University of BRISTOL

Billiards, Pretzels

… and Chaos

Inaugural lecture

Professor Corinna Ulcigrai

Wednesday 30 November 2016

Please note the building's fire exits. There are no planned fire alarms taking place today, so if you hear the alarm sound, please leave via the fire exits and gather at the meeting point outside the Merchant Venturers' Building.

Find out more about public events at the University of Bristol:
Web: bristol.ac.uk/events Twitter: @BrisUniEvents

Chaotic systems are everywhere:

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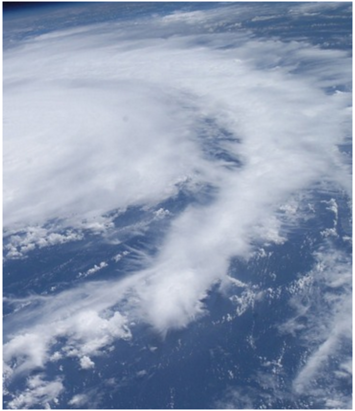

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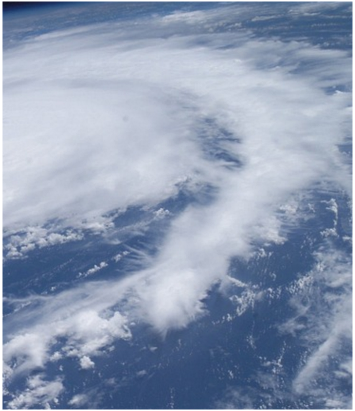

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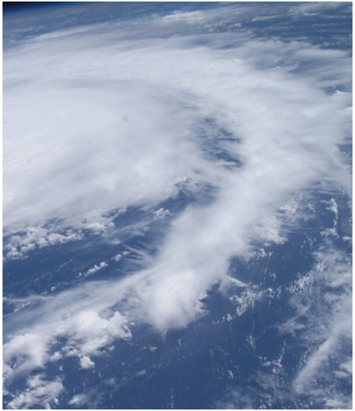

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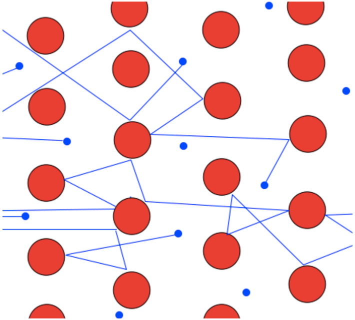

molecules of a gas,
electrons in a metal...

Deterministic Systems

These systems is deterministic, they obey "*rules*":

- laws of physics,
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Pierre-Simon Laplace
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" An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, [. . .] for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes."

Laplace, *A Philosophical Essay on Probabilities*

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Sensitive dependence: the "Butterfly effect"

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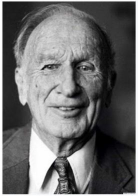

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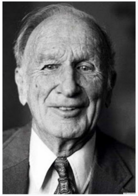

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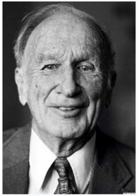

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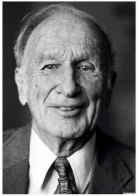

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How *quickly* does divergence in the Butterfly Effect happen? 

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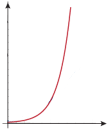

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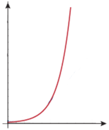

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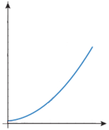

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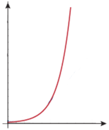

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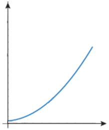

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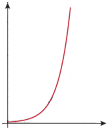

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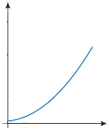

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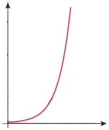

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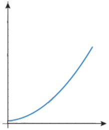

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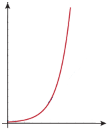

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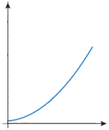

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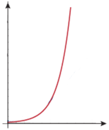

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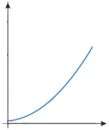

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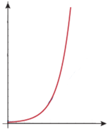

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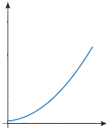

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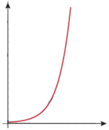

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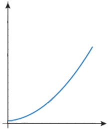

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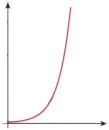

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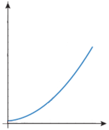

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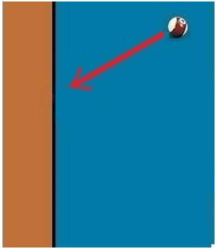

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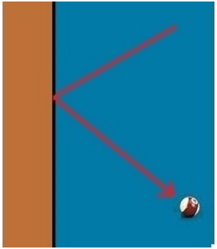

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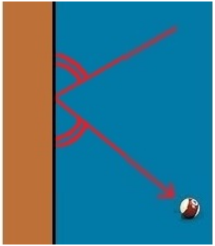

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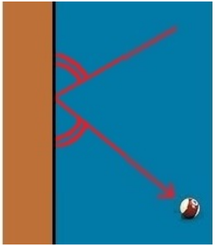

law of optics:

angle of incidence
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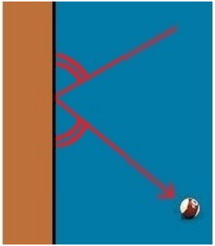

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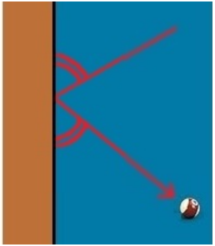

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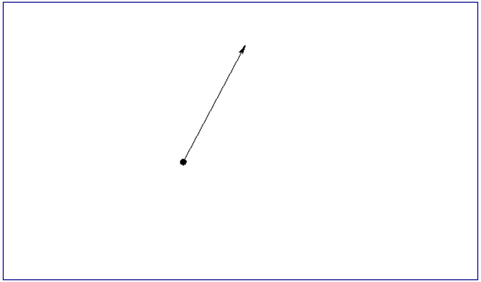

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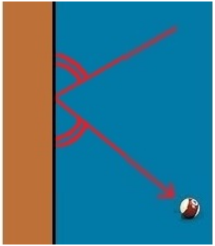

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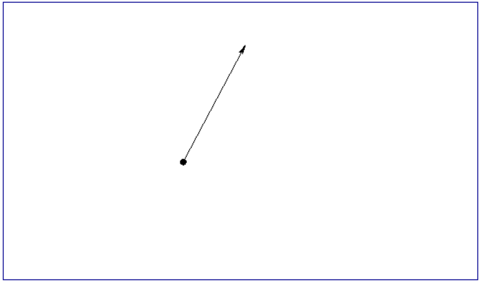

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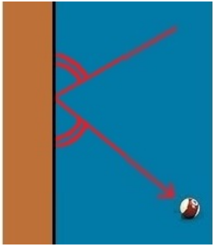

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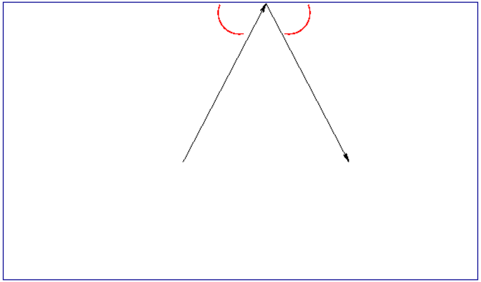

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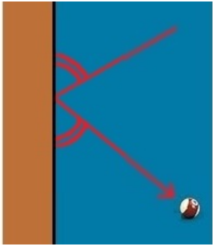

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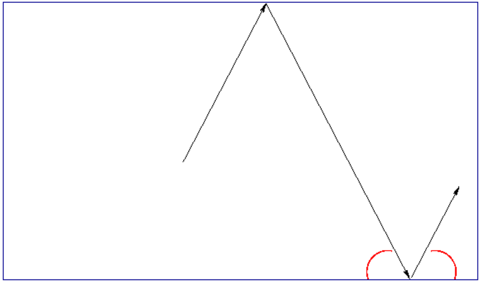

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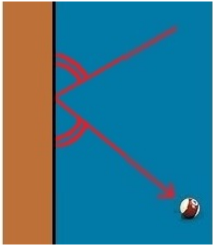

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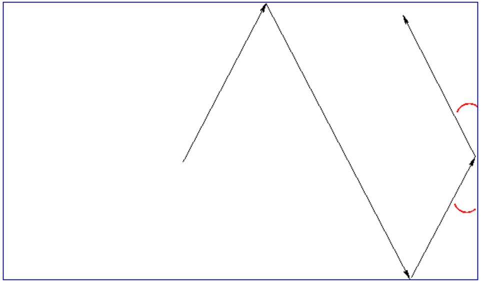

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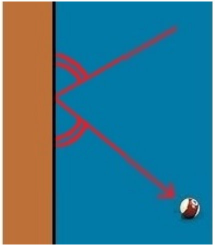

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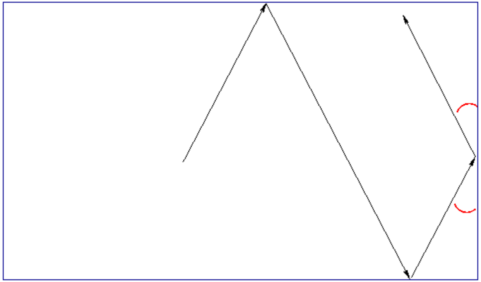

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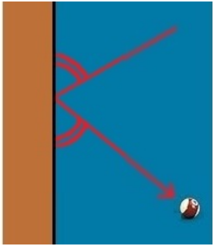

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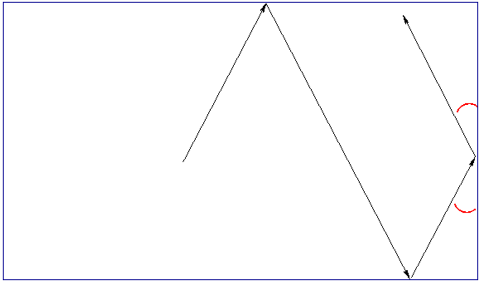

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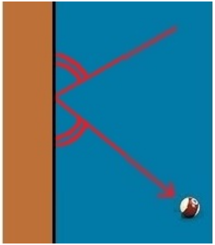

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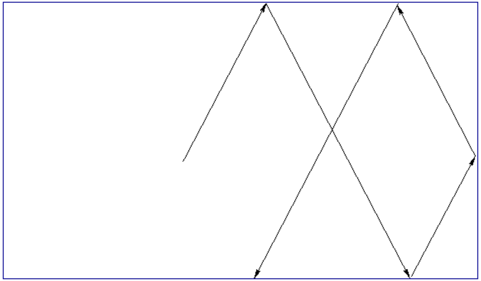

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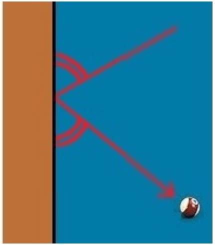

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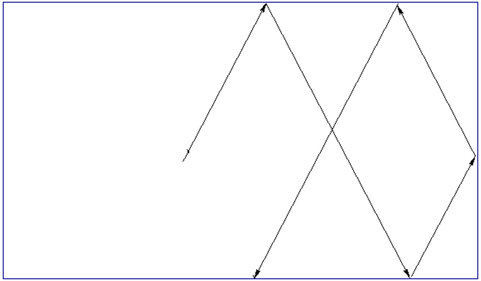

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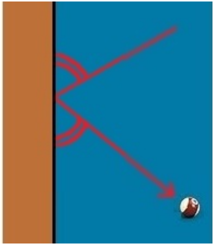

law of optics:

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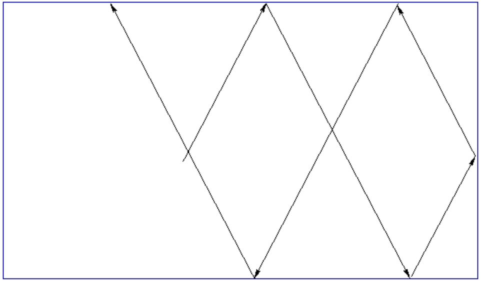
angle of reflection



