Computational modelling of complex bubble interactions

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Abstract

We consider the direct numerical simulation of gas bubbles in viscous fluid flows. The behavior of both single bubbles and bubble swarms is studied as well as bubble coalescence and breakage phenomena. For single bubbles, we are able to simulate the bubble behavior in all relevant flow regimes, including the von Karman type vortex shedding and bubble wobbling at intermediate and high Reynolds numbers. Various numerical results, obtained by simulations in different flow regimes for, e.g., bubble swarm rise velocity, are combined in statistical regression models which can be compared with the existing correlations available in the literature.

1. Introduction

Multiphase flows are frequently observed in natural phenomena and in various areas of engineering. However, a detailed study of bubbly flows becomes fairly incomplete without gaining an insight into the phenomenon of motion of a single bubble or a small group of bubbles. The computational investigation of this phenomenon constitutes a primary goal of the present work. This is a continuation of our previous studies on the dynamics of a single gas bubble rising in a viscous liquid (see (Smolianski et al.)), and we use the same, finite-element/level-set/operator-splitting method that was proposed in (Smolianski 2001). The numerical method allows to simulate a wide range of flow regimes, accurately capturing the shape of the deforming interface of the bubble and the surface tension effect, while maintaining a good mass conservation. Using the computational method we can provide a systematic study of diverse shape regimes for a single buoyant bubble, recovering all main regimes in a full agreement with available experimental data. In this work, we numerically evaluate the bubble rise velocity for single bubbles and bubble pairs. This can be considered as an extension of the earlier results of (Krishna et al. 1999). We also present the results on bubble swarm velocities.

2. Numerical method

As a simulation tool we employ a general computational strategy proposed in (Smolianski 2001) (see also (Smolianski et al.) and (Smolianski et al. 2003)) that is capable of modelling any kind of two-fluid interfacial flows. Our model is a combination of the Navier-Stokes equations and a free boundary problem for the bubble shape. The bubble boundaries are treated with the level set method, that allows the topology changes typical for the bubble coalescence and breakage. This, in addition to FEM methods implemented in the operator-splitting framework, has proved to allow an accurate prediction of flows with moving interfaces in a wide range of physical parameters.

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3. Bubble rise velocities in different shape regimes

The notion of the terminal velocity of a rising bubble is, to some extent, vague. In reality, the Archimedian force, the drag, the lift and the virtual mass forces exerted on the bubble never balance each other, and, thus, the bubble motion always remains unsteady. However, if the bubble motion is considered in an infinite fluid-medium, after some period of time we may speak of a certain average “rise velocity”, whose change in time can be neglected. The averaging is meant here over a time interval much shorter than the period of time passed since the beginning of the bubble motion. Therefore, under the terminal velocity we understand such a time-averaged (“smoothed”) rise velocity of the bubble.

The terminal velocity of a rising bubble has been experimentally measured by many different investigators. The measured terminal velocities differ widely, mainly due to the differences in system’s purity. For practical predictions, it is useful to have the terminal velocity correlated explicitly in terms of system variables. Clift et al. (Clift et al. 1978) determined the terminal velocity for a rising bubble as $U_T = \frac{\mu}{\rho d_e} M^{-0.149}(J - 0.857)$, where $J = 0.94H^{0.757}$ for $2 < H \leq 59.4$ and $J = 3.42H^{0.441}$ for $H > 59.4$. Here $H = \frac{4}{3} Eo\, M^{-0.149}(\frac{\mu}{\mu_w})^{-0.14}$ and μ_w is the viscosity of water, which may be taken as 0.0009 $kg/m \cdot s$. It is important to note that the density and viscosity of the fluid inside of the bubble are not included in the correlation; ρ and μ denote here the parameters of the surrounding fluid.

A more recent equation for the terminal velocity has been reported by Rodrigue (Rodrigue 2001) as $U_T = V/(\frac{d_e \rho^2}{\sigma\mu})^{1/3}$, where $V = \frac{aF^b}{1+cF^d}$ and $F = g(\frac{d_e^8 \rho^5}{\sigma\mu^4})^{1/3}$ with fitted parameter values $a = 1/12, b = 1, c = 0.049, d = 3/4$.

