Homework 5

Please turn these in on Monday, Nov. 23 (in class or in the problem session). You should be prepared to present these problems on the board during the problem session.

1. Let $X \subset \mathbb{P}^n(k)$ be a closed algebraic subset, $U \subset \mathbb{P}^n(k)$ an open subset and let $i : X \cap U \to U$ be the inclusion. Show that i is a morphism of algebraic sets.

2. Let $\pi : k^{n+1} \setminus \{0\} \to \mathbb{P}^n(k)$ be the map of sets,

$$\pi(x_0, \ldots, x_n) = [x_0 : \ldots : x_n].$$

Show that π is a morphism.

3. (a) Let $p = [0 : \ldots : 0 : 1] \in \mathbb{P}^{n+1}$ and let $q : \mathbb{P}^{n+1}(k) \setminus \{p\} \to \mathbb{P}^n(k)$ be the map $q([x_0 : \ldots, x_{n+1}]) = [x_0 : \ldots, x_n]$. Show that q is a morphism.
(b) Let $f(x_0, \ldots, x_{n+1}) \in k[x_0, \ldots, x_{n+1}]$ be a homogeneous polynomial of degree $d > 0$ with $f(0, \ldots, 0, 1) \neq 0$ (i.e., the coefficient of X_{n+1}^d in f is non-zero). Let $X \subset \mathbb{P}^{n+1}(k)$ be the closed subset $V^h((f))$. Let $q_X : X \to \mathbb{P}^n(k)$ be the restriction of q from (a) to X. Show that q_X is a morphism.

4. (a) Show that $\mathcal{O}_{\mathbb{P}^n(k)}(\mathbb{P}^n(k)) = k$ for all $n \geq 0$.
(b) $\mathcal{O}_{\mathbb{P}^n(k)}(\mathbb{P}^n(k) \setminus \{[0 : \ldots : 0 : 1]\}) = k$ for all $n \geq 0$.
(c) Let $F \subset \mathbb{P}^3(k)$ be the closed subset $V^h((X_0, X_1))$. Show that
$\mathcal{O}_{\mathbb{P}^3(k)}(\mathbb{P}^3(k) \setminus F) = k$.