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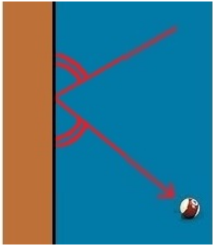

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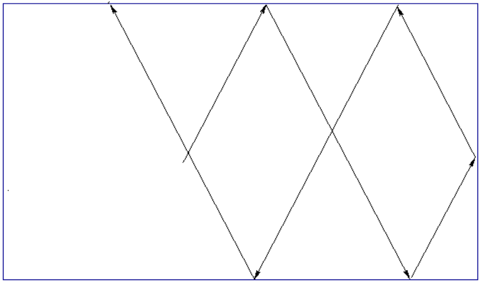

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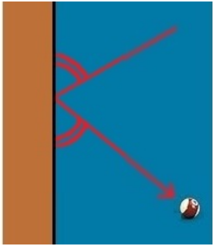

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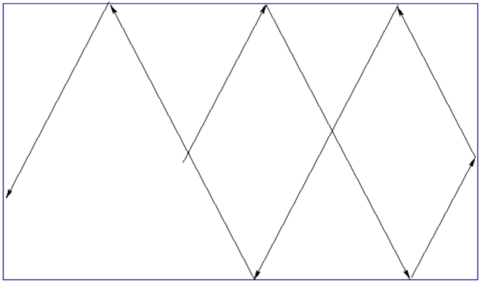

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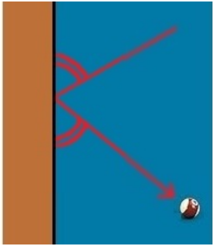

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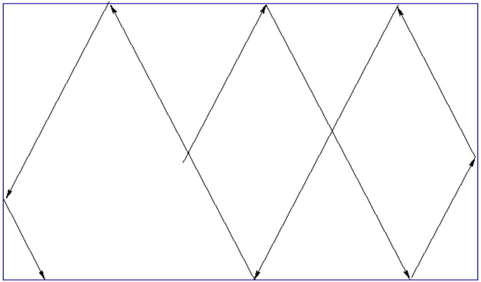

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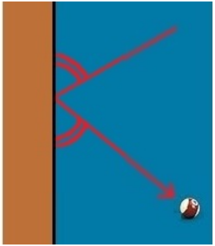

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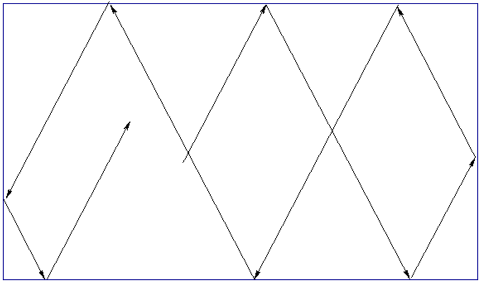

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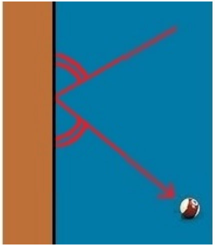

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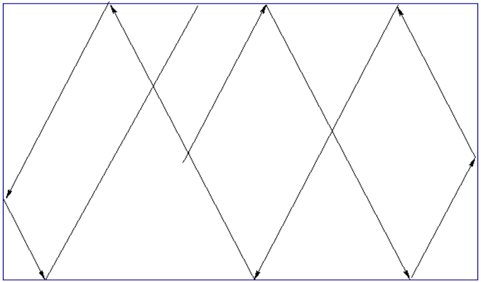

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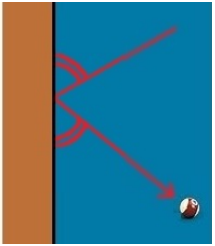

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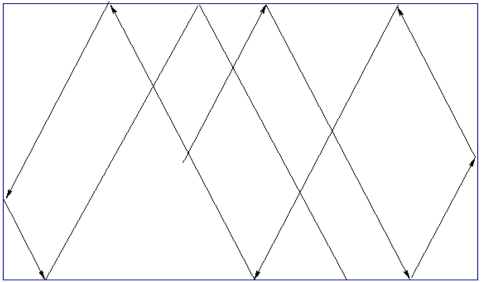

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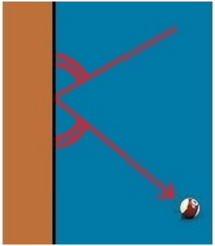

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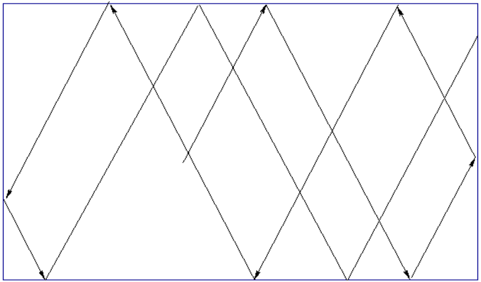

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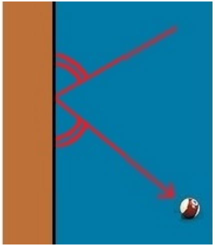

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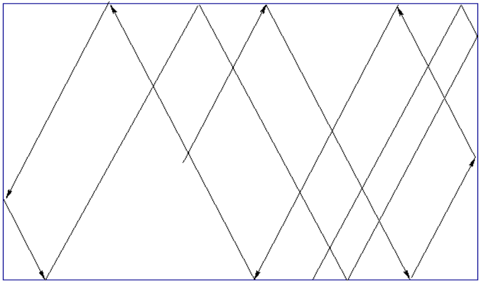

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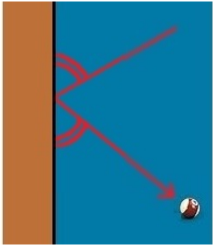

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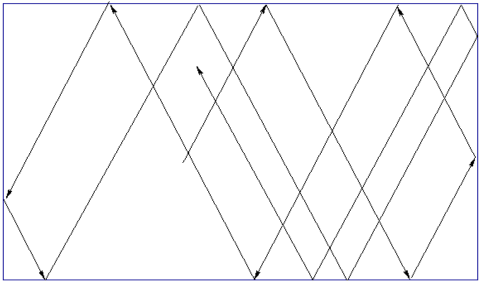

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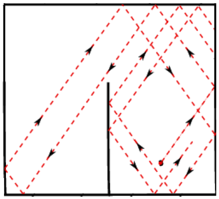

with barrier

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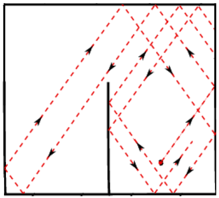

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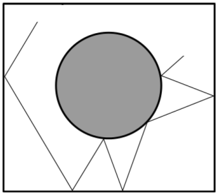

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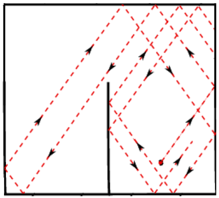

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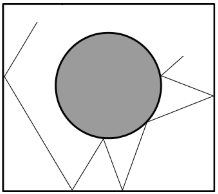

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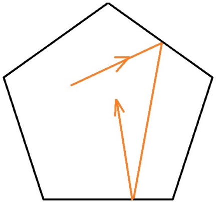

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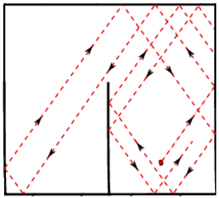

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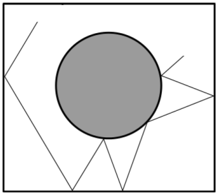

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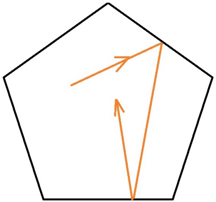

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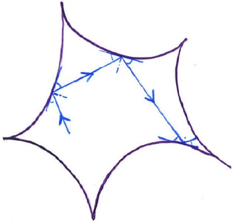

concave

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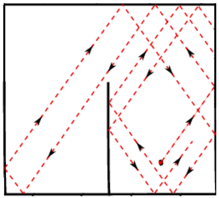

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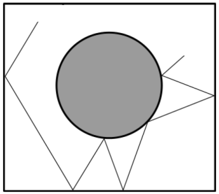

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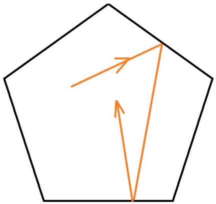

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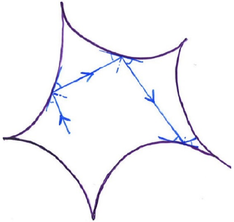

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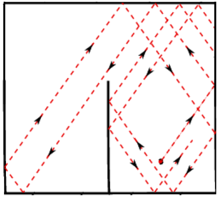

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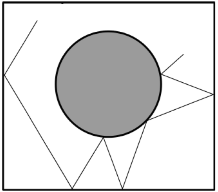

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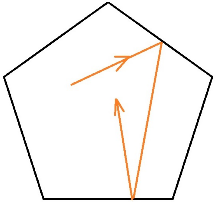

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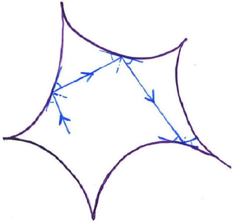

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Motivation to study Mathematical Billiards

Billiards are models of many systems in mechanics, optics, acoustics, termodynamics . . .

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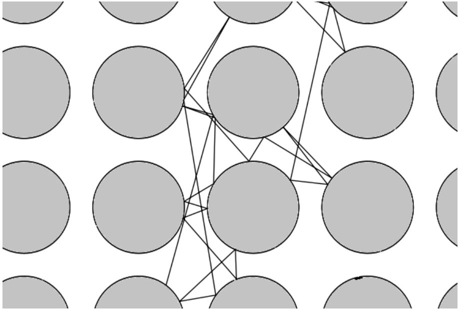
Periodic Lorentz Gas

1905, H. A. Lorentz



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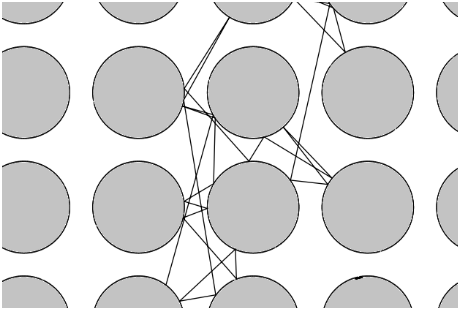
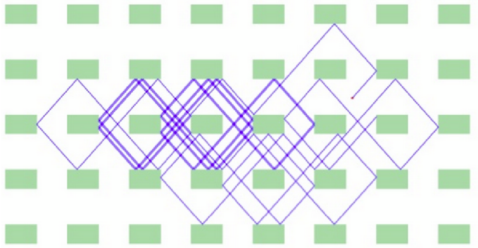
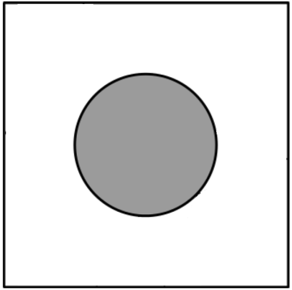


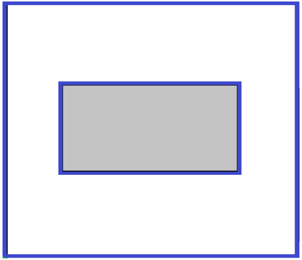
Ehrenfest Model, 1912
Tatjana and Paul Ehrenfest
Periodic version: Hardy-Weber



(image by V. Delecroix)

Sensitive Dependence: circular vs rectangular scatters





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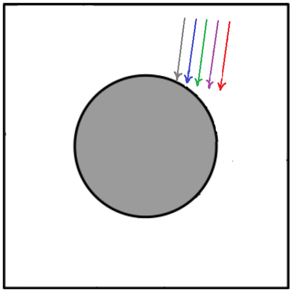

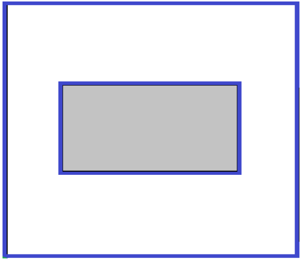

defocusing mechanism:

fast chaos (*hyperbolic* billiard)

