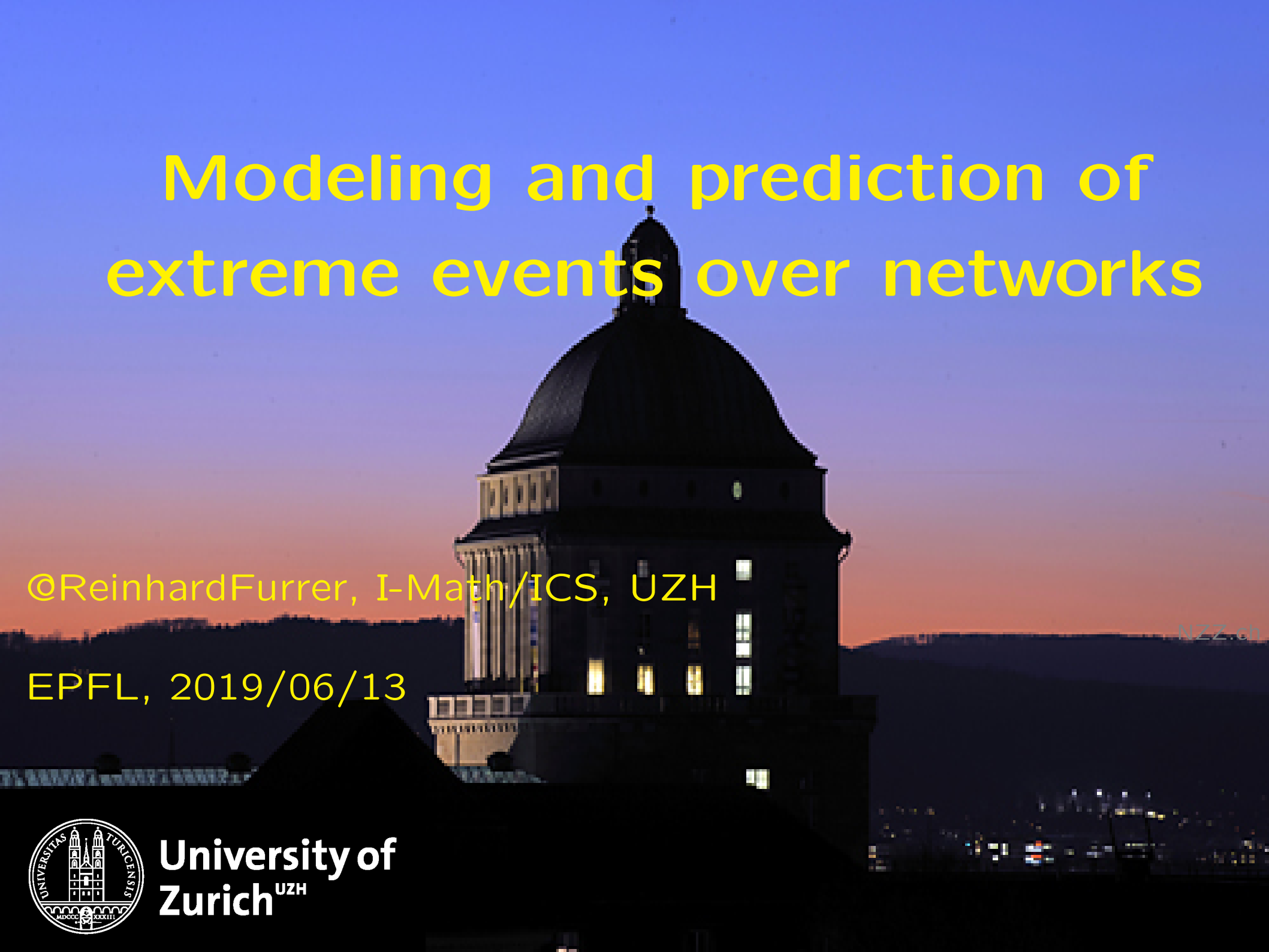
Modeling and prediction of extreme events over networks



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EPFL, 2019/06/13

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Joint work with

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- Philippe Naveau (LSCE, CNRS)

and others!

University of Zurich[UZH]

Outlook

- Propagation of "extremes" in a small river network

  Modeling the spatial dependence of floods using the Fisher copula
  doi:10.5194/hess-23-107-2019

- Prediction of "extremes" in MGP based processes

  work in "extreme" progress

Slides at: http://www.math.uzh.ch/furrer/slides/

Motivation

Asadi et al. (2015):