In (Smolianski et al. 2003) we computed the shapes of a single rising bubble in all main flow regimes, in full agreement with the ’bubble map’ given in (Clift et al. 1978). Here we want to study the single bubble rise velocities in those shape regimes. In Table I we compare the results computed by the above mentioned formulae of Clift et al. and of Rodrigue with the results of our calculations. We computed the bubble rise velocity as the velocity of the top of the bubble.

TABLE I

Terminal velocity for different shape regimes. The comparison with the formulae of Clift et al. and of Rodrigue.

Bubble shape	Clift et. al	Rodrigue	our results
Spherical cap	0.3717	0.5349	0.58
Dimpled ellipsoidal	0.8271	0.9581	0.62
Skirted	0.5996	0.7957	0.71
Spherical	0.3253	0.0806	0.04
Ellipsoidal	0.6064	1.5011	0.42

If compared to Rodrigue’s results, our terminal velocities differ most in ellipsoidal and dimpled ellipsoidal shape regimes, where it is experimentally difficult to find accurate predictions. Otherwise, the results seem to agree reasonably well. In all but one (spherical) case the values of Clift et al. are also rather close to ours. It is worth noticing again that in the formulae of Clift et al. and of Rodrigue the dependence of the terminal velocity on the viscosity/density of the internal fluid is neglected; this is not the case in our simulations.

4. Bubble rising in high Reynolds number regime and shedding vortices

A numerical experiment with a vortex shedding phenomenon is shown in Figure 1. First, one can see the bubble forming a skirted shape. Later, the skirt breaks off due to the action of the vortices in the bubble wake, and the remaining part of the bubble rapidly develops a spherical-cap shape. This qualitative picture of the bubble break-up is in a full agreement with experimental observations of (Hnat & Buckmaster 1976).

After the break-up, the bubble starts to rise rather steadily, forming a large wake behind the bottom. While the wake grows, the bubble starts to 'wobble' from side to side. The wobbling motion might be explained by the asynchronous separation of the vortices from the bubble sides, which ultimately leads to the wake instability. To the best of our knowledge, this kind of nonsymmetric vortex shedding has not been simulated by other groups.

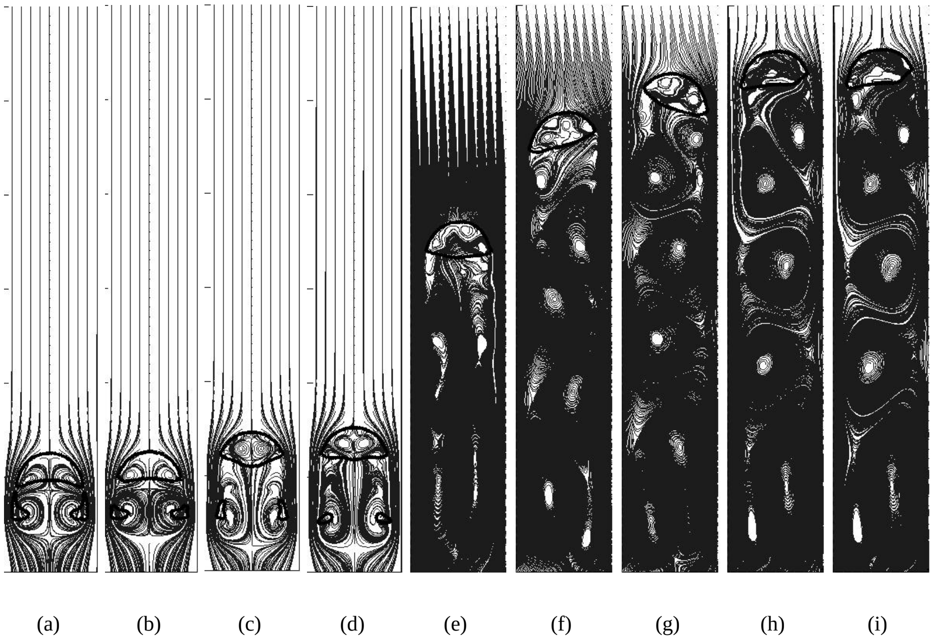


Figure 1. Bubble break-up and nonsymmetric wake forming behind the bubble.$Re = 316$, $Eo = 10$, $M = 10e-8$, $\rho_1/\rho_2 = 10^3$, $\mu_1/\mu_2 = 100$, $h = 1/80$.