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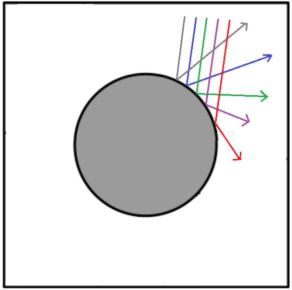

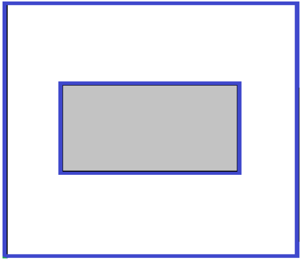

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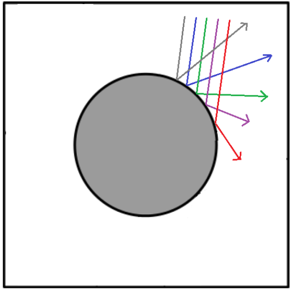

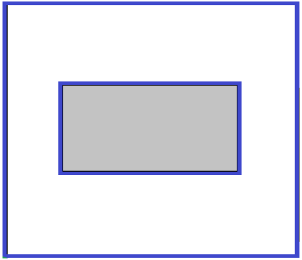

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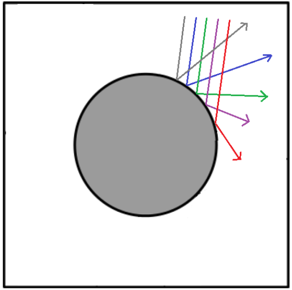

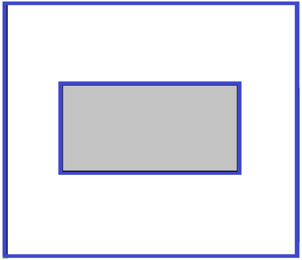

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Abel Prize 2014)

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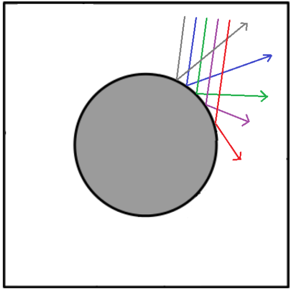

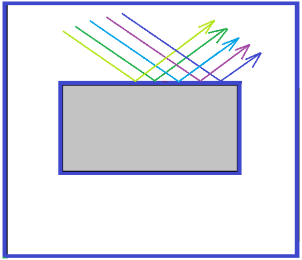

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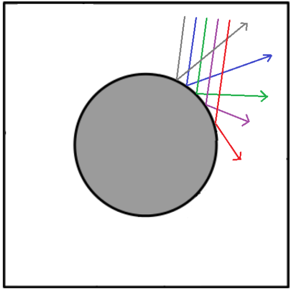

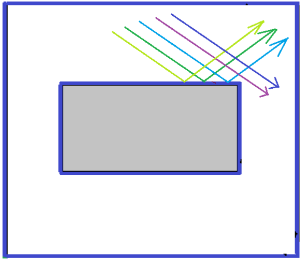

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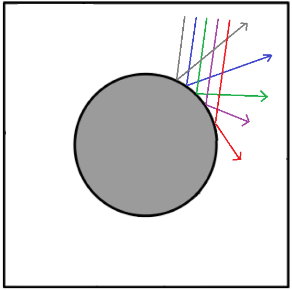

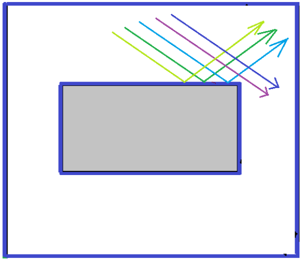

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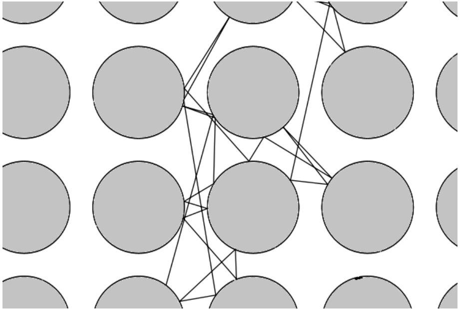
Ehrenfest vs Lorentz: fast vs slow chaos

Periodic Lorentz Gas, 1905



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much studied since the Seventies...

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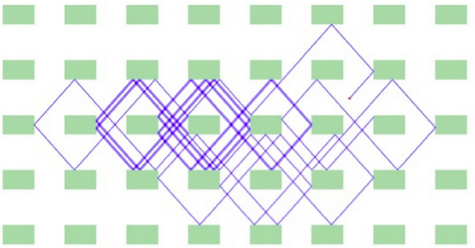

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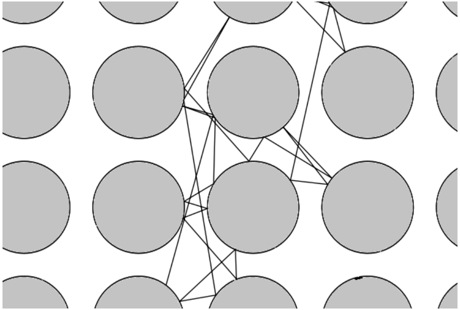

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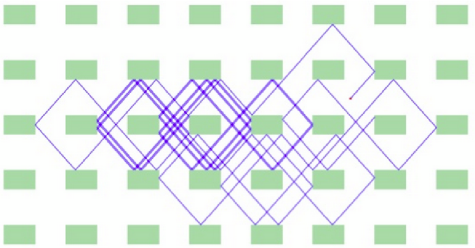

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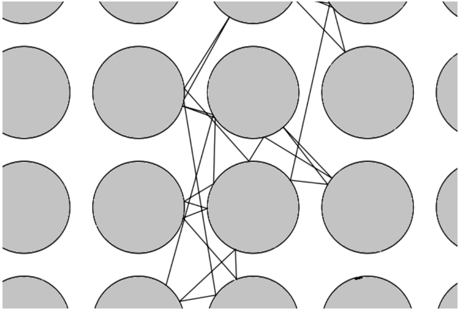

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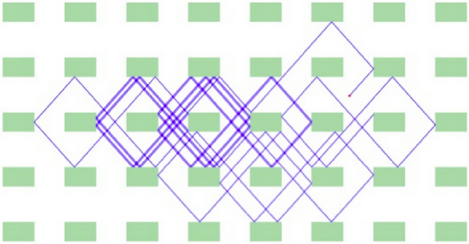

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Ehrenfest vs Lorentz: fast vs slow chaos

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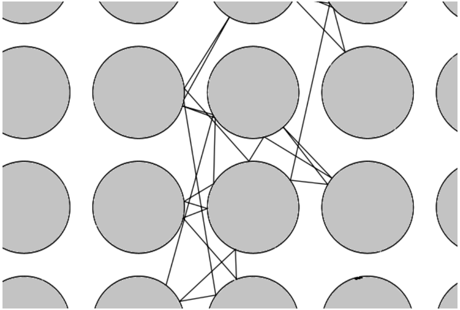

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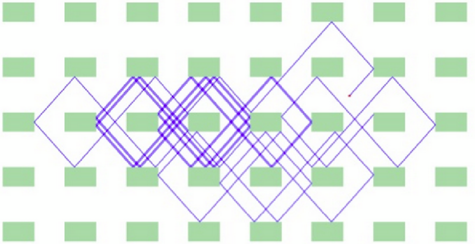

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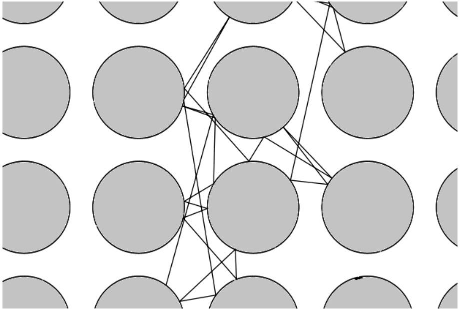

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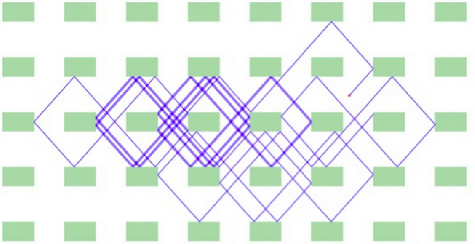

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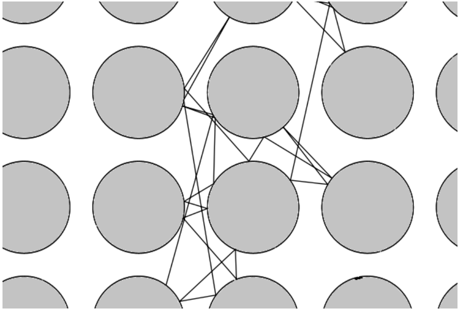

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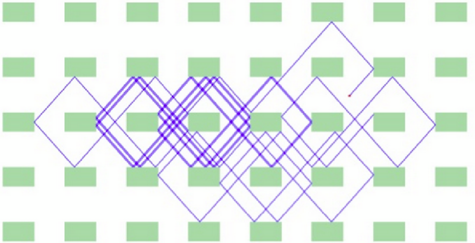

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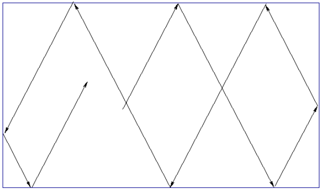

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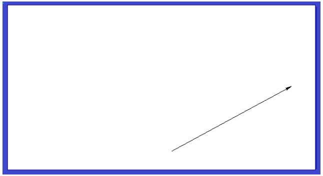

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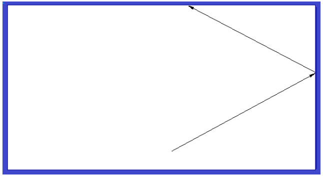

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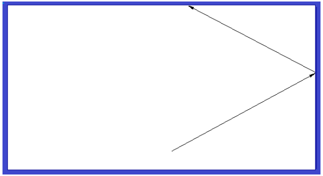

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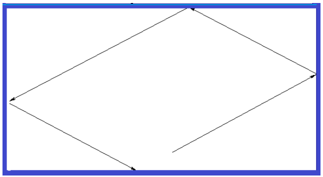

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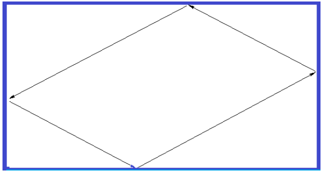

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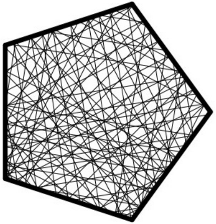

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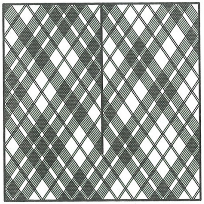

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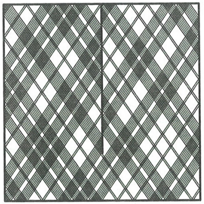

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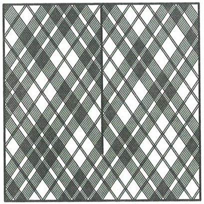

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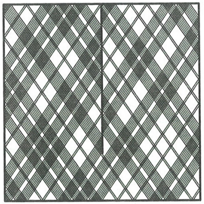

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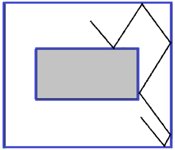

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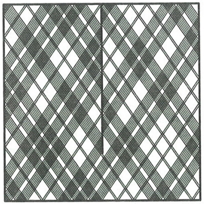

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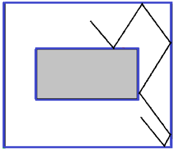

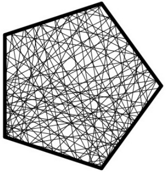

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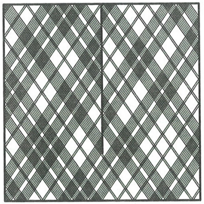

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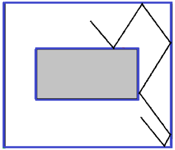