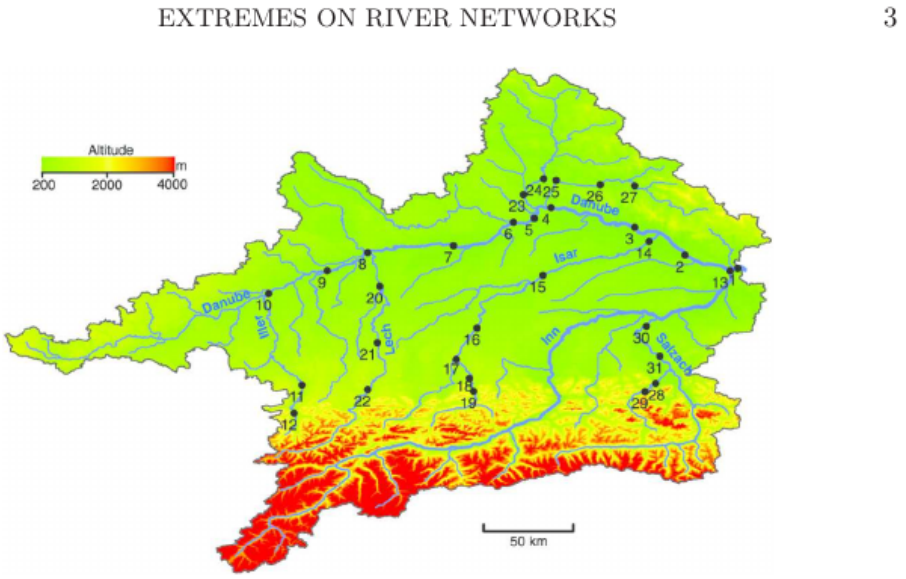


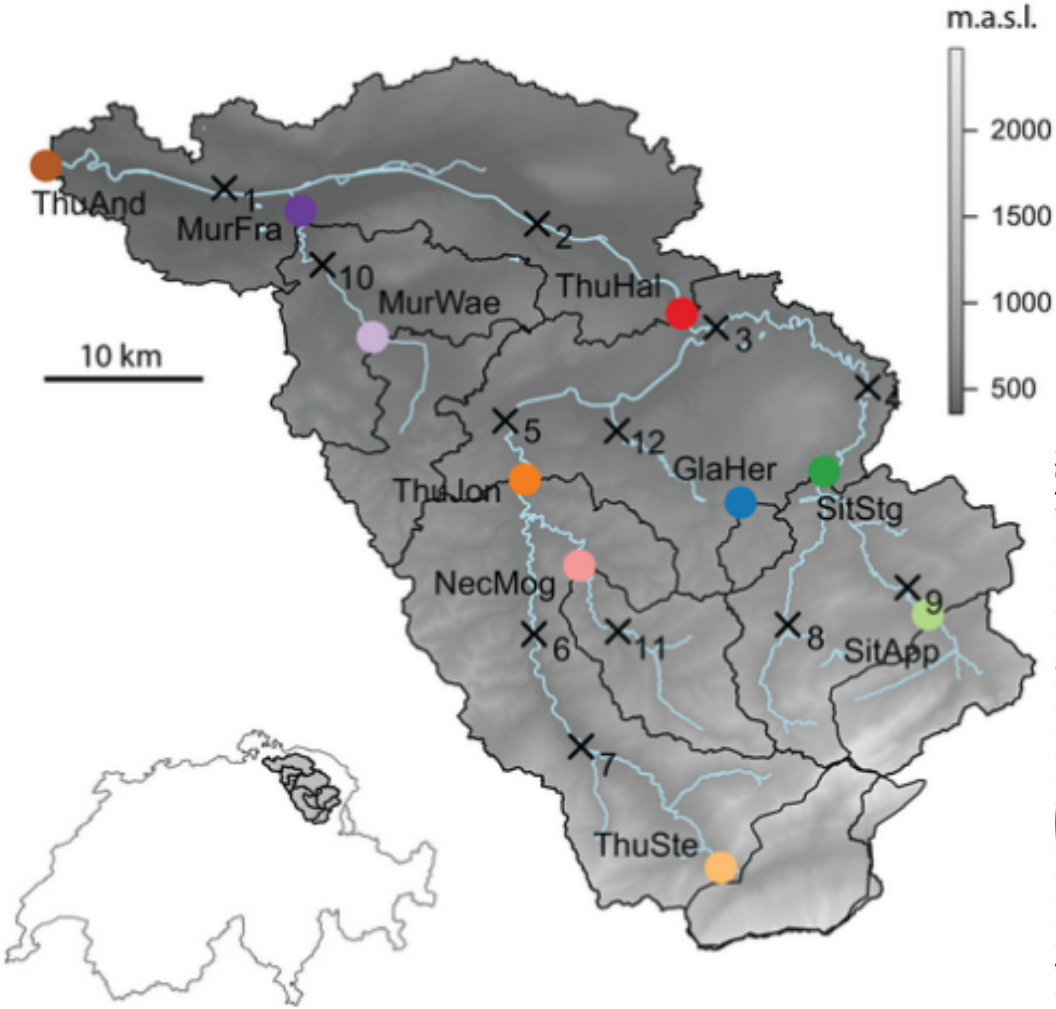
FIG. 1. *Topographic map of the upper Danube basin, showing sites of 31 gauging stations (black circles) along the Danube and its tributaries. Water flows toward gauging station 1.*

Source: Asadi et al. (2015) doi:10.1214/15-AOAS863

Does it work for a small(er) network as well?

