\infty, n)$ -category with duals  $\mathcal{C}$ , there is a canonical homotopy action<sup>2</sup> of  $O(n-1) \subset O(n)$  on the collection of 1-morphisms of  $\mathcal{C}$ . He then describes boundary conditions for an unoriented fully extended TQFT given by a fully dualizable object  $C$  which is a  $O(n)$ -homotopy fixed point as objects in the arrow category  $\text{Mor}(1, C)$  which are moreover homotopy fixed points for the  $O(n-1)$ -action; a similar description is provided for defects.

The  $O(n-1)$ -action described in [Lu09] remains to this date very mysterious. In this project with Nils Carqueville, we plan first to study in details the  $O(n-1)$ -action described above for  $n = 2$ , with the example of 2d unoriented Dijkgraaf-Witten theory in mind. In this case, the fact that  $O(1)$  is isomorphic to  $\mathbb{Z}_2$  renders the level of higher categorical complexity quite low, but at the same time functions as a useful playground to explore the properties of boundary conditions and defects in low dimensions. We plan next to investigate the case of oriented fully extended 3d TQFTs taking values in  $\text{Tens}$ , the tricategory of tensor categories, bimodule categories, module functors and natural transformations. It was conjectured in [DSPS13] that homotopy fixed points of the  $SO(3)$ -action on the fully dualisable objects of  $\text{Tens}$  correspond to spherical fusion categories. We expect then to construct an homotopy action of  $SO(2)$  on the bicategory of module categories (with finiteness conditions) over a spherical fusion category. This  $SO(2)$  action, which we expect to be not homotopically trivial, has not yet been explored in the literature, in particular in the area of representation theory. Moreover, according to the cobordism hypothesis for manifolds with singularities, boundary conditions for an oriented TQFT associated to a spherical fusion category can be described as module categories which are also homotopy fixed points under the  $SO(2)$ -action above. As shown in detail in [HSV16, HV17], being a homotopy fixed point for a group action on a bicategory equips the objects with an additional structure. Moreover, the work in [HSV16, HV17, FSV12] provides a solid technical foundation for this project.

**Invertible field theories and their homotopy type.** Topological quantum field theory which are **invertible** play a crucial role in the study of anomalous field theories, as recently investigated [FT12, Fr14a, Fr14b, FV14]. In particular, a fully extended invertible TQFT with values in a  $(\infty, n)$ -category  $\mathcal{C}$  factors through its  $\infty$ -**Picard groupoid**  $\text{Pic}(\mathcal{C})$ , which can be regarded as a topological space. Even better, it has the structure of an  $E_\infty$ -spectrum. This

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<sup>2</sup>Notice that  $O(n)$  does *not* act on  $\mathcal{C}$ , but only on the subcategory of fully dualizable objects.

allows to study invertible field theories as a space of maps in homotopy theory: in particular, it has been suggested recently that the set of components of such a topological space is an invariant of **topological gapped phases** [Fr14b]. An important case of study is given by choosing  $\mathcal{C}$  to be  $n\text{-Vect}$ , the  $(\infty, n)$ -category of  $n$ -Vector spaces. In [FV14] it was conjectured that  $\text{Pic}(n\text{-Vect})$  has the homotopy type of an Eilenberg-MacLane space  $K(\mathbb{C}^*, n)$ , for any  $n$ . By using an explicit construction of  $n\text{-Vect}$  given in [CS15], I plan to prove the conjecture above, which can be proven to hold for  $n = 1, 2$ . such a results would provide important information on the topology of the space of fully extended  $n$ -dimensional TQFTs, which in [Fr14b] have been suggested to have possible applications to condensed matter physics.

**Cobordisms with constraints and Higher Segal spaces.** Together with Claudia Scheimbauer, I have been working on formalising the notion of a **constrained TQFT** via higher Segal sets/spaces, as developed in [DK12]. Roughly, a higher Segal set models mathematical structures that behave like categories where the compositions of (higher) morphisms may not be unique, and where the space of such composition may not be contractible. A simple example of such occurances is given by a *partial monoid*, which can be regarded as a 2-Segal set with a single vertex. As a more refined example, we consider the case of constrained cobordisms, which allows us to formalise and study topological modular functors at genus 0, and quantum field theories with bounded volume. In the latter case, I expect the condition of boundedness of the volume to act in the same way as a cutoff, hence to obtain a family of QFTs with “finite-dimensional” properties. It would then be very interesting to formalize the limit of such a family in the framework of 2-Segal sets: in particular, the case of 2d Yang-Mills theory could provide a particularly rich and manageable example of such a construction.

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