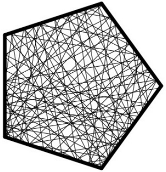

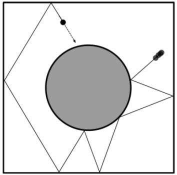

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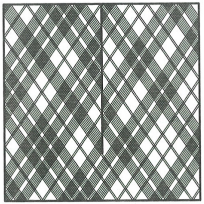

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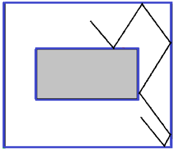
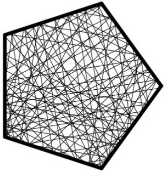
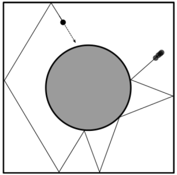
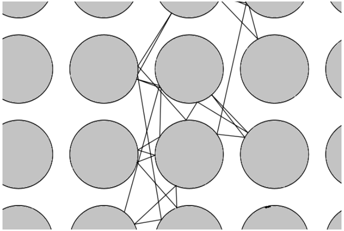

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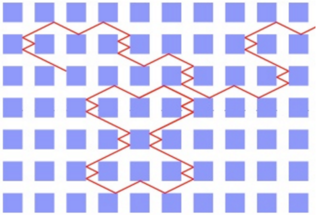

courtesy of V. Delecroix

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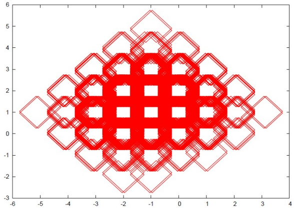

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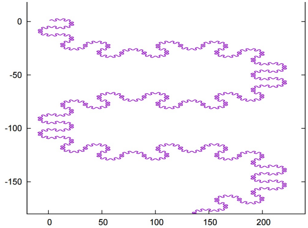

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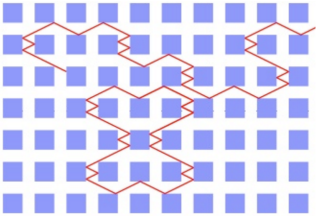

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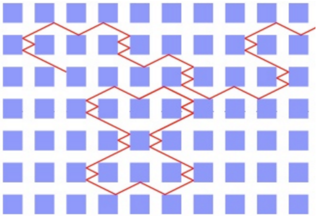

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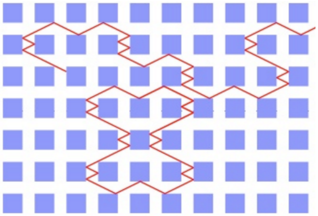

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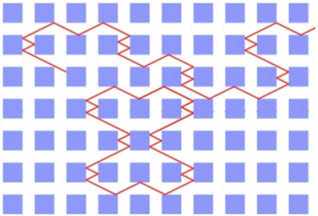

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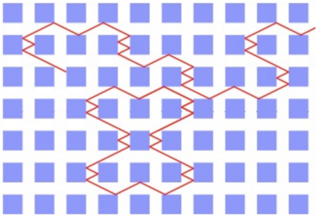

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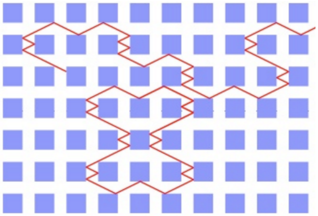

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Pretzels

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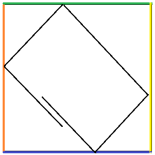

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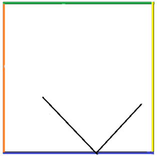

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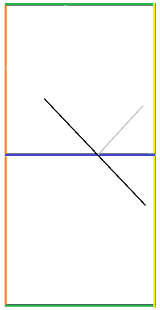

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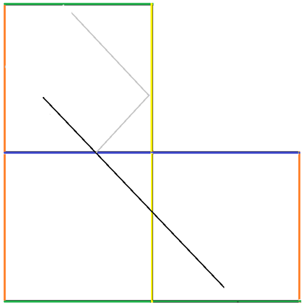

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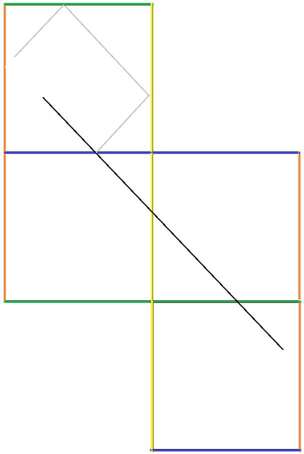

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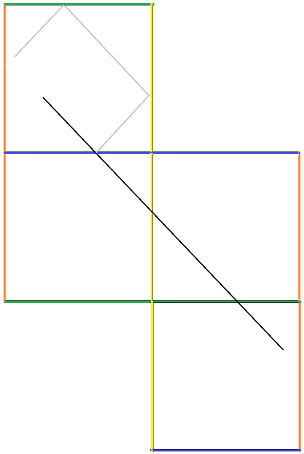

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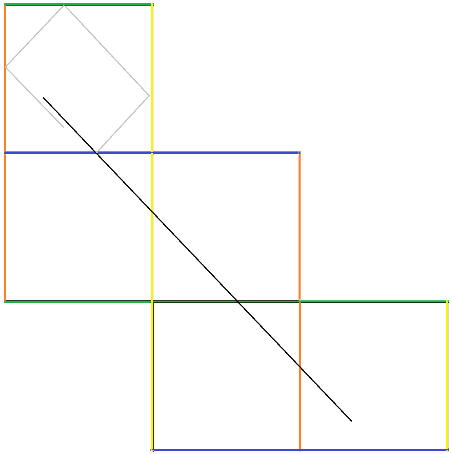

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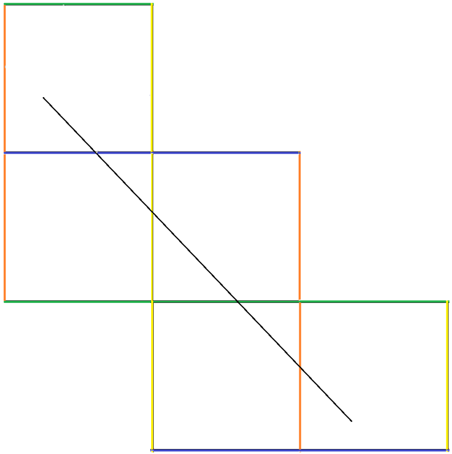

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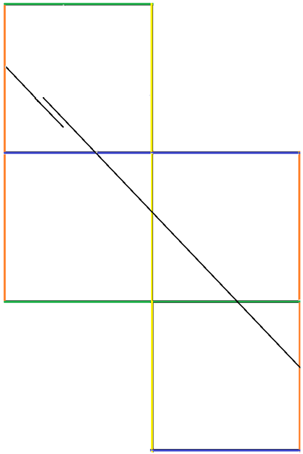

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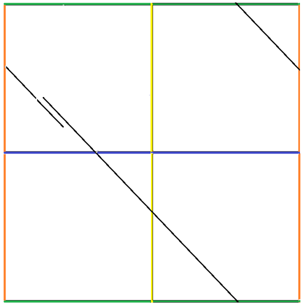

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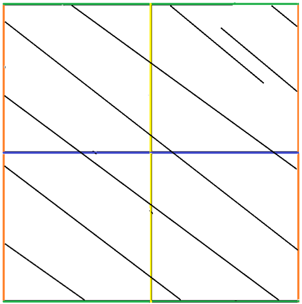

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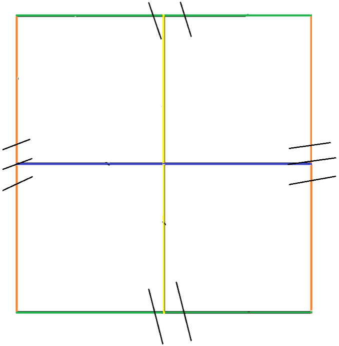

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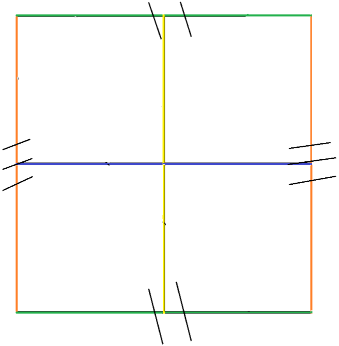

4 copies are enough;
glue opposite sides:

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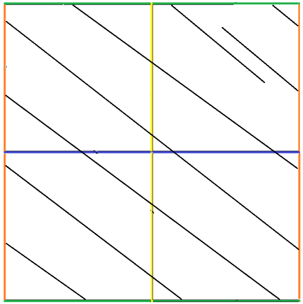

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surface of a bagel!

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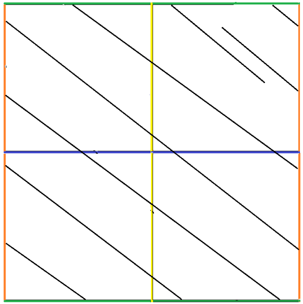

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Gain: one can show trajectories are either closed or dense;

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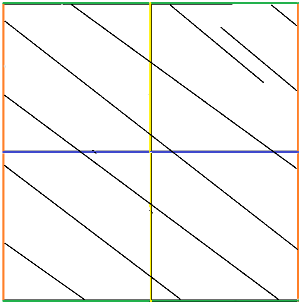

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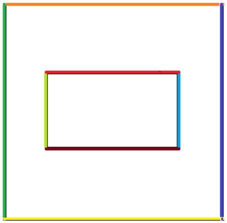

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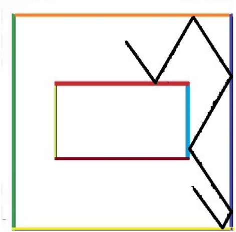

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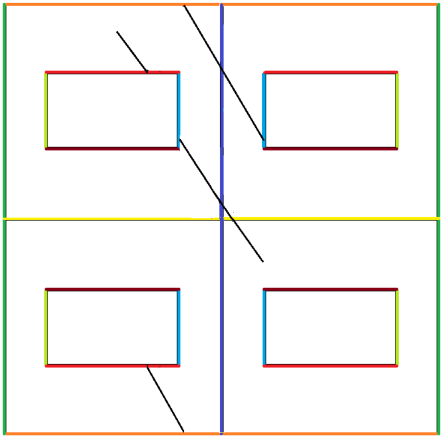

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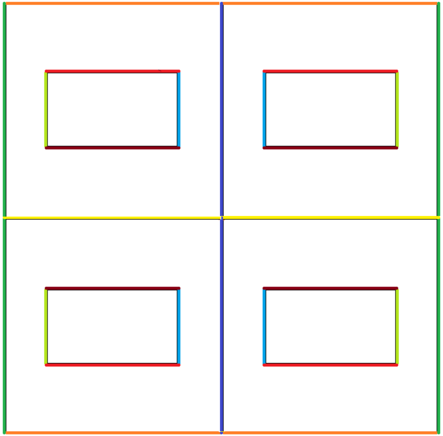

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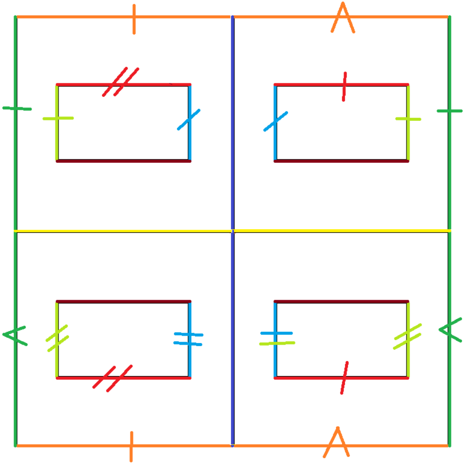

