The Arrow of Time

—

Images of Irreversible Behavior

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Credits and Contents

Credits

Various collaborations with, among others, V. Bach, J. Faupin, I. M. Sigal, M. Merkli, W. K. Abou Salem, W. De Roeck, A. Pizzo, B. Schubnel, D. Ueltschi, Gang Zhou, and others. – Late nineties until recently.

Contents

This lecture is about *Irreversibility* in *Quantum Mechanics* and is based on the use of (functional and hard) *analysis*.

1. Relative Entropy, etc.
2. The Second Law of Thermodynamics – Clausius and Carnot
3. Quantum Brownian Motion
4. Hamiltonian Friction
5. "L'insoutenable irréversibilité de l'évolution quantique"; (questo pomeriggio)

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n. Irreversibility in cosmology; (lezione di lunedi e martedi scorso)

Riassunto: dinamica effetiva e comportamento irreversibile

- *Dynamics in Limiting Regimes:* Mean-field limit ..., kinetic limit, Boltzmann-Grad limit, etc.; (semi-classical methods, Lindblad dynamics, preservation of molecular chaos, ...)
- *Effective dynamics of matter coupled to wave field, $T = 0$:* Decay of Resonances & dissolution of one-particle states, Relaxation to a Groundstate, Ionization of Atoms by Laser Pulses, Emission of Cerenkov Radiation $\rightarrow$ Hamiltonian Friction, Ren. of Mass and dispersion law of charged particles & tracing out of em field, etc.
- *Small systmes coupled to thermal reservoir:* Return to Equilibrium, Thermal Ionization; (Fermi Golden Rule, Feshbach-Schur Map, ...)
- *Coupled Thermal Reservoirs:* Approach to a NESS or to a NE time-periodic state, Second Law of Thermodynamics
- *Particle(s) moving in thermal bath:* (Sub-, super-, &) Diffusive Motion – (Quantum) Brownian Motion; effect of thermal noise on Anderson localization, - on Boltzmann dynamics; Mott transition,...
- *Foundations of QM:* Dynamics of Quantum Systems subjected to repeated measurements/observations of events

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1. Relative Entropy

Let ρ be a density matrix acting on a Hilbert space $\mathcal{H}$. The von Neumann entropy of ρ is defined by

$$S(\rho) := -Tr(\rho \ln\rho) \tag{1}$$

It has the following properties:

1. $S(\rho) \geq 0$, $\forall\rho$, with "$=$" only if ρ is pure.
2. $S(\cdot)$ is strictly concave.
3. $S(\cdot)$ is *subadditive* and *strongly subadditive*.

Item 1. is obvious. Item 2. and *subadditivity* follow from the following *general inequality*: Let f be a real-valued, strictly convex function on the real line, and let A and B be self-adjoint operators on $\mathcal{H}$. Then

$$Tr(f(B)) \geq Tr(f(A)) + Tr(f'(A) \cdot (B - A)), \tag{2}$$

with "$=$" only if $A = B$. (Set $f(x) = x \ln x$ to prove 2. & SA.)

Proof of inequality (2), due to Klein:

Let $\{\psi_j\}_{j=0}^{\infty}$ be a CONS of eigenvectors of B corresponding to e.v.'s β_j. Let ψ be a unit vector in $\mathcal{H}$, and $c_j := \langle \psi_j, \psi \rangle$. Then

$$\langle \psi, f(B)\psi \rangle = \sum_j |c_j|^2 \; f(\beta_j) \geq f(\sum_j |c_j|^2 \beta_j) = f(\langle \psi, B\psi \rangle). \quad (3)$$

Convexity of f also implies that

$$f(\langle \psi, B\psi \rangle) \geq f(\langle \psi, A\psi \rangle) + f'(\langle \psi, A\psi \rangle) \cdot \langle \psi, (B - A)\psi \rangle.$$

If ψ is an eigenvector of A then the R.S. is

$$= \langle \psi, [f(A) + f'(A) \cdot (B - A)]\psi \rangle. \quad (4)$$

Eq. (2) follows by summing eqs. (3) and (4) over a CONS of eigenvectors of A!