Thur network

10 stations total for 30-40 years, ≈1700 km^2



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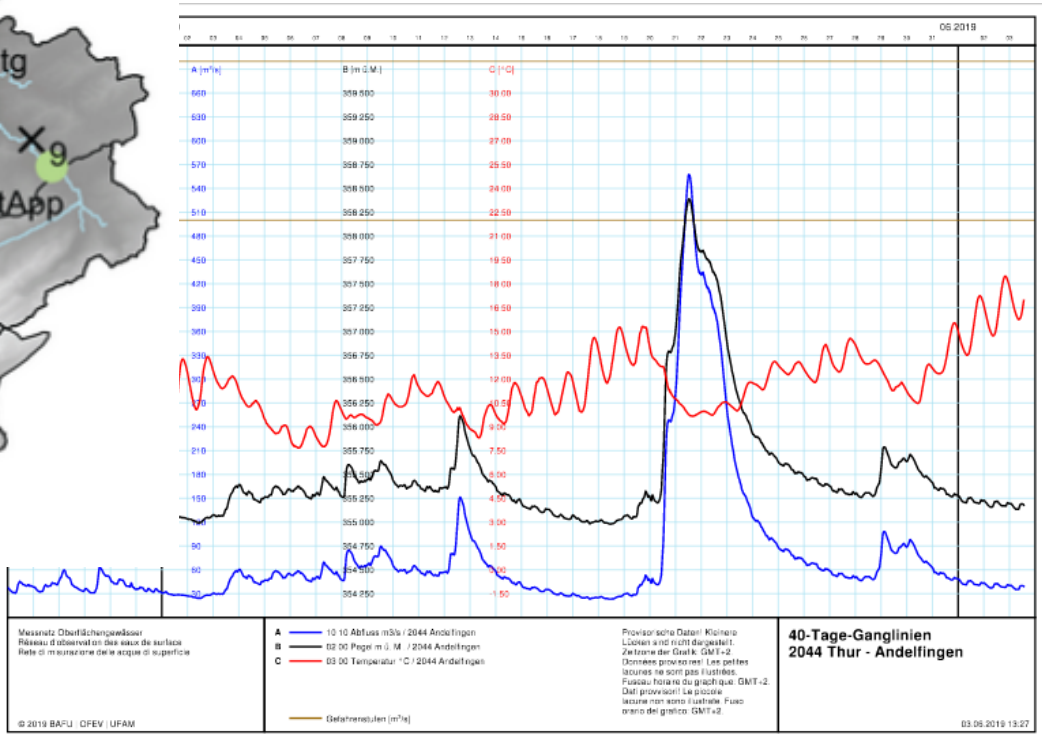
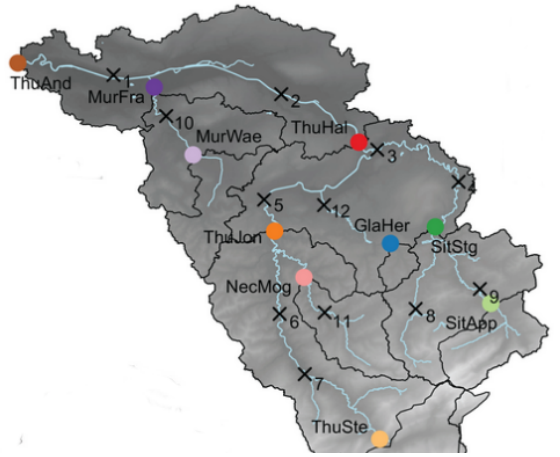
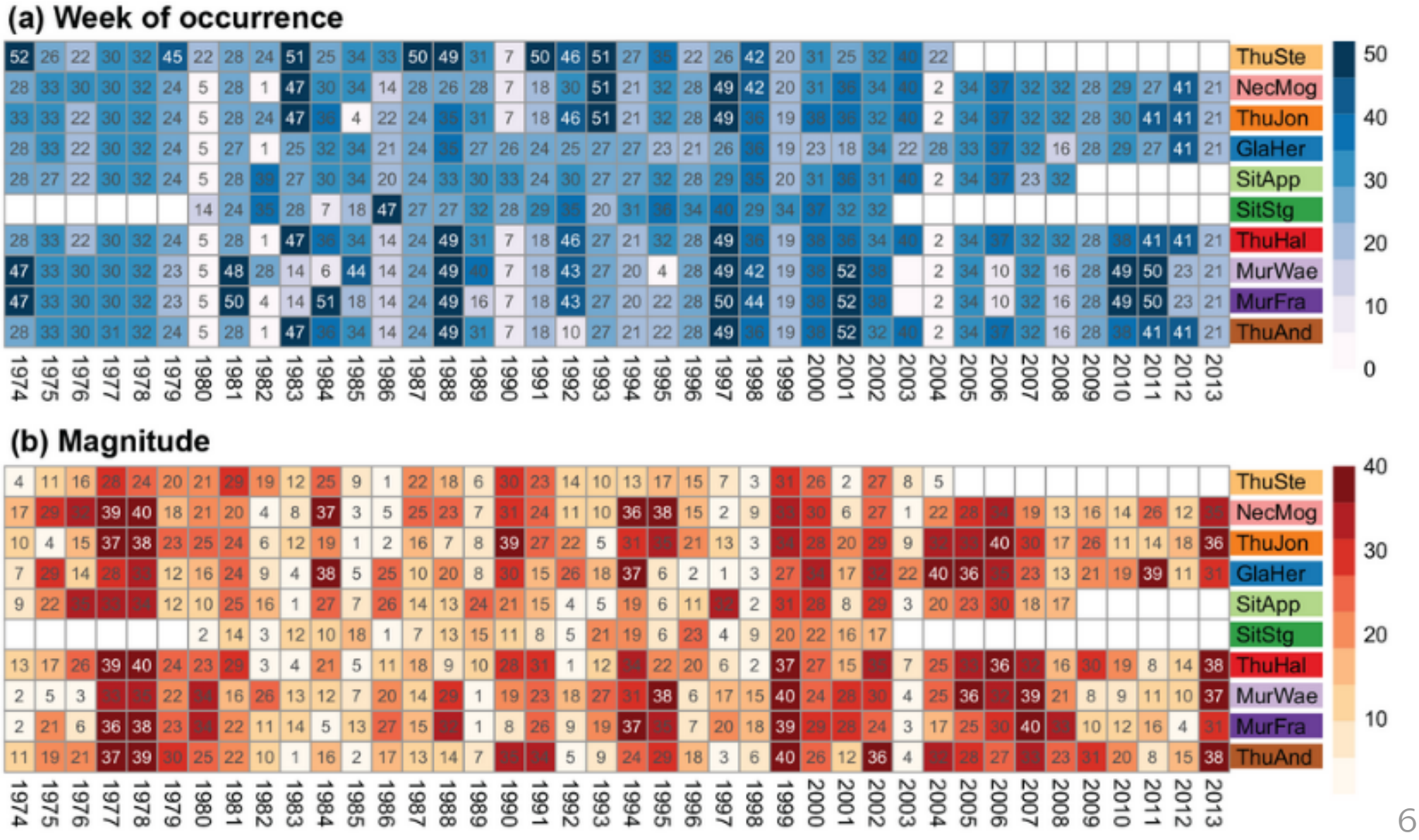