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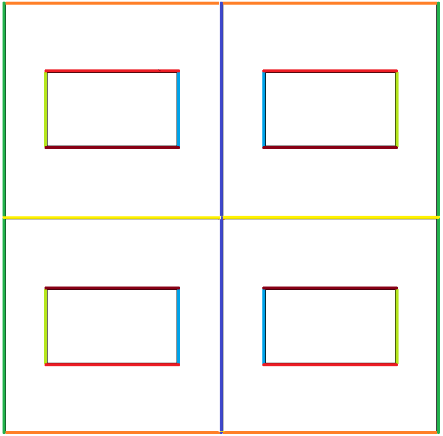

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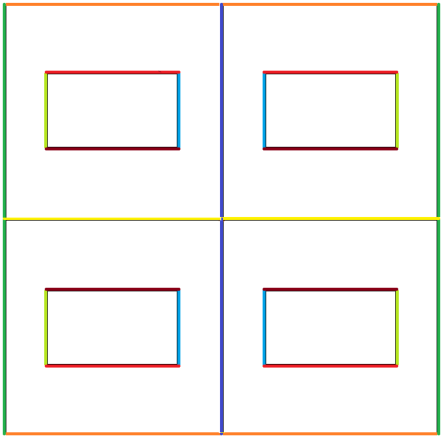

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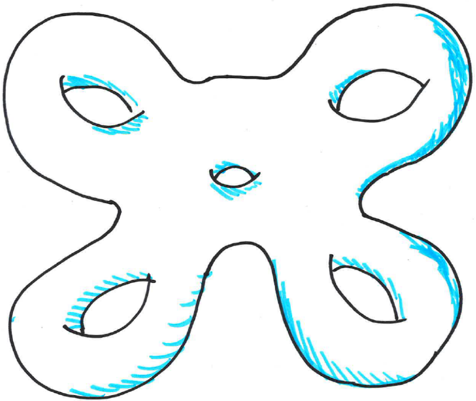

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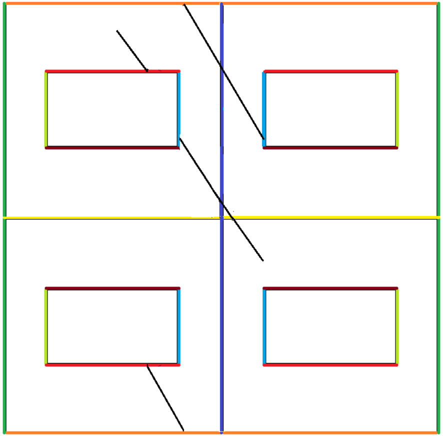

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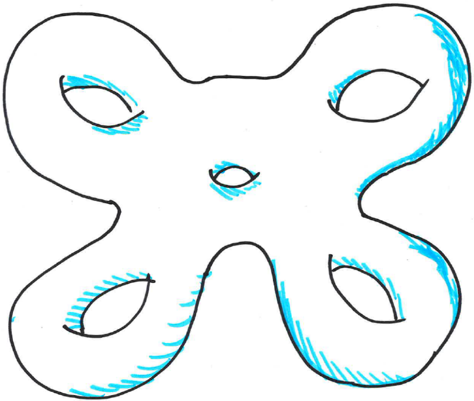

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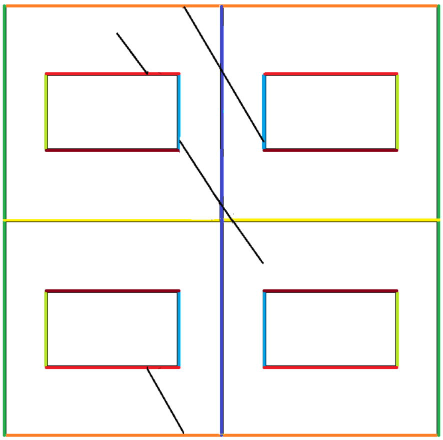

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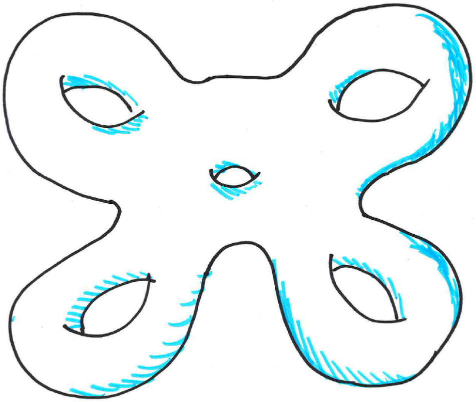

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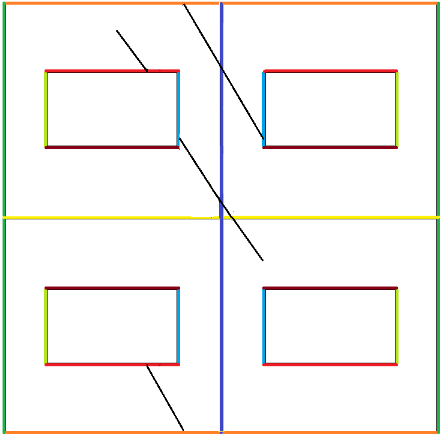

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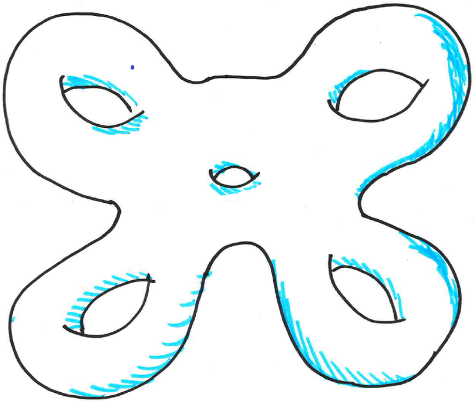

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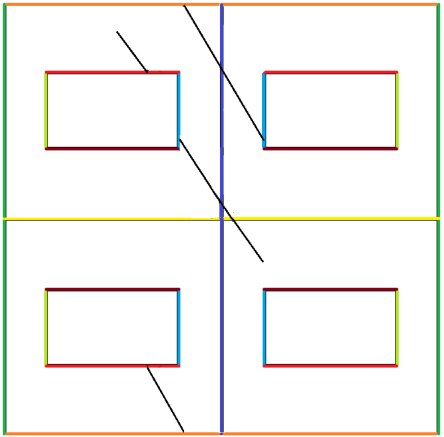

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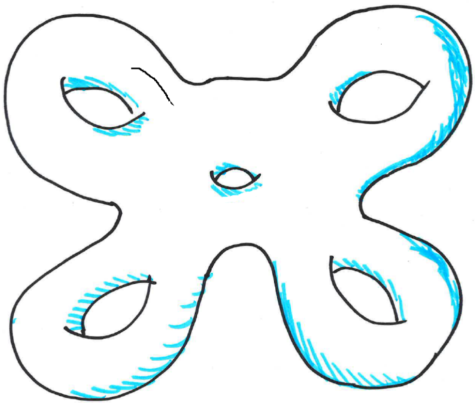

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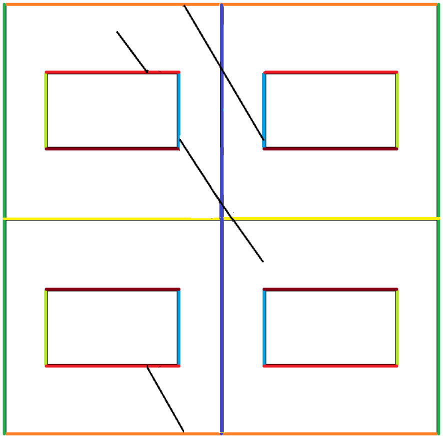

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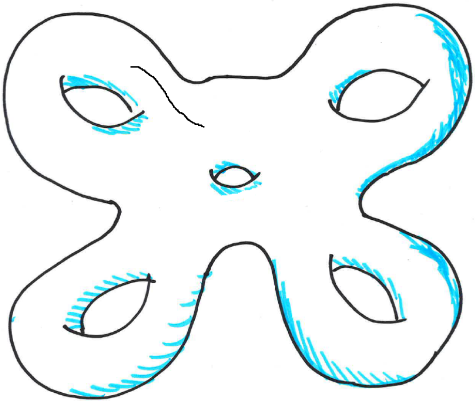

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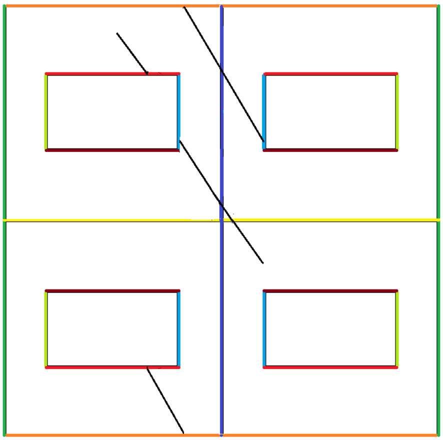

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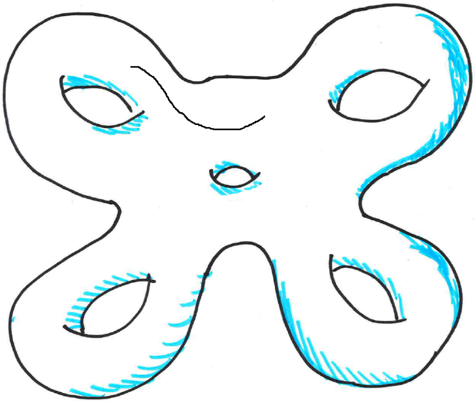

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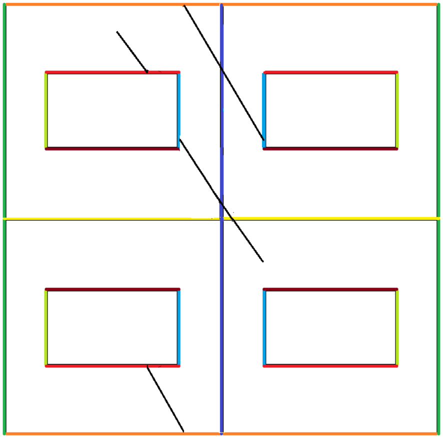

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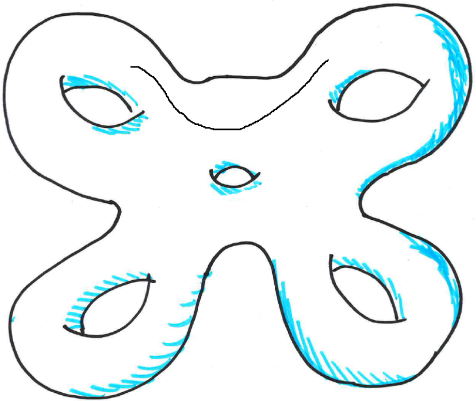

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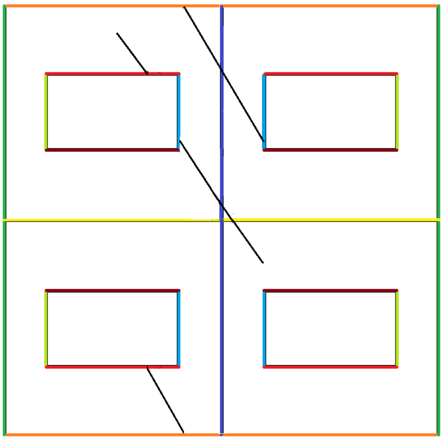

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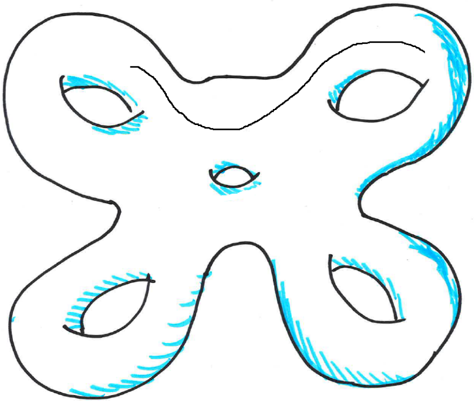