Properties of relative entropy

Let ρ and σ be density matrices on $\mathcal{H}$. The relative entropy of ρ with respect to σ is defined by

$$S(\rho||\sigma) := Tr(\rho(\ln\rho - \ln\sigma)), \tag{5}$$

assuming that $\ker(\sigma) \subseteq \ker\rho$.
Crucial properties of $S(\rho||\sigma)$ are:

- Positivity: $S(\rho||\sigma) \geq 0$, as follows from inequality (2). (6)
- Convexity: $S(\rho||\sigma)$ is jointly convex in ρ and σ.

Next, let $\mathcal{T}$ be a trace-preserving, completely positive map on the convex set of density matrices on $\mathcal{H}$. Then

$$S(\rho||\sigma) \geq S(\mathcal{T}(\rho)||\mathcal{T}(\sigma)).$$

Exercise: Show that this inequality, due to Lindblad and Uhlmann, implies Strong Subadditivity, (first established by Lieb and Ruskai):

$$S(\rho_{12}) + S(\rho_{23}) - S(\rho_{123}) - S(\rho_2) \geq 0.$$

Jost's warning

$\Rightarrow$ Existence of TD limit of entropy for quantum systems, etc.
Remark: S has a somewhat tantalizing *homological interpretation* ($\nearrow$ Baudot & Bennequin).

I have learned the neat proof of (2) and the right way of introducing the 2^{nd} Law of thermodynamics from *Res Jost*. He warned some of us that, at a party, one should never start a conversation about

- Irreversibility and the arrow of time
- The interpretation of quantum mechanics
- Religious faith

because most people mistakenly believe that they know something about these topics and get emotional if they are proven wrong or confused.

Goals of lecture

Indeed, there is much confusion – even among grown-up physicists – about the origin of *time's arrow* and of irreversible behavior; and there is *enormous* confusion about the meaning of quantum mechanics! Not having to make a career, anymore, I can afford to get entangled with some of these confusions and to try to alleviate them. In today's lecture, I attempt to uncover *origins of the arrow of time*. *Irreversible behavior of a physical system* can arise from:

- Choice of unlikely initial conditions far from thermal equilibrium for a physical system; dispersive properties of its environment (e.g., a macroscopic thermal reservoir).
- Time evolution of a "small system", such as a particle, under the influence of noise coming from its environment.
- System shedding energy and "information" into massless modes that propagate to "∞"; entanglement of system with degrees of freedom no longer accessible to observation.
- Time evolution of qm systems producing detectable events.

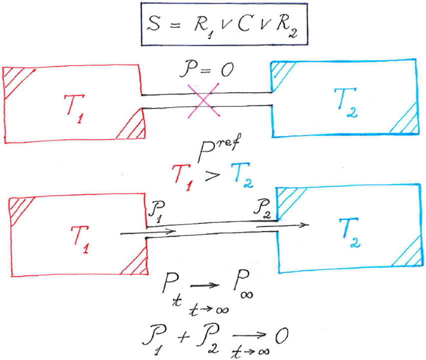
2. The Second Law of Thermodynamics – Clausius and Carnot

Rudolf Clausius

Consider qm system, S, consisting of two nearly infinite thermal reservoirs, R_1 and R_2, at temperatures T_1 and T_2, joined by a thermal contact, C. The state of S is given by a density matrix P_t; the Hamiltonians of R_1 and R_2 are denoted by H_1 and H_2, resp., the Hamiltonian of C, which includes interaction terms between C and $R_1 \vee R_2$, by H_C. For simplicity, state space of C is finite-dim. Before C is opened, the state of S is the Gibbs state (L-v N), P^{ref}:

Sketch of system

$$P^{ref} := \Xi^{-1} exp(-\beta_1 H_1) \otimes \mathbf{1}_C \otimes exp(-\beta_2 H_2), \ (\beta_i := 1/k_B T_i). \ (7)$$



Heat power of $R_i, i = 1, 2$:

$$\mathcal{P}_i(t) := " \frac{d}{dt} \mathrm{Tr}(P_t H_i)" = -i \mathrm{Tr}(P_t [H_i, H_C])$$

Positivity of entropy production

Assuming that R_1 and R_2 are filled with an ideal quantum gas or w. black-body radiation and exploiting dispersive props. of reservoirs, one proves that, in TD limit, P_t approaches a so-called non-equilibrium stationary state (NESS), P_∞, as $t \to \infty$. (See Dirren & F; Jaksic & Pillet; F, Merkli & Ueltschi, ...) Consider relative entropy

$$S(P_t || P^{ref}) = \text{Tr}(P_t(lnP_t - lnP^{ref})$$

It is easy to see that

$$\dot{S}(P_t || P^{ref}) = \beta_1 \mathcal{P}_1(t) + \beta_2 \mathcal{P}_2(t) \tag{8}$$

If P_t approaches a NESS P_∞, which is time-translation-invariant, then

1. $\dot{S}(P_t || P^{ref})$ has a limit, σ_∞, as $t \to \infty$, and it follows from (6) that

$$\sigma_\infty \geq 0, \quad \text{(positivity of entropy production)} \tag{9}$$

2nd Law according to Clausius

2. $\mathcal{P}_1(t)$ has a limit, denoted $-\mathcal{P}_\infty$, as $t \to \infty$, and

$$\mathcal{P}_1(t) + \mathcal{P}_2(t) \to 0, \text{ as } t \to \infty. \tag{10}$$

From Eqs. (8) through (10) we derive the 2^{nd} Law in the formulation of Clausius:

$$\mathcal{P}_\infty \underbrace{\left(\frac{1}{T_2} - \frac{1}{T_1}\right)}_{>0} \geq 0, \tag{11}$$

i.e., heat flows from the warmer reservoir R_1 to the colder one R_2. If atoms can flow through C, from R_1 to R_2, then

$$\sigma_\infty \equiv \mathcal{P}_\infty(\beta_2 - \beta_1) - \mathcal{I}_\infty(\beta_2\mu_2 - \beta_1\mu_1) \geq 0, \tag{12}$$

where $\mathcal{I}_\infty$ is the particle current flowing from R_1 to R_2, and μ_i is the chemical potential of R_i.

Onsager, Ohm,..., and the 0^{th} Law of Thermodynamics

Additional results:

- Return to Equilibrium (R, J-P, B-F-S,...); Isothermal Theorem (AS-F) $\Rightarrow$ e.g., $\Delta F = W$, in quasi-static, isothermal proc.
- $\sigma_\infty > 0$ (weak coupling; F-M-Ue), Onsager relations (J-O-P)
- Ohm's Law (weak coupling; F-M-Ue): If $T_1 = T_2 = T$ then

$$\mathcal{I}_\infty \approx R^{-1} \cdot (\mu_1 - \mu_2) \tag{13}$$

- Universal current fluctuations; full counting statistics, etc.