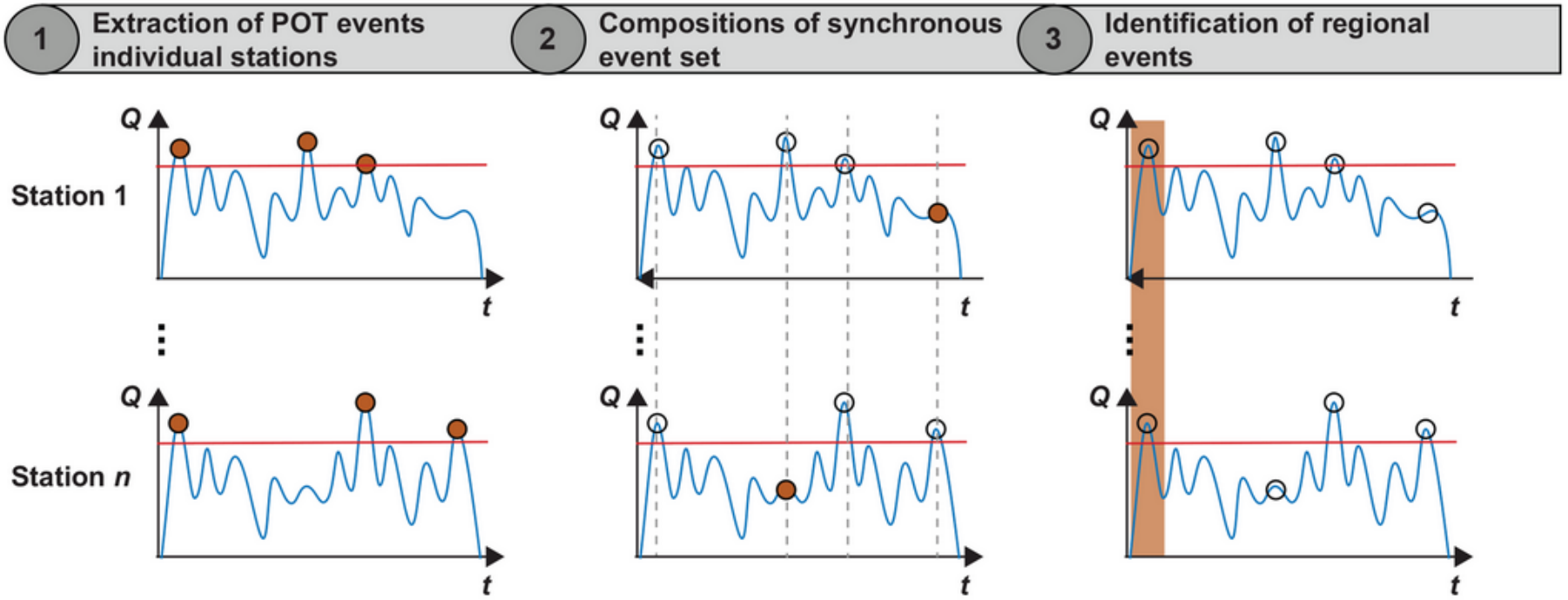
Source: www.hydrodaten.admin.ch/de/2044.html

Thur network





Event definition



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- High rank sum, low rank variability
- Only regional events relevant (63 events)

Model choices/questions

1. Threshold selection?

2. Distance for spatial dependence?

3. Model for the parameters?

4. Dependence structure?

5. Validation of the model?

Results

1. Threshold selection:
   regional events (definition/construction as outlined)
2. Distance for spatial dependence:
   best based on river distance (then mean catchment elevation)
3. Model for the parameters:
   marginal "GEV" fitting (covariate based)
4. Dependence structure:
   Fisher copula is best (vs $\mathcal{N}$, t, R-vine . . . , & max-stable processes)
5. Validation of the model:
   via F-madograms, cross-validation

Fisher copula?

Introduced by Favre, Quessy, Toupin (2018), Environmetrics.

- Asymmetry, upper tail dependence, simulation and "extrapolation"
- Obtained via squaring t-random vectors parameterized by ν and $\mathbf{\Sigma}$ (nasty closed form expression)
- Estimation via two-step pseudo-mle

Now?

What about a direct approach?

Assume a some multivariate distribution and

propose a (spatial) prediction scheme.

. . . which is (at least theoretically) useful

MGP (fast-forward slide)

Notation: Rootzen, Segers, Wadsworth, JMVA (2018)

- Let $\mathbf{Y}$ random vector in $\mathbb{R}^d$
  (in the max-domain of attraction of a multivariate max-stable distr G)
- For $\mathbf{Y}_1, \ldots, \mathbf{Y}_n$, indep, $\exists\ \mathbf{a}_n, \mathbf{b}_n$ such that

$$\mathbb{P}\left(\max_i \frac{\mathbf{Y}_i - \mathbf{b}_n}{\mathbf{a}_n} \leq \mathbf{x} \right) \xrightarrow{\mathsf{d}} G \qquad \text{(GEV)}$$

  with non-degenerate margins G_j, having lower endpoint η_j
- Interest in exceedances:

$$\underbrace{\mathbb{P}\left(\max\left(\frac{\mathbf{Y}_i - \mathbf{b}_n}{\mathbf{a}_n}, \boldsymbol{\eta}\right) \leq \mathbf{x} \,\middle|\, \mathbf{Y} \not\leq \mathbf{b}_n \right)}_{=: \mathbf{X}_n} \xrightarrow{\mathsf{d}} H \qquad \text{(GP)}$$