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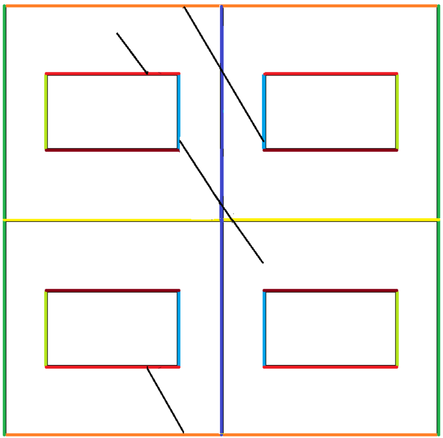

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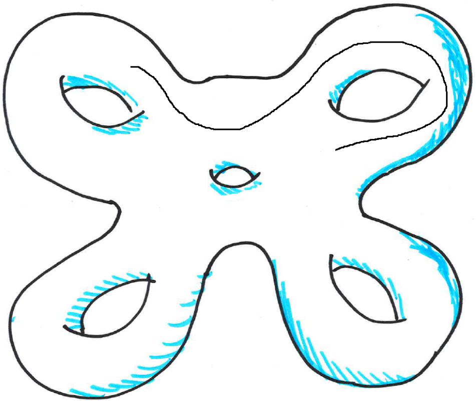

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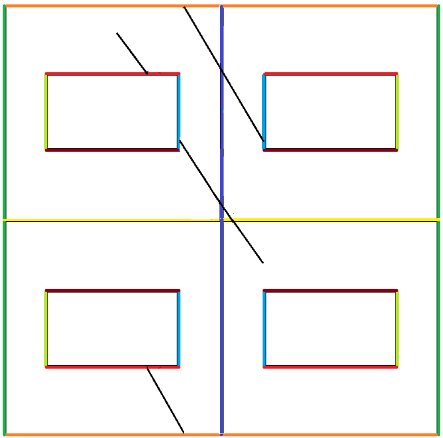

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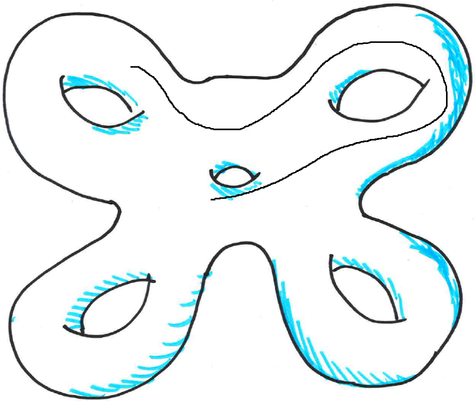

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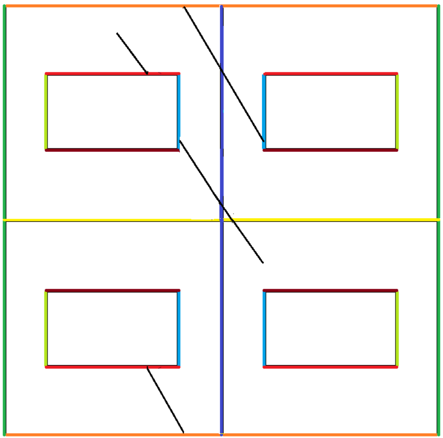

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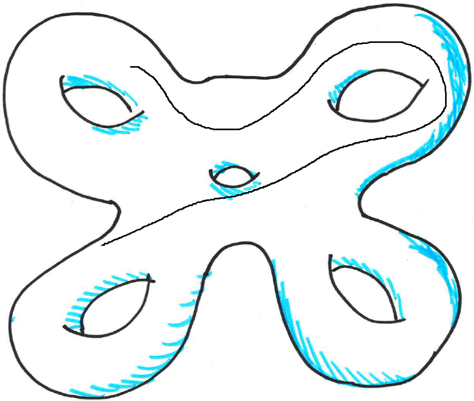

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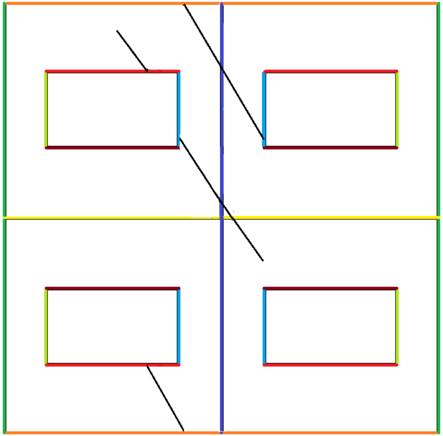

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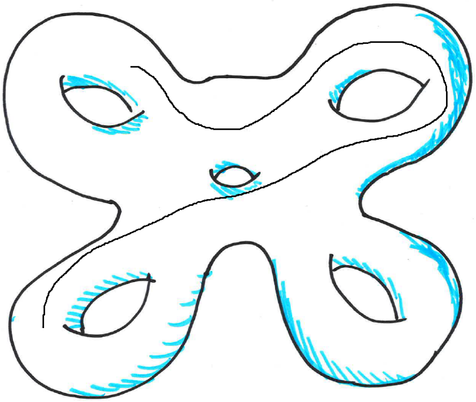

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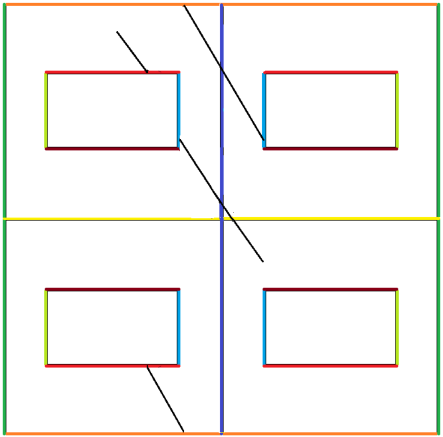

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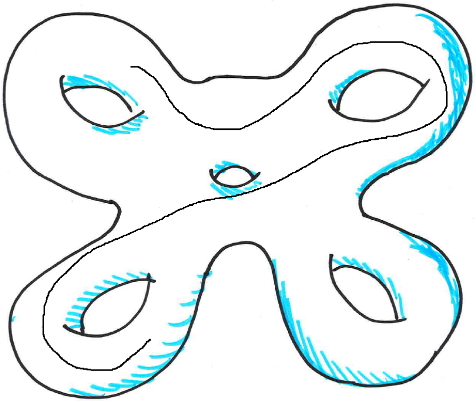

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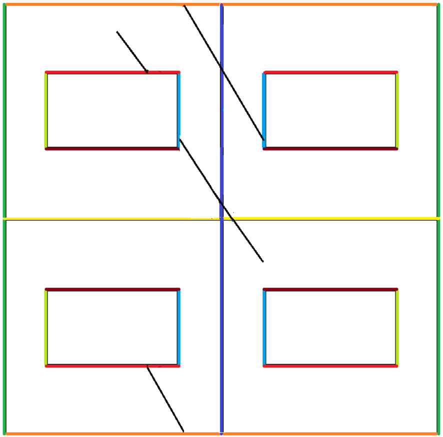

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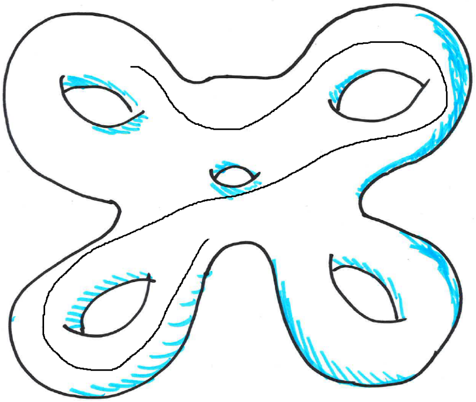

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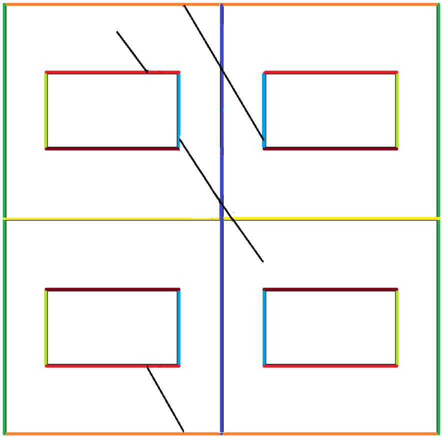

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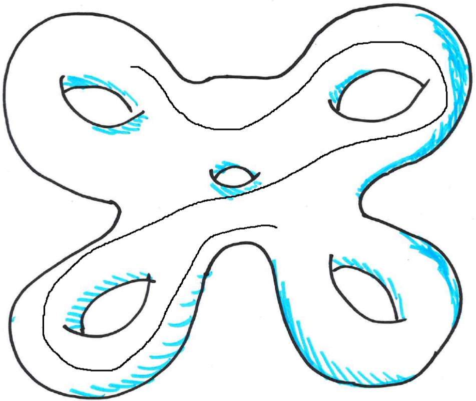

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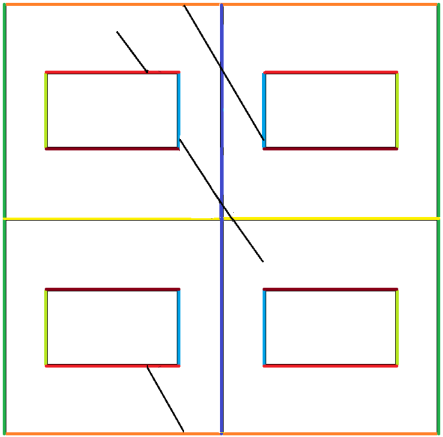

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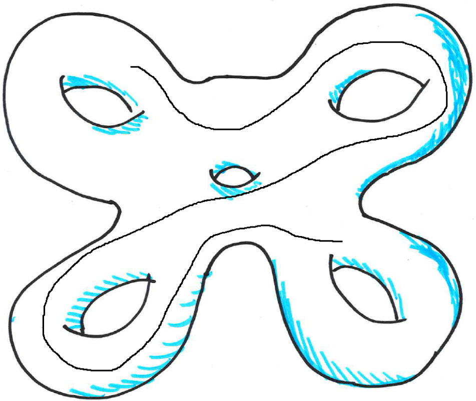

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Surfaces

Unfolding (rational) polygonal billiards one gets surfaces:

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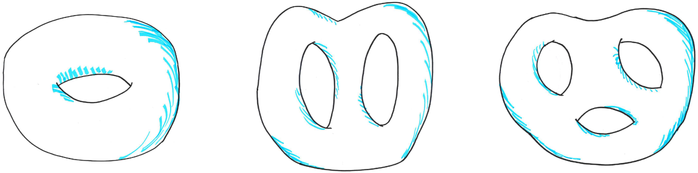

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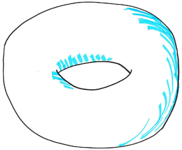
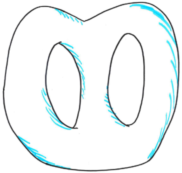
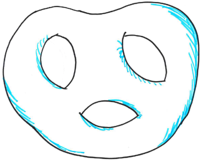
genus 1 genus 2 genus 3 ...

   ...