A fundamental problem: 0^{th} Law of TD

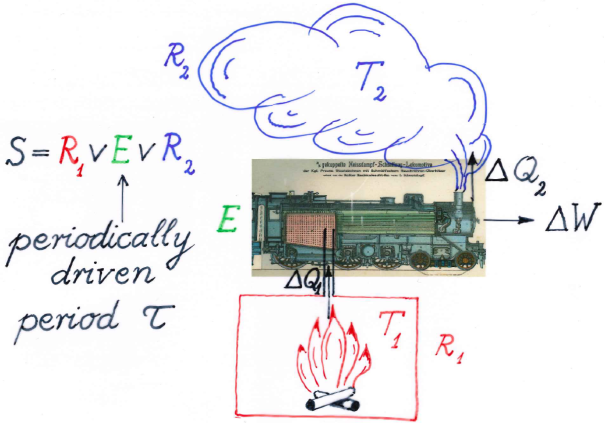
Existence of ∞ heat baths, local equilibration of macroscopic systems? – "eth" versus many-body localization; (preliminary results by Goldstein, Hara, Lebowitz, Tasaki, Tumulka, Zanghi; DeRoeck, Huveneers,...)

Carnot's formulation of the 2nd Law

Sadi Carnot

Replace C by a heat engine (e.g., a "quantum locomotive"), E, that extracts heat energy from R_1, releases part of it into R_2 (with $T_1 > T_2$), and performs work; $S = R_1 \vee E \vee R_2$.
E is driven periodically in time, with period $\tau > 0$. Thus, its Hamiltonian, $H_E(t)$, is time-dependent, with period τ. Assuming that R_1 and R_2 have good dispersive properties (ideal quantum gases or black-body radiation) and applying Floquet theory, one proves that the true state, P_t, of S approaches a time-periodic state, $P^{asy}(t)$, with period τ (Abou Salem & F,...); P^{ref} as above.

The example of a steam locomotive



Let ΔQ_i denote the heat energy extracted from R_i, ΔW the work done by E, and ΔS the change in relat. entropy, during one cycle.

Carnot's bound on the degree of efficiency of E

Pos. of rel. entr. & approach to *time-periodic state* $(t \to \infty) \Rightarrow$

$$0 \leq \Delta S = -\frac{\Delta Q_1^{\nearrow}}{T_1} + \frac{\Delta Q_2^{\swarrow}}{T_2}, \text{ per cycle.} \tag{14}$$

Note that because of periodicity in t (period τ)

$$\Delta U_E := \mathrm{Tr}(P^{asy}(t+\tau)H_E(t+\tau)) - \mathrm{Tr}(P^{asy}(t)H_E(t)) = 0 \tag{15}$$

By (14), (15) and 1^{st} *Law of TD*,

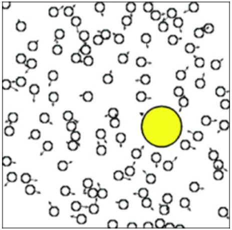
$$\eta := \frac{\Delta W}{\Delta Q_1^{\nearrow}} = \frac{\Delta Q_1^{\nearrow} - \Delta Q_2^{\swarrow}}{\Delta Q_1^{\nearrow}} \leq \frac{T_1 - T_2}{T_1} \equiv \eta_{\text{ Carnot}} \tag{16}$$

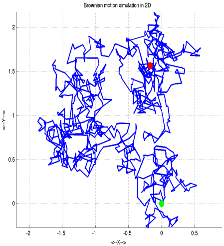
$$= \text{ iff } \Delta S = 0$$

This is Carnot's formulation of the 2^{nd} Law.

3. Quantum Brownian Motion

Albert Einstein *Marian Smoluchowski*





Sketch of a model

Yellow disk is a *tracer particle* immersed in an (ideal) quantum Bose gas, which it interacts with.
The small white disks are atoms of the quantum gas.
Tracer particle hops on a lattice $\mathbb{Z}^3$; Hilbert space of pure state vectors is $\ell^2(\mathbb{Z}^3) \otimes \mathbb{C}^2$; Hamiltonian given by

$$H_P := -\frac{\Delta_X}{2M} \otimes \mathbf{1} + \mathbf{1} \otimes \sigma^z, \qquad X \in \mathbb{Z}^3. \tag{17}$$

Atoms in Bose gas are free, non-relativistic particles with mass $m \ (= \frac{1}{2}) \ll M$ moving in $\mathbb{R}^3$. Interaction of Tracer particle with atoms in Bose gas given by operator

$$H_I := g \sum_j W(X - x_j), \qquad x_j \in \mathbb{R}^3 : \text{position of } j^{th} \text{ atom}, \tag{18}$$

$W(x)$ a "suitable" 2×2 -matrix-valued function on $\mathbb{R}^3$; (FGR!).