MGP

Properties

- Margins of H_j are not necessary GP: $\{Y_j > b_j\}$ vs $\{\mathbf{Y} \not\leq \mathbf{b}\}$
- H and G are associated: $H(\cdot) = \log G(\min(\cdot, \mathbf{0})) - \log G(\cdot) \Big/ \log G(\mathbf{0})$
  $\mathbf{X} \sim \text{MGP}(G) = \text{MGP}(\boldsymbol{\sigma}, \boldsymbol{\xi}; \dots)$, $\boldsymbol{\sigma} = \boldsymbol{\alpha} - \boldsymbol{\xi}\boldsymbol{\mu}$
- Transformations to arbitrary scales and shapes:
$$\mathbf{X} = \boldsymbol{\sigma} \frac{\exp(\boldsymbol{\xi}\mathbf{X}_0) - \mathbf{1}}{\boldsymbol{\xi}}$$
- Weak conditions on a random vector $\mathbf{Z}$
$$\mathbf{X} = E + \mathbf{Z} - \max \mathbf{Z} \qquad \text{“T” characterization}$$
  with $E \sim \mathcal{E}xp(1)$ indep of Z.

  $\mathbb{P}(Z_i > \infty) > 0$, $\forall i$ and $\mathbb{P}(\max \mathbf{Z} > \infty) = 1$

  $\mathbf{S} := \mathbf{Z} - \max \mathbf{Z}$ is a spectral random vector

MGP process based approach

- Let $\left\{Z(\mathbf{s}), \mathbf{s} \in \mathcal{D}\right\}$ an (intrinsically stationary) spatial process.

  For $\mathbf{s}_0, \mathbf{s}_1, \ldots, \mathbf{s}_d$, let

$$X_j = E + Z(\mathbf{s}_j) - \max_{i=0,\ldots,d} Z(\mathbf{s}_i)$$

  $\mathbf{X} \sim \text{MGP}\Big(\mathbf{1}, \mathbf{0}; \mathcal{L}(Z)\Big)$ in $\mathbb{R}^{d+1}$

Predict X_0 based on $x_1, \ldots, x_d$

Kriging (excursion)

- Let $\left\{Z(\mathbf{s}), \mathbf{s} \in \mathcal{D}\right\}$ an (intrinsically stationary) spatial process.
- Variogram: $2\gamma(\mathbf{h}) = \text{Var}\left(Z(\mathbf{s}+\mathbf{h}) - Z(\mathbf{s})\right)$
- Kriging predictor:

$$p\Big(Z(\mathbf{s}_0), \mathbf{Z}\Big) = \boldsymbol{\lambda}^\top \mathbf{Z}, \qquad \boldsymbol{\lambda} = \boldsymbol{\Gamma}^{-1}\Big(\boldsymbol{\gamma} + \mathbf{1}\frac{1 - \mathbf{1}^\top\boldsymbol{\Gamma}^{-1}\boldsymbol{\gamma}}{\mathbf{1}^\top\boldsymbol{\Gamma}^{-1}\mathbf{1}}\Big)$$

  where $\boldsymbol{\Gamma} = \Big(\gamma(\mathbf{s}_i - \mathbf{s}_j)\Big)$, $\boldsymbol{\gamma} = \Big(\gamma(\mathbf{s}_0 - \mathbf{s}_i)\Big)$.

- Kriging predictor is BLUP
  (under Gaussianity assumption, kriging predictor is BP).

MGP process based approach

- Let $\left\{Z(\mathbf{s}), \mathbf{s} \in \mathcal{D}\right\}$ an (intrinsically stationary) spatial process.

  For $\mathbf{s}_0, \mathbf{s}_1, \ldots, \mathbf{s}_d$, let

  $$X_j = E + Z(\mathbf{s}_j) - \max_{i=0,\ldots,d} Z(\mathbf{s}_i)$$

  $\mathbf{X} \sim \text{MGP}\Big(\mathbf{1}, \mathbf{0}; \mathcal{L}(Z)\Big)$ in $\mathbb{R}^{d+1}$

- $X_i - X_j \overset{\text{d}}{=} Z(\mathbf{s}_i) - Z(\mathbf{s}_j)$

- $\boldsymbol{\lambda}^\top \mathbf{X} \,\Big|\, \boldsymbol{\lambda}^\top \mathbf{X} > 0 \sim \text{GP}(\boldsymbol{\lambda}^\top \boldsymbol{\sigma}, 0)$

Does it work?

Simulation study (excerpt here):

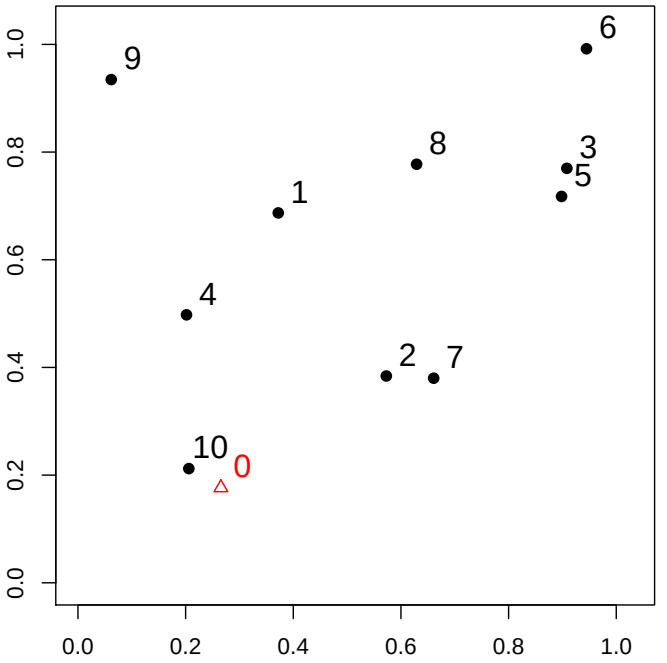
- Gaussian process $Z(\cdot)$ with:

