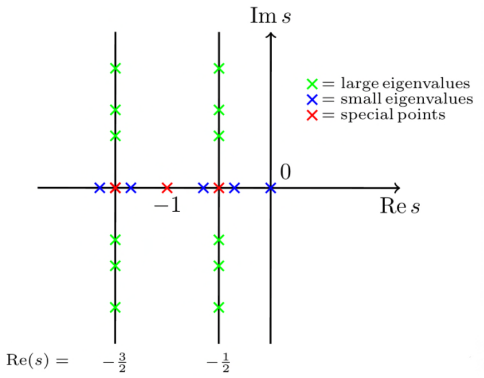
HS 2022: MICROLOCAL METHODS IN DYNAMICAL SYSTEMS – MAT644

LECTURER: DR. MIHAJLO CEKIĆ

Tuesdays 10-12 at Irchel campus, room Y36-J-33.

Who? The course is aimed at Masters students. PhD students/Postdocs are also welcome.

What? In this course we will introduce important tools in modern mathematical analysis with a strong emphasis on applications to dynamics. Originating in the study of partial differential equations, microlocal analysis has found recent exciting applications in the areas of dynamical systems and geometry, a very active field of mathematics.



Syllabus. In the first half of the course we will introduce the tools of *microlocal analysis*: the geometric study of distributions (singular functions) in the phase (cotangent) space. In the other half, we will show how these tools apply to chaotic dynamics (as in the left figure) and draw consequences on some of their classical properties: e.g. ergodicity and how fast the flow '*mixes*'.

The covered topics will include:

- **Pseudodifferential operators.** Natural extension of differential operators, where we will e.g. be able to include the inverse of the Laplace operator, through quantisation of non-polynomial *symbols*. *Symbol classes*. *Stationary phase lemma*.
- **Calculus, mapping properties.** Algebra of pseudodifferentials operators on manifolds. Mapping properties on *Sobolev spaces*. *Elliptic operators*. *Wavefront set*.
- **Anisotropic Sobolev spaces and resonances.** Introduce Sobolev spaces *tailored to the dynamics* of e.g. the geodesic flow