5. Two rising bubbles in different shape regimes

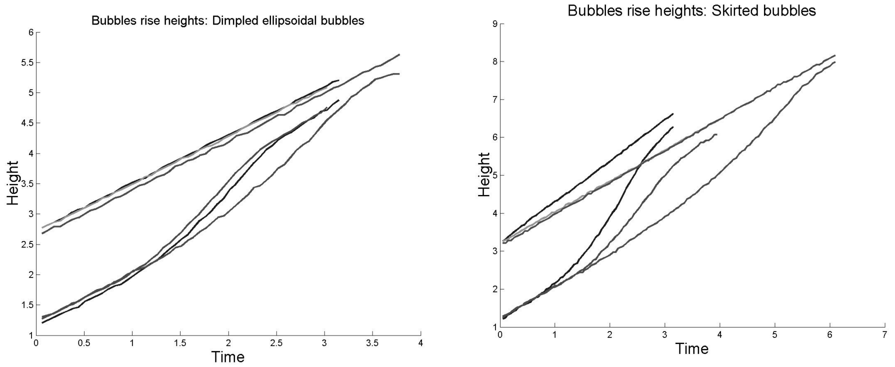
Here we study the rise of bubble pairs. We consider three different cases. The radii of the bubbles R_u and R_l are either equal or have a ratio 1.5. In the first case the upper bubble radius R_u is larger, in the second equal and in the third case is smaller than the lower bubble radius R_l.

Figure 2a presents the rise heights of dimpled ellipsoidal bubbles in all three cases. After a rapid initial acceleration (the bubbles are supposed to have zero initial velocity in the computations), the bubbles reach an almost steady rise velocity, but, later, the lower bubble starts rising faster. After the merger, the newly formed bubble rises slower than any of the two bubbles did. In cases $R_u > R_l$ and $R_u = R_l$ the rise is slightly faster than in the case $R_u < R_l$.

Rise heights for skirted bubbles are shown in Figure 2b. In all cases one can clearly see how the lower bubble starts to rise faster when it catches up with the upper bubble wake. We can also observe that the rise is faster in the case when the lower bubble is smaller. In the dimpled ellipsoidal shape regime the wake is not that strong and such kind of behavior was not observed.

6. Bubble swarm in high-Morton regime

As an example of more complex bubble dynamics, we consider the array consisting of nine bubbles. In the initial setting, the lower and middle 'rows' were composed of three bubbles side



(a)

(b)

Figure 2. Rise heights for two bubbles in dimpled ellipsoidal(a) and skirted(b) bubble shape regimes

to side, in the upper 'row' one bubble was placed slightly above others to break the symmetry. All bubbles were of equal size, the relative initial radius of the bubbles with respect to the width of the computational domain was 0.25.

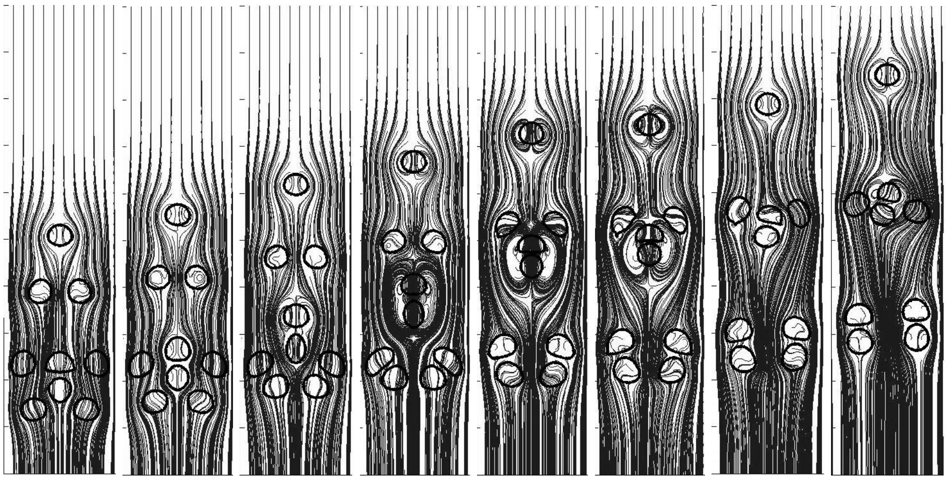
In the simulations with high Morton number regime, where bubbles do not coalesce, we used slightly different values than in the previous calculations. The density ratio of gas and liguid was the same (1000) but viscosity ratio was $\mu_1/\mu_2 = 20$. Morton, Eötvös and Reynolds numbers were given as $M = 0.25$, $Eo = 0.61$ and $Re = 4.59$. As seen from the values, we are clearly in non-coalescing regime. The transition from "coalescing" to "non-coalescing" corresponds to the difference between "low-M" and "high-M" liquids and takes place at the value $M \approx 4 \times 10^{-4}$ (Stewart 1995).