  (a) equicorrelation ($d = 2$)

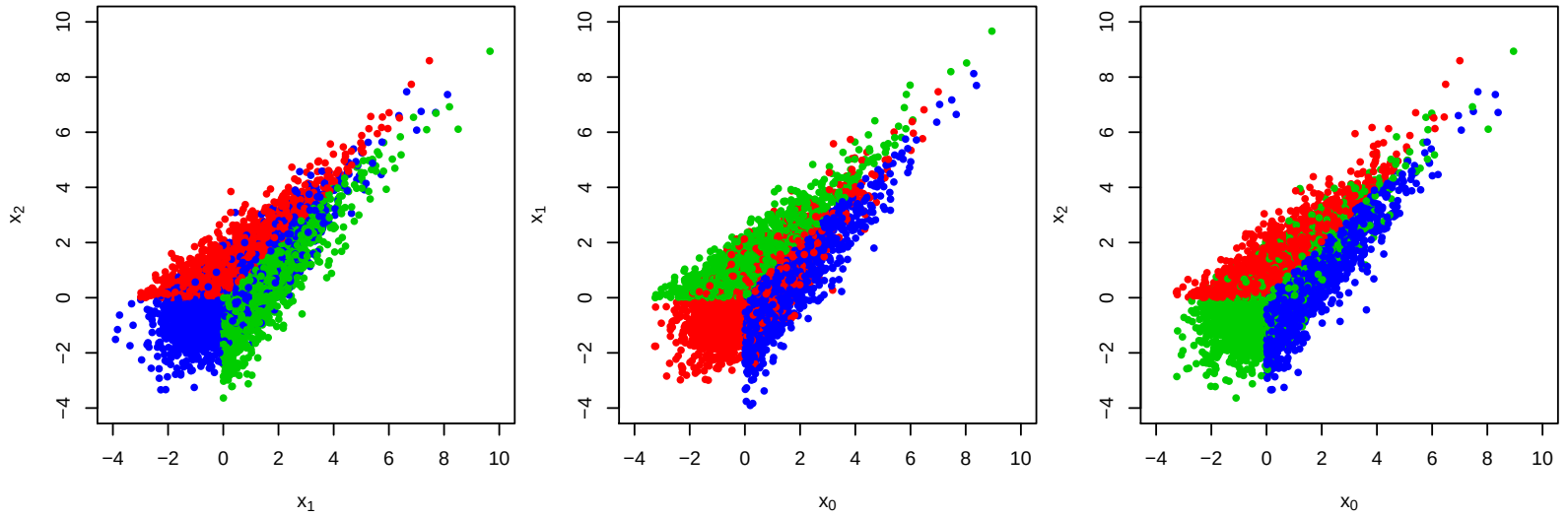
  (b) exponential model ($d = 10$)

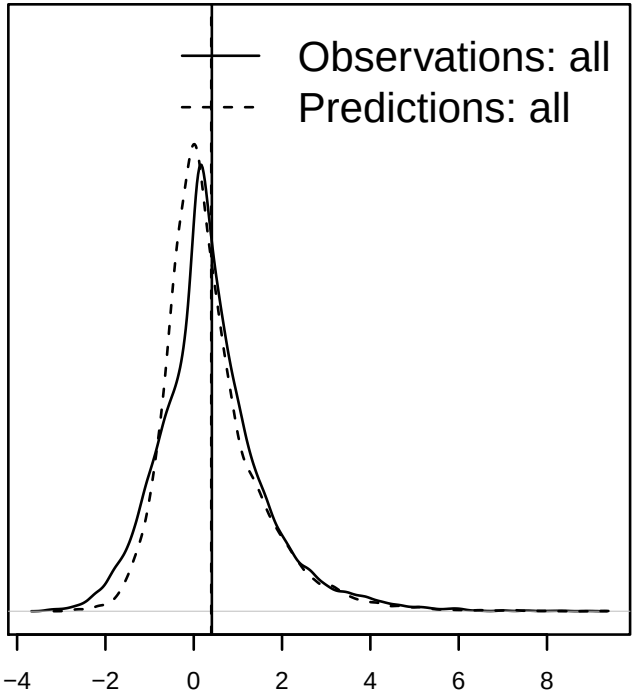
- $n = 10'000$

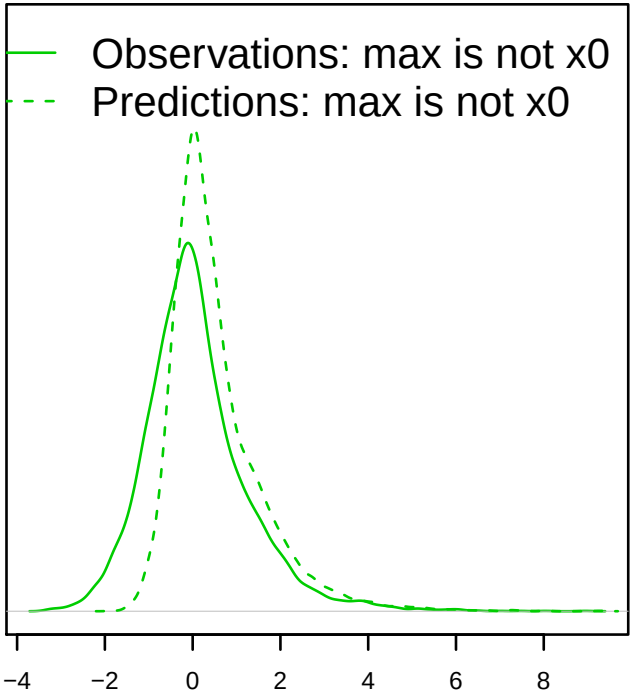
- Prediction of X_0 via kriging (known covariance)

