Hints[1] for exercise sheet 3

Exercise 1

The idea of this exercise is to apply the Stone-Weierstrass theorem.

As mentioned in exercise class, a function of the form $f(z) = \sum_{k=-n}^{n} c_k z^k$ is equivalent to the data of a sequence $(c_k)_{k\in\mathbb{Z}}$ such that only finitely many elements of this sequence are non-zero. Such a sequence can be viewed as a function $c_f : \mathbb{Z} \to \mathbb{C}$ with finite support (where $c_f(k) = c_k$ for all $k \in \mathbb{Z}$).

This perspective might be useful for example when showing that the product of two such functions again has such a form; one has the formula $c_{fg} = c_f \star c_g$, where $\star$ denotes the convolution product.

For treating the conjugate of such a function, remember that $\overline{z} = z^{-1}$ if $z \in S^1$...

Exercise 2

a. Try by contradiction using the sequence

$$x_n^N = \begin{cases} 1/N, & n \leq N \\ 0, & n > N \end{cases}$$

b. If you're stuck, try with $X = L^1([0,1])$ and $K = \{f \in L^1([0,1]) : \int_0^1 xf(x)dx = 0\}$.

Exercise 3

a. Compare with exercise 3 from sheet 1!
b. You can do it!

Exercise 4

a. Go for a proof by contradiction. It's short!
b. Don't panic. Look at $X_{n,j} = ((j-1)/2^n, j/2^n)$ and

$$(Y_m)_{m\in\mathbb{N}} = (X_{1,1}, X_{1,2}, X_{2,1} \cdots X_{2,4}, X_{3,1} \cdots).$$

Show that $\limsup_{m\to\infty} \chi_{Y_m}(x) = 1$ a.e..

Then look at subsequences. Choose a subsubsequence such that the "n" indices (there are "n" and "j") are monotonely increasing...

[1] Try by yourself first!