Accessible regimes

The density of the Bose gas is

$$\rho = \rho_0 g^{-2}, \text{ where } \rho_0 \text{ is a constant, } g \text{ as in (18).} \tag{19}$$

Bogolyubov limit: $g \to 0$. In this limit, Hamiltonian given by

$$H := H_P + H_{BG} + \nu \int_{\mathbb{R}^3} dx \; W(X - x)\{b^*(x) + b(x)\}, \tag{20}$$

where $\nu := \sqrt{\rho_0/2}$, and $b^*(x)$ and $b(x)$ are creation- and annihilation operators satisfying the usual canonical commutation relations.

Two regimes:

(A) ν small, $M = \nu^{-2} M_0$, M_0 const., (kinetic regime);

(B) ν large, with $M = \nu^2 M_0$, $\nu^{-2} \leftrightarrow \hbar$, (mean-field regime)

We first study regime (A), ν small, and assume Bose gas is in thermal equ. at temperature $T > 0$: Quantum Brownian Motion!

Properties of model & results

Model is lattice-translation invariant. Let $\mathcal{Z}_t^\nu$ denote the effective dynamics of state ρ of particle *after* having taken a trace over degrees of freedom of Bose gas; (*not* a semi-group). Then

$$\mathcal{Z}_t^\nu \approx \exp[t(-i \text{ ad}_{\sigma^z} + \nu^2 \mathcal{M})], \tag{21}$$

where $\mathcal{M}$ is the (explicitly known) generator of a *semigroup of completely positive maps* (related to linear Boltzmann eq. for Wigner distr. of ρ ...). Using a cluster expansion of $\mathcal{Z}_t^\nu$ around the right side of Eq. (21), we control the diffusion constant, D:

$$\langle([X(t) - X(0)]^2\rangle_T \sim D \cdot t, \text{ as } t \to \infty, \tag{22}$$

with $D \approx (\overline{v}_\nu \cdot \overline{t}_\nu)^2/\overline{t}_\nu \propto \nu^2$; $(\overline{v}_\nu \propto \nu^2, \overline{t}_\nu \propto \nu^{-2})$.
Idea of proof of (22):

$$\langle([X(t) - X(0)]^2\rangle_T = \int_0^t d\tau \int_0^t d\sigma \langle \dot{X}(\tau) \cdot \dot{X}(\sigma)\rangle_T \tag{23}$$

Diffusion and equipartition

If $\langle \dot{X}(\tau) \cdot \dot{X}(\sigma) \rangle_T$ decays integrably fast in $|\tau - \sigma|$ then right side of (23) grows *linearly* in t, as $t \to \infty$. Int. decay (with decay rate of $\mathcal{O}(\nu^2)$) can be established starting from (21) and applying a cluster expansion in time. Note that $\|\dot{X}(t)\| < \mathcal{O}(\nu^2)$. (This is in marked contrast to ordinary Brownian motion for which $\dot{X}(t)$ does not exist.). Furthermore, distribution of functions of $\dot{X}(t)$ approaches Maxwell velocity distr.; i.e., the *Equipartition Theorem* holds.
These are the first and only results on the derivation of diffusive motion from fundamental quantum dynamics, (De Roeck-F, De R-Kupiainen)!