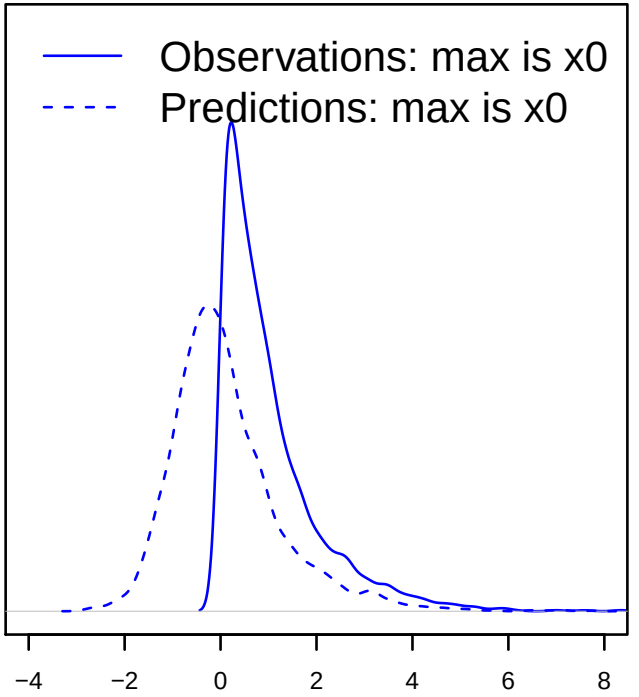


(a) Prediction of X_0 via kriging

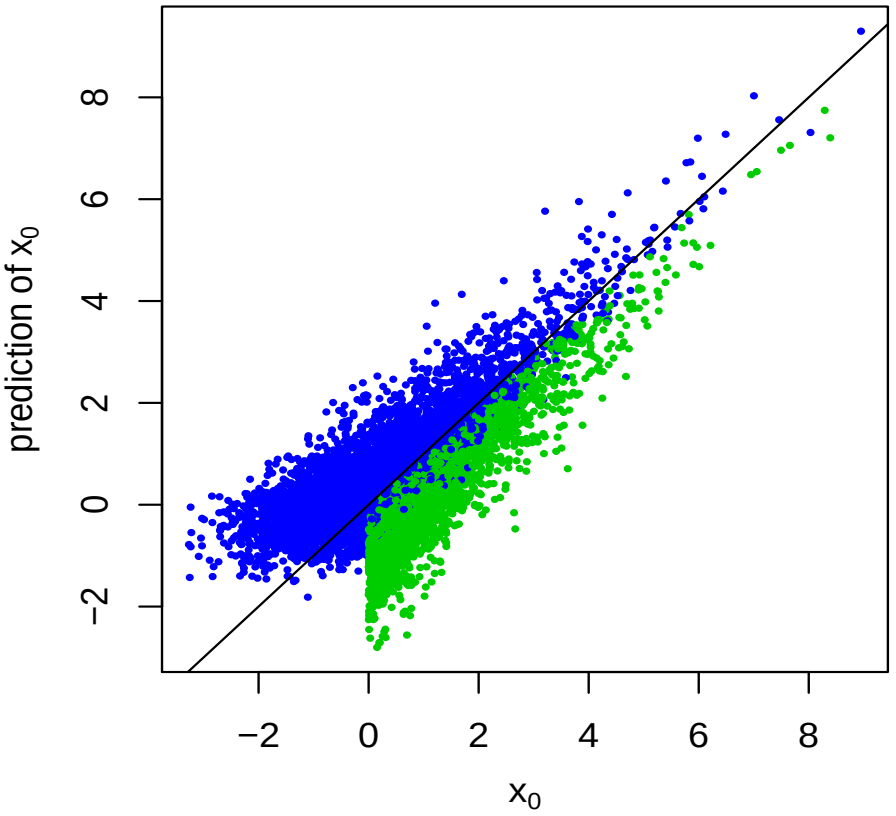


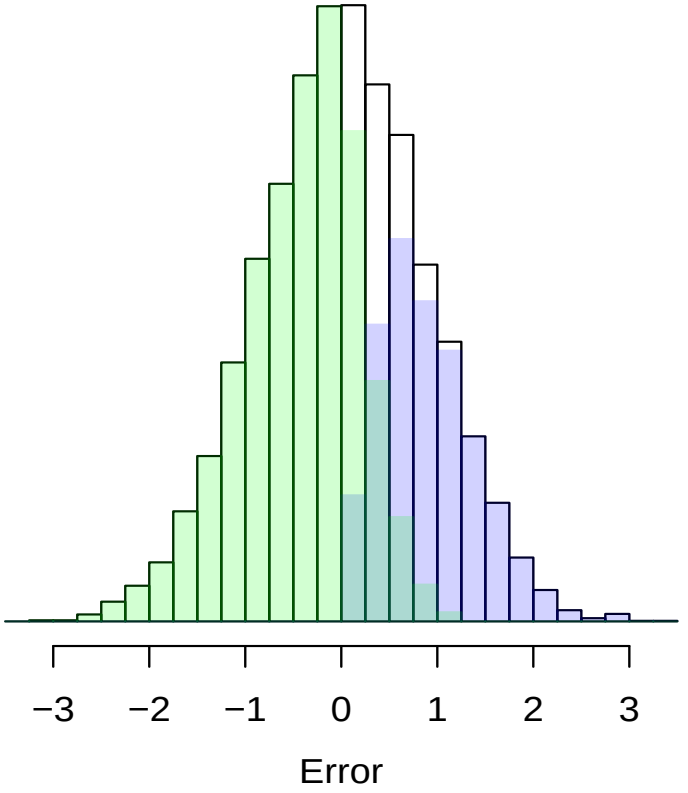






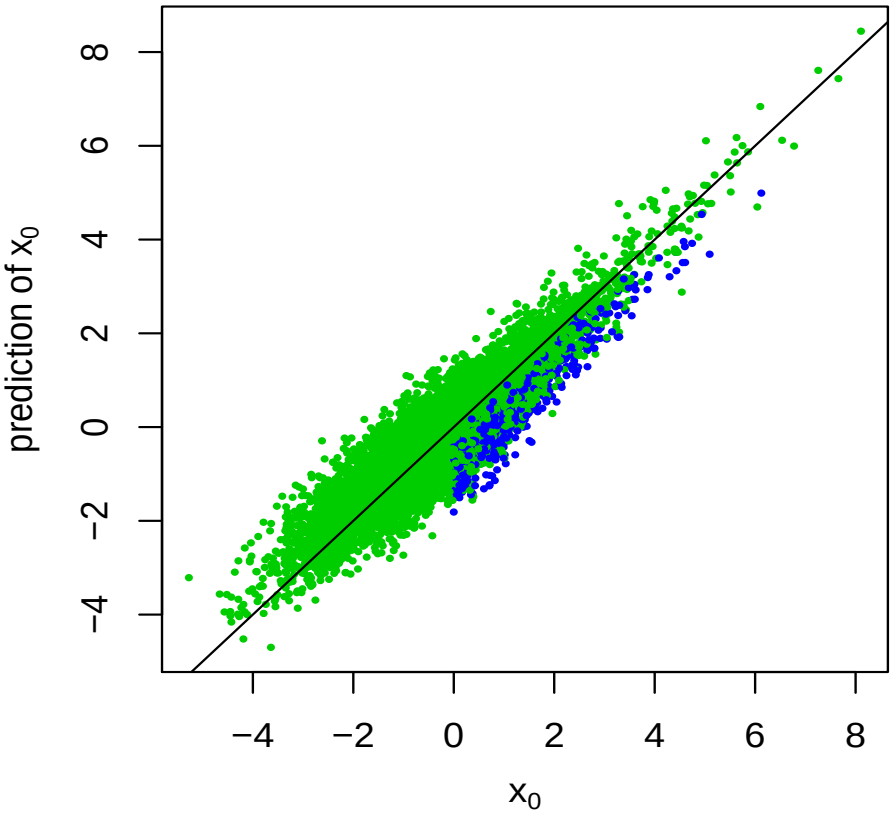
(a) Prediction of X_0 via kriging

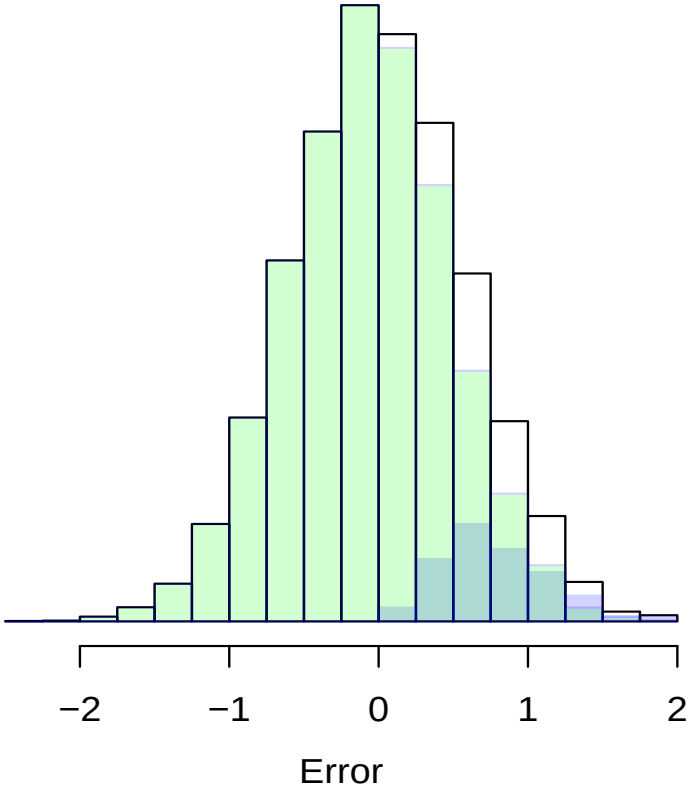




	n	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.	var	s2BLUP
overall	10000	-3.18839	-0.569	0.0139	0.0139	0.60855	3.26	0.767	0.75
notX0	6582	-3.18839	-0.836	-0.3635	-0.4491	0.00117	1.23	0.403	
X0	3418	0.00974	0.527	0.8194	0.9056	1.21431	3.26	0.259	

(b) Prediction of X_0 via kriging





	n	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.	var	s2BLUP
overall	10000	-2.331	-0.394	-0.00778	-0.00161	0.393	1.96	0.342	0.336
notX0	9085	-2.331	-0.440	-0.07215	-0.08024	0.288	1.93	0.296	
X0	915	0.121	0.512	0.72951	0.77909	1.017	1.96	0.130	