As seen in Figure 3, the bubbles tend to form clusters but they do not merge. The bubbles at the center of the computational domain start rising much faster than the bubbles near the boundaries. The uppermost bubble at the center rises faster than others, leaving them behind. Also, other two center bubbles form a cluster and reach the upper bubbles near the boundaries rather fast. Bubbles in clusters of two seem first to get closer, then again take distance from each other. This kind of back and forth motion was observed with several two-bubble clusters. When a two-bubble cluster reaches another one, they form together a four-bubble cluster and start to rise as a whole.

Bubble rise velocities are given in Figure 4. The average rise velocity of all bubbles is indicated with a thick line. As seen from the figure, just before the 'collision', the bubbles accelerate due to bubble interactions. Further, the cluster of two bubbles in the middle rises much faster than any other bubbles. This clearly demonstrates the fact that bubbles rise faster in clusters. Also, the upper bubbles attract the lower ones, making them rise faster. No significant wake was observed in this case. In Figure 4 we have fitted the average bubble swarm velocity to an exponential type curve to be able to quantify the bubble swarm terminal velocity in this flow regime (the terminal velocity is approximately equal to 0.44).

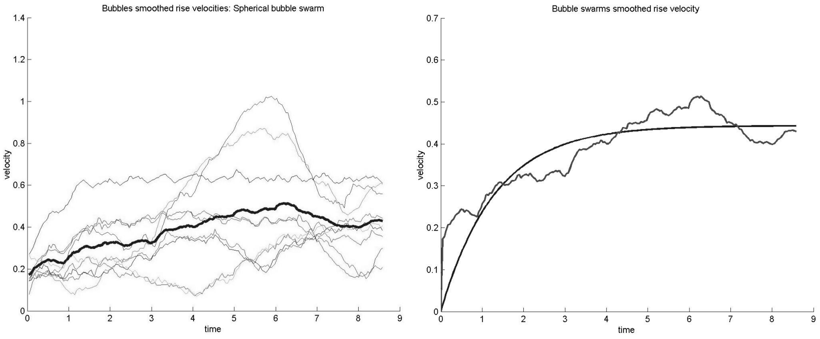
7. Discussion

We have presented the results of a computational study on two-dimensional bubble dynamics. Despite the seeming insufficiency of a two-dimensional model for the quantitative analysis of three-dimensional bubble evolution phenomena, we have been able to obtain a good qualitative agreement with the available experimental data. With our numerical method we managed to



(a) (b) (c) (d) (e) (f) (g) (h)

Figure 3. Bubble swarm in high Morton number regime ($M = 0.25$*);* $Re = 4.59$, $Eo = 0.61$, $\rho_1/\rho_2 = 10^3$, $\mu_1/\mu_2 = 10^2$, $h = 1/40$.



(a) (b)

Figure 4. a) Bubble swarms rise velocities in high Morton number regime; thick line is average rise velocity; b) Bubble swarms smoothed rise velocity in high Morton number regime; $M = 0.25$, $Re = 4.59$, $Eo = 0.61$,$\rho_1/\rho_2 = 10^3$, $\mu_1/\mu_2 = 10^2$, $h = 1/40$.

simulate unsymmetric vortex shedding, bubble coalescense and break-up and, also, bubble swarms rising without coalescense. In addition, we computed bubbles rise velocities in all the cases. A more thorough study of bubble swarms is a topic of our forthcoming paper.

In many cases, a good quantitative agreement with the experimental results has been observed (see (Smolianski et al.) for a detailed comparison of our computational results with available experimental data). This, probably, means that a two-dimensional modelling of bubble dynamics is not so far from being realistic.

The preliminary study on the bubble coalescence phenomena also shows that plausible results can be obtained already with two-dimensional simulations.

Acknowledgments

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