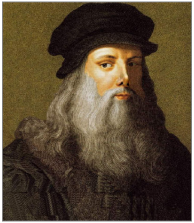
Further Results:

1. Add a random potential to H_P: At large disorder, D tends to 0, as $\nu \to 0$; (F-S; thermal noise modeled by Lindblad dynamics)
2. Add ext. force pushing tracer part. $\Rightarrow$ *Einstein relation*, i.e., $\partial v / \partial F|_{F=0} = \beta D$, holds in *very simple* models! (DeR-F-Schnelli)
3. Gas of tracer particles suspended in heat bath: NL Boltzmann eq. with "good" properties, such as R to E; (F-Gang Zhou)

4. Hamiltonian Friction

"A moving body will come to rest as soon as the force pushing it no longer acts on it in the manner necessary for its propulsion."

(Aristotle)



Leonardo Da Vinci

Guillaume Amontons

In this section we study *friction* in a model of a particle moving through an ideal Bose gas, as described by the mean-field regime (B), $M = \nu^2 M_0$, $\nu^{-2} \leftrightarrow \hbar$ with $\hbar \to 0$, of the model ($\mathbb{Z}^3 \to \mathbb{R}^3$, no int. deg. of freedom) introduced in Sect. 3, at zero temperature, $T = 0$.

Mean-Field-, or Classical Limit

The limit $\nu^{-2} \equiv \hbar \to 0$ corresponds to the classical (Hamiltonian) limit of the quantum system:

$$(X, -i\nu^{-2}\nabla_X) \to (X, P),\ \ b(x) \to \beta(x), b^*(x) \to \overline{\beta}(x),$$

where β is a complex-valued (c-number) field in $\mathcal{H}^1(\mathbb{R}^3)$. The phase space of the classical system is given by $\mathbb{R}^6 \times \mathcal{H}^1(\mathbb{R}^3)$. It's symplectic structure is encoded into the Poisson brackets:

$$\{\beta(x), \overline{\beta}(y)\} = i\delta(x-y),\ \ \{X^i, P_j\} = -\delta^i_j \tag{24}$$

Other Poisson brackets $= 0$. The Hamilton functional is given by

$$H_{cl}(X, P; \beta, \overline{\beta}) :=$$

$$= \frac{P^2}{2M_0} + 2\int_{\mathbb{R}^3} dx\ W(X-x)\mathrm{Re}\beta(x) + \int_{\mathbb{R}^3} dx\ (\nabla\overline{\beta})(x)\cdot(\nabla\beta)(x) \tag{25}$$

Equations of Motion and an Egorov-type Theorem

The equations of motion of the particle and of the Landau-Ginzburg order parameter field β of the Bose gas are given by

$$\dot{X}_t = M_0^{-1} P_t, \qquad \dot{P}_t = F - 2 \int_{\mathbb{R}^3} dx\, \nabla W(X_t - x)\, \mathrm{Re}\beta_t(x), \quad (26)$$

where F is an external force acting on the particle, and

$$i\dot{\beta}_t(x) = -\Delta\beta_t(x) + W(X_t - x) \quad (27)$$

Remark:
Canonical quantization of the Hamiltonian system (24)-(25), with $\hbar = \nu^{-2}$, reproduces the quantum system we started from.

One can prove a *Egorov-type theorem* (see F-K-S): Quantization and time-evolution commute, up to errors of order ν^{-2}!
This means that insights into the dynamics of the classical system reveal features of the qm dynamics in a regime of large values of ν.

Friction by Emission of Cherenkov Radiation

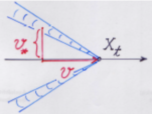
We first study "stationary" solus. of eqs. (26) and (27); i.e., we set

$$\dot{P}_t = 0 \text{ and } \beta_t(x) = \gamma_v(x - vt - X_0), \text{ with } X_t = X_0 + vt.$$

Eq. (26) then tells us that the external force F must be cancelled by the second term on the right side of (26), which describes a *friction force* arising from the particle's emission of Cherenkov radiation of sound waves into the Bose gas.