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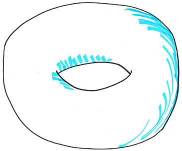

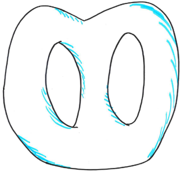

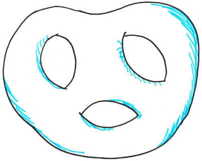

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Pretezels and bagels in the presentation by T. Hansson of the 2016 *Physics Nobel Prize* work by Thouless, Haldane and Kosterlitz

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motion of bodies

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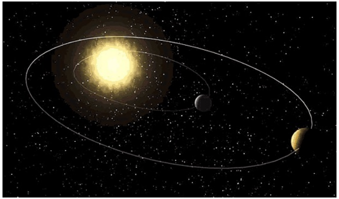

H. Poincaré

1854-1912

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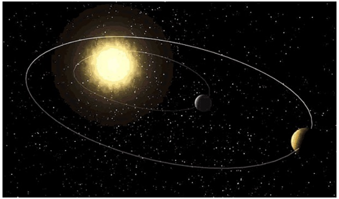
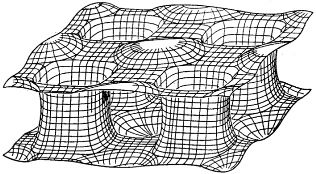


H. Poincaré
1854-1912

electrons in metals in
solid state physics
(Fermi surfaces)



Novikov
model
(1990s)

Mixing and Arnold's conjecture

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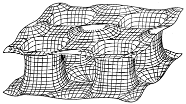

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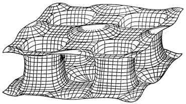

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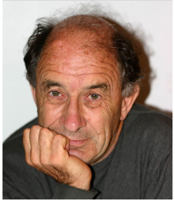

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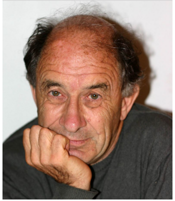

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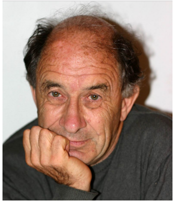

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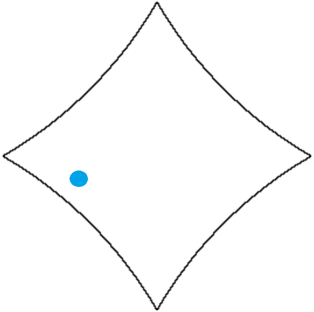

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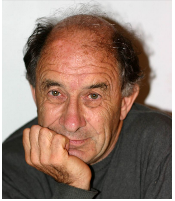

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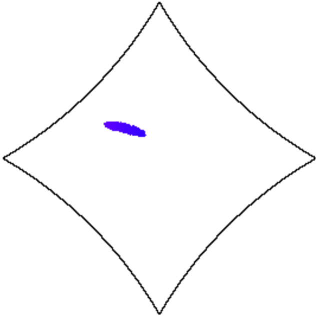

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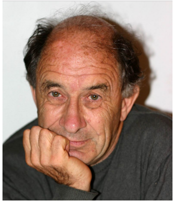

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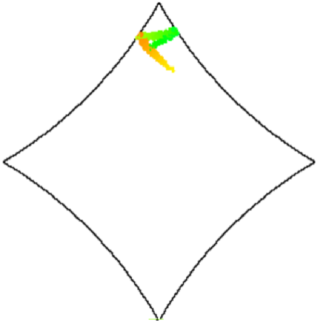

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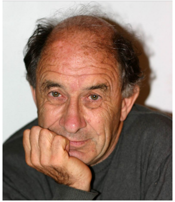

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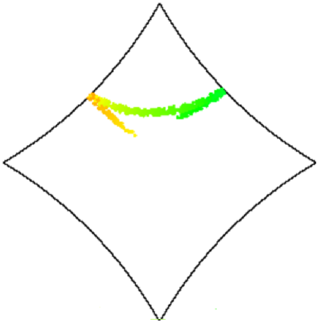

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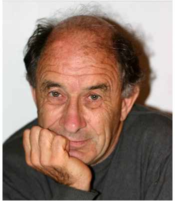

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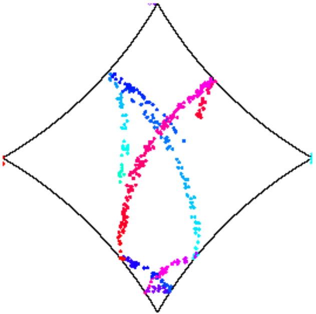

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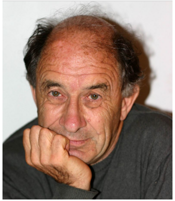

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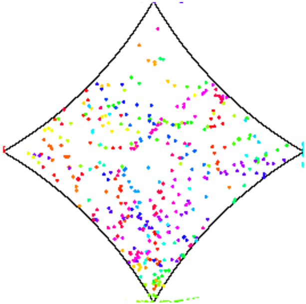

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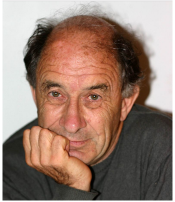

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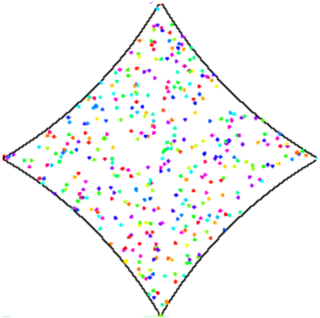

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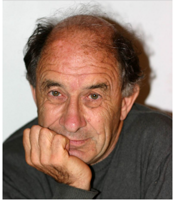

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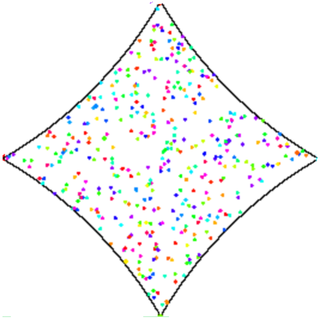

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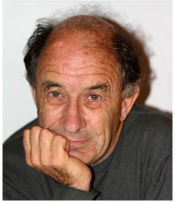

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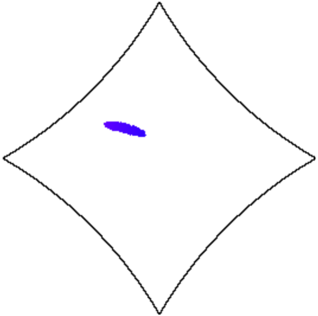

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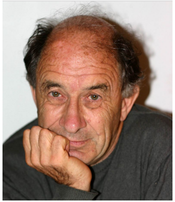

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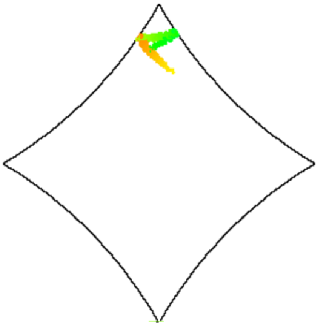

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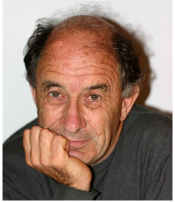

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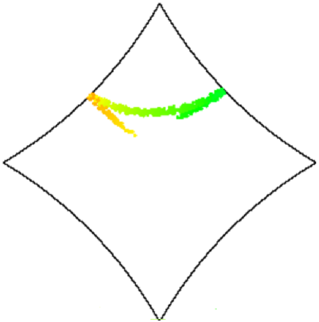

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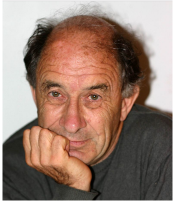

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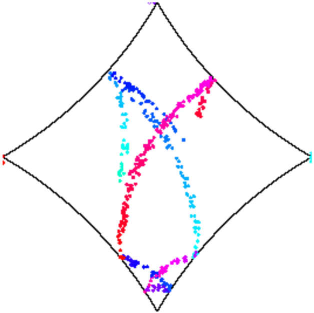

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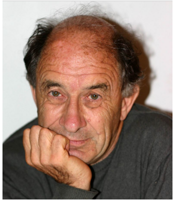

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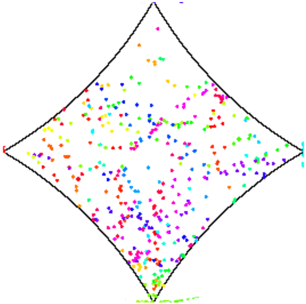

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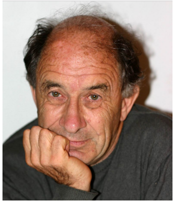

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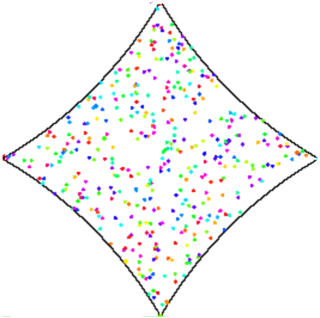

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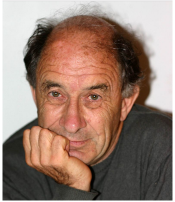

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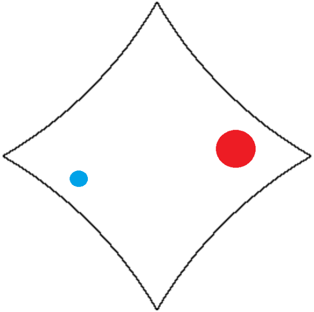

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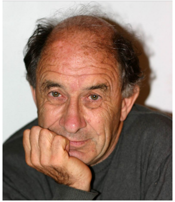

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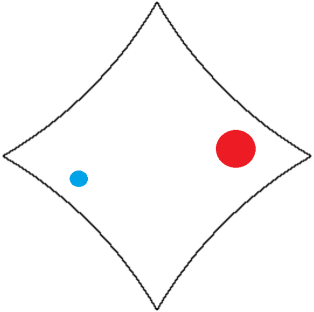

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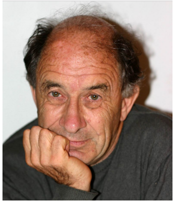

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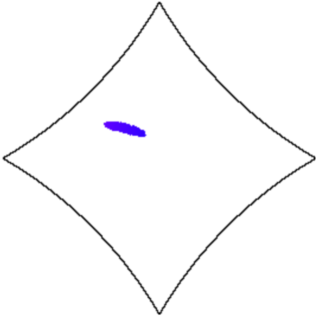

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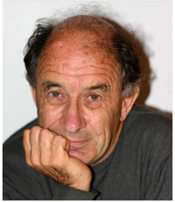

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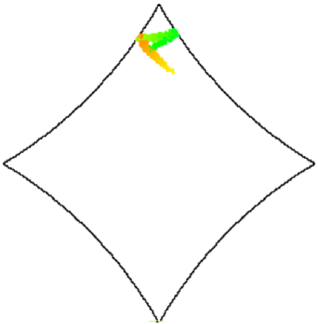

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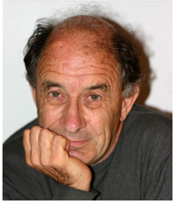

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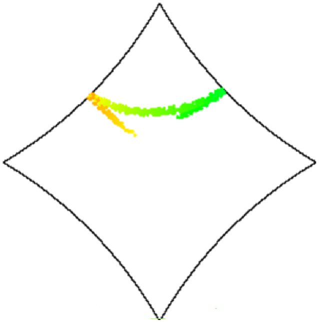

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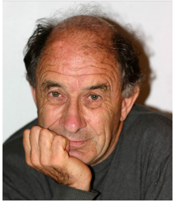

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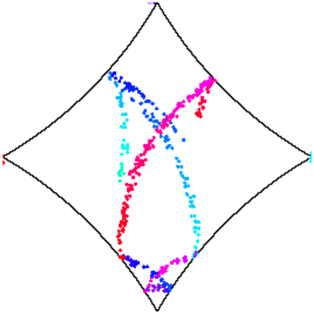

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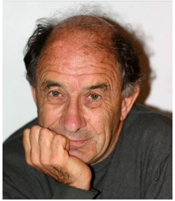

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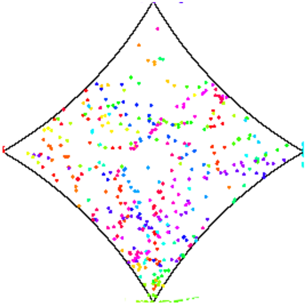

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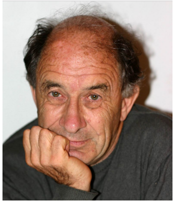

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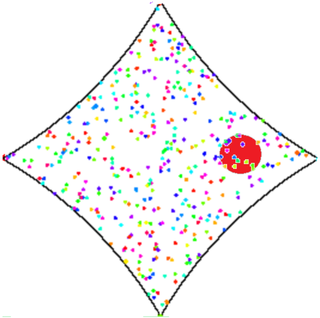

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Answer to Arnold's question on mixing

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- For bagels (*genus one*):
  Yes (Khanin-Sinai, 1992)

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- For bagels (*genus one*):
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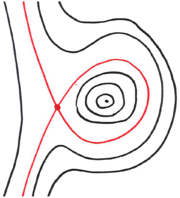

