MAT 636 ODES AND FOURIER SERIES
SOLUTIONS TO HOMEWORK 9

Problem 1. *Let* $\{\phi_j\}$ *be an orthonormal system in a Hilbert space* H*. Prove that* $\{\phi_j\}$ *is a complete orthonormal system if and only if the only vector* $u \in H$ *for which* $\langle u, \phi_j \rangle = 0$ *for all* j *is* $u = 0$*.*

Note added later: Here $\{\phi_j\}$ is to be taken as a countable set. However, it is an interesting fact that the statement remains true if the orthonormal system, $\{\phi_\alpha\}$ say, is uncountable. I've added some words about proving this more general claim at the end of the solution posted on piazza.

Problem 2. *Let* f *of period* 1 *be an element of* $L^1[0,1)$ *and differentiable such that* $f' \in L^2[0,1)$*. Prove that*

$$\sum_{n\in\mathbb{Z}} |\hat{f}(n)| \leq \|f\|_1 + \Big(\frac{1}{2\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\Big)^{1/2}\|f'\|_2.$$