Result: If W is smooth and of short range then there is a positive constant $F_{max} < \infty$ (max. strength of ext. force) such that:

1. For $|F| < F_{max}$ there are two solus. propagating with speeds v_- (stable solu.) and $v_+ > v_-$ ("run-away" solu.).
2. For $|F| > F_{max}$, stationary solutions do not exist.



$F = 0, v_* = 0 \Rightarrow$ Aristotle was right!

Next, we study what happens to the particle when $F = 0$. Well, as long as speed of particle is larger than speed of sound, v_*, in Bose gas it keeps loosing energy into sound waves, which, thanks to the dispersive properties of the gas, propagate outwards to ∞. For an ideal Bose gas, $v_* = 0$. In this case, *particle will come to rest, as time t tends to ∞*. Here is a theorem:

Theorem (see F-GZ)
In an ideal Bose gas, if W is smooth and of short range then, given an arbitrary $\delta \in (0, \delta_*)$, with $\delta_* \approx 0.66$, there exists an $\varepsilon = \varepsilon(\delta) > 0$ such that, for initial conditions with $\|(1 + |x|^2)^{\frac{3}{2}} \beta_0(x)\| < \varepsilon, \; |P_0| < \varepsilon$:

$$|P_t| \leq \mathcal{O}(t^{-\frac{1}{2}-\delta}), \; \|\beta_t - \Delta^{-1} W(X_t - \cdot)\|_\infty \to 0, \text{ as } t \to \infty.$$

Choosing $\delta > \frac{1}{2}$, then $X_t \to X_\infty$, as $t \to \infty$, with $|X_\infty| < \infty$.

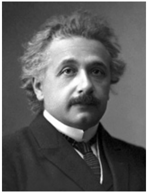
Remark: Similar results for interacting Bose gases with $v_* > 0$, and in the kinetic limit ($\nearrow$ Bauerschmidt-De Roeck-F).

5. L'insoutenable irréversibilité de l'évolution quantique

"Alle Naturwissenschaft ist auf die Voraussetzung der vollständigen kausalen Verknüpfung jeglichen Geschehens begründet." –
Albert Einstein (Zurich, 1910)

Well, is it?

Answering this question from the point of view of Quantum Mechanics is the goal of this part of my lecture. I propose to discuss the *fundamental irreversibility* (that many physicists find "unbearable") of the evolution of *isolated*, but *open* physical systems, as featured by *Quantum Mechanics*.



A. Einstein

W. Heisenberg

N. Bohr

How do we describe an isolated physical system?

It is the irreversibility of quantum-mechanical time evolution that mirrors the basic difference between *Past* and *Future*:

- Past = a factual history of events
- Future = a branching tree of potentialities

We start with a

Pedestrian Definition of an Isolated Physical System

According to quantum theory, an isolated physical system, S, is specified by the following data:

1. A list, $\mathcal{O}_S$, of directly observable/detectable physical properties represented by abstract self-adjoint operators $\hat{X}$;
2. self-adjoint operators, $X(t)$, on a Hilbert space $\mathcal{H}$ representing props. $\hat{X} \in \mathcal{O}_S$ at time t, with $X(t) = U(s,t)X(s)U(t,s)$, where $U(t,s)$, $t, s \in \mathbb{R}$, is a unitary propagator on $\mathcal{H}$ describing time-evolution of operators in the *Heisenberg picture*.

Properties potentially observable at times $\geq t$

Let $\mathcal{E}_{\geq t} \subseteq B(\mathcal{H})$ denote the von Neumann algebra generated by all the operators $\{X(s) | \hat{X} \in \mathcal{O}_S, s \geq t\}$. By definition

$$B(\mathcal{H}) \supseteq \mathcal{E}_{\geq t} \supseteq \mathcal{E}_{\geq s} \supseteq \{X(s) | \hat{X} \in \mathcal{O}_S\}, \text{ for } s > t. \qquad (28)$$

$\neq \leftarrow$ loss of access to information (!)