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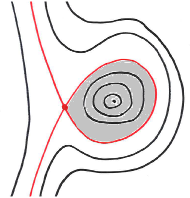

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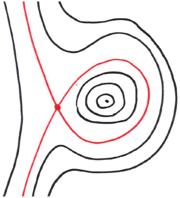

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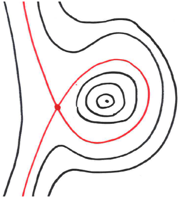

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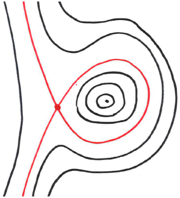

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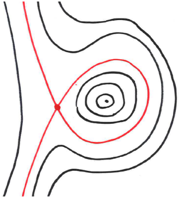

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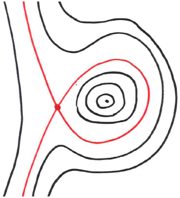

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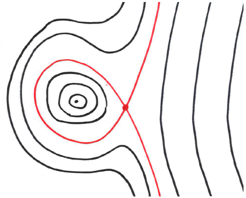

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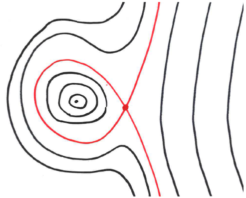

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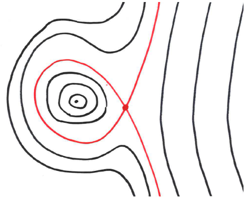

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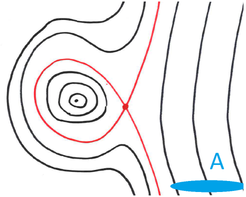

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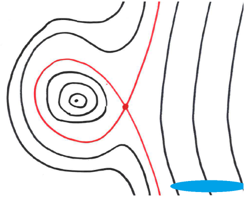

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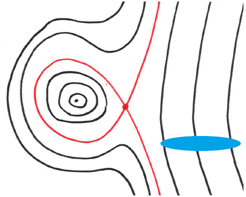

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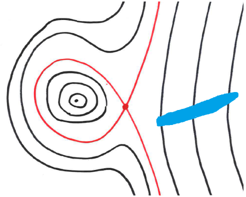

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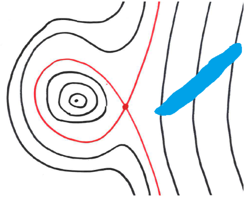

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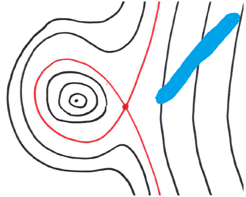

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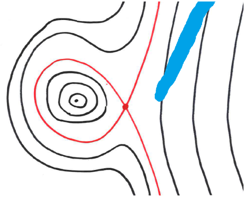

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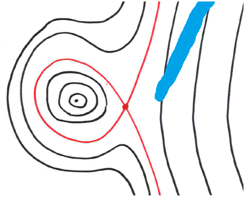

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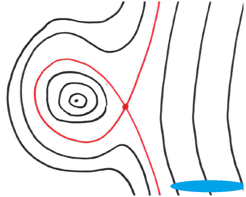

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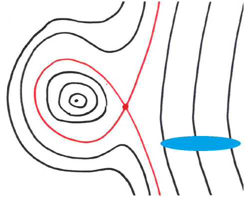

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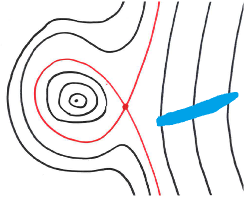

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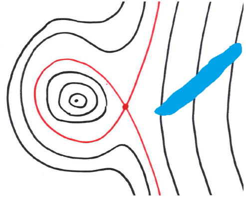

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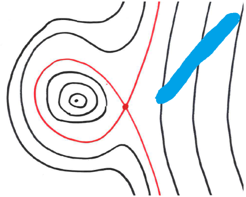

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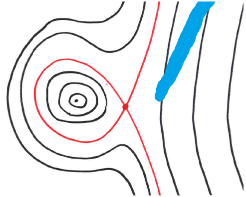

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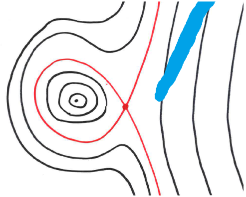

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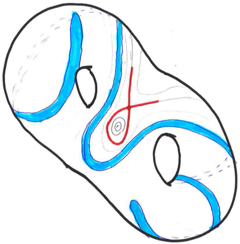

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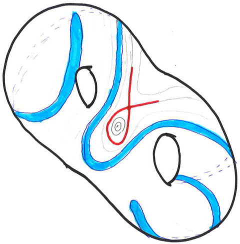

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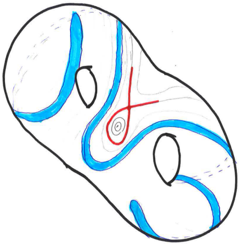

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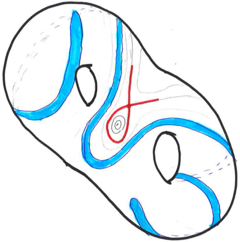

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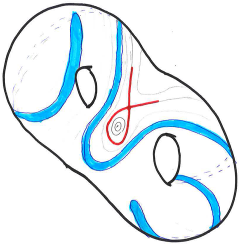

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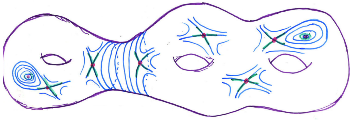

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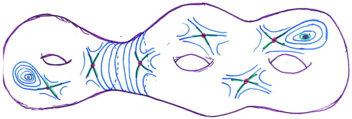

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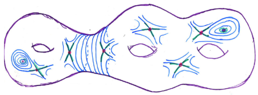

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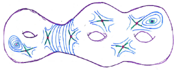

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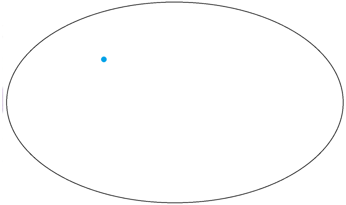

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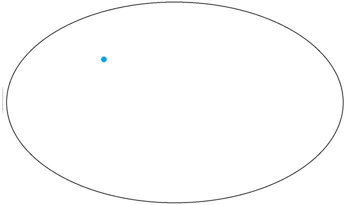

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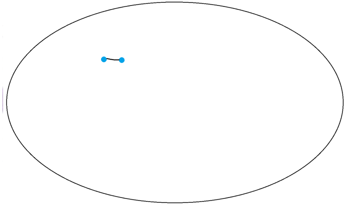

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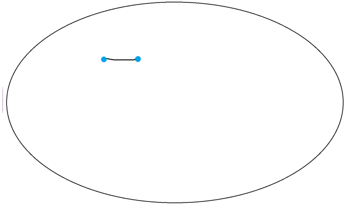

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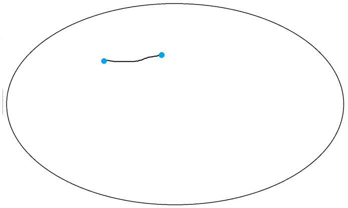

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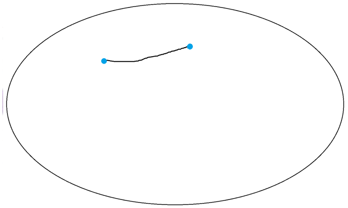

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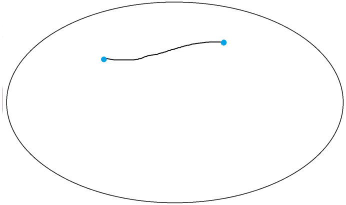

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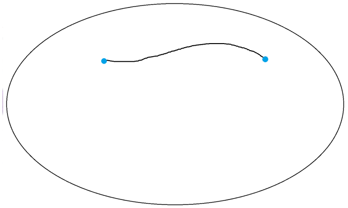

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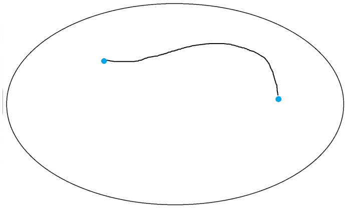

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- space of all systems of the same type (e.g. flows on surfaces):
- system studied (e.g. flow) is a *point*;
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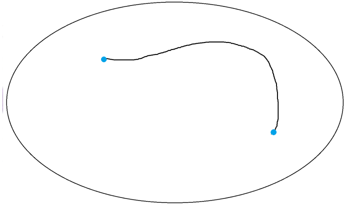

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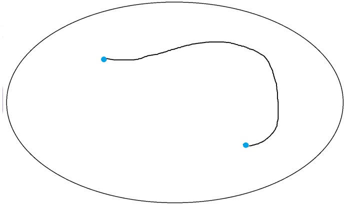

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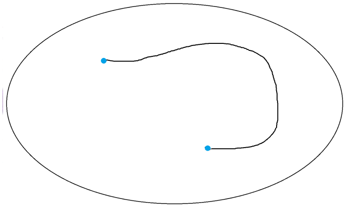

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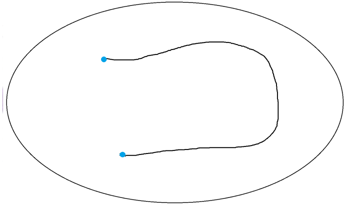

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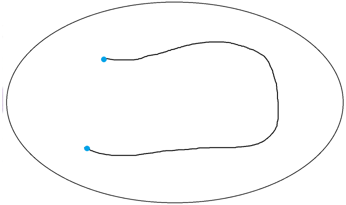

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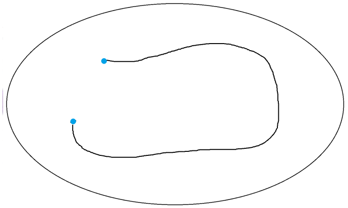

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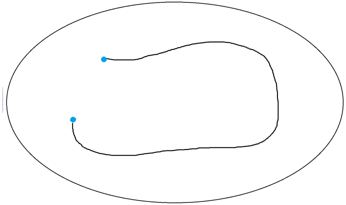

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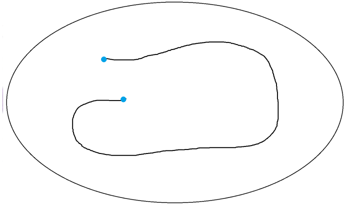

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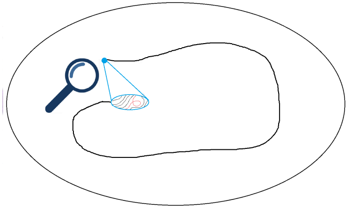

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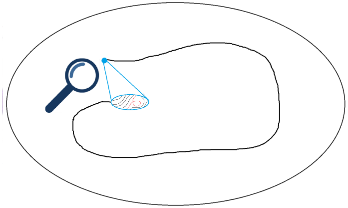

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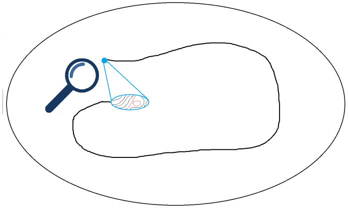

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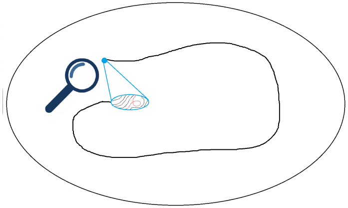

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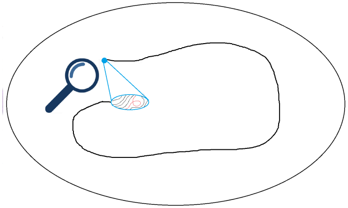

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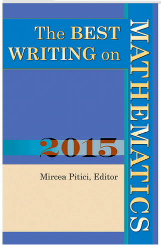

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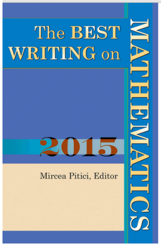

doughnut pool table
by Cleon Daniel