Loss of access to information is a *fundamental feature* of relat. local quantum theory with massless particles, such as QED ($\nearrow$Buchholz and Roberts): *"2^{nd} Law of the quantum-mech. measurement process"!*
Suppose that S has been prepared in a state ρ at a time t_0. For $t > t_0$, we set

$$\rho_t(A) := \rho(A), \quad \forall A \in \mathcal{E}_{\geq t}.$$

By (28) and the phenomenon of entanglement, ρ_t can be a *mixed state* on $\mathcal{E}_{\geq t}$ *even* if ρ might be a *pure* state on $\mathcal{E}_{\geq t_0}$.

The Fundamental Axiom of Observations/Measurements

Given that S has been prepared in a state ρ at some time t_0, it may happen that, around some later time t, the state ρ_t on the algebra $\mathcal{E}_{\geq t}$ is very close to an *incoherent superposition* of eigen-states of an operator $X(t)$, for some $\hat{X} \in \mathcal{O}_S$; (more precisely, that "$X(t)$ is a function of the density matrix rep. ρ_t".) If this happens then – *Axiom* –

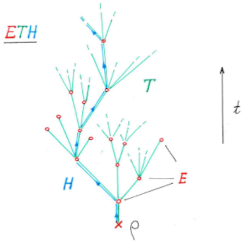
1. $\hat{X}$ is observed/measured around time t;
2. $\hat{X}$ then has a value $\xi \in \sigma(\hat{X})$;
3. to improve prediction of future events, the state ρ_t must then be replaced by the state $\rho_{t,\xi}$ defined by

$$\rho_{t,\xi}(A) := \frac{\rho(\Pi_\xi(t) A \Pi_\xi(t))}{\rho(\Pi_\xi(t))}, \quad \forall A \in \mathcal{E}_{\geq t}, \tag{29}$$

where $\Pi_\xi(t)$ is spect. proj. of $X(t)$ corresp. to the ev. ξ.

"ETH" and Irreversibility

Obviously, eqs. (28) and (29) are quantum-theoretical expressions of a *fundamental irreversibility*: Whenever an event that amounts to the detection of a physical quantity $\hat{X} \in \mathcal{O}_S$ happens in a system S its state does *not* evolve according to a Schrödinger eq., but according to eq. (29). Pictorially:



E: "events" (proj. measnts.), T: "trees" (of states),
H: "histories" ; probs. of "histories" are det. by QM

Along "histories" – whenever events happen – quantities such as energy or angular momentum are *not* conserved.

... n. The irreversible evolution of the Universe

Five basic enigmas:

I. The universe expands (rather than contracts).

II. There appears to be a basic asymmetry in the content of matter and of anti-matter in the universe.

III. Existence of Dark Matter: $p \ll \rho$.

IV. Existence of Dark Energy :$p = \rho$.

V. Ex. tiny, highly homogeneous intergalactic magnetic fields.

Suspicion: There must be a common root for these phenomena!

Possible explanation: Besides $g_{\mu\nu}$, introduce complex axion field, φ, with renormalizable $g\varphi^4$- interaction. Write

$$\varphi = e^{\sigma + i\theta}, \text{ with}$$

$\dot{\sigma}$: "chemical potential" tuning matter – anti-matter asym;
$\dot{\theta}$: couples to helicity, $A \wedge F_A$, of very heavy abelian gauge field, A, conj. to j_{B-L}; $\sigma \leftrightarrow$ Dark Matter; $\theta \leftrightarrow$ Dark Energy.

Initial inflation caused by relaxation of initial configuration, φ_0, of axion towards equilibrium point in field space. Etc., (see blackboard!).

The unbearable arrow of time we can't escape from:

$\Rightarrow$

40 years!

Yet, this creates an "immense feeling of liberty" and gives hope:

the past – a history of facts

$\downarrow$

the future – a "garden of forking paths" (*Borges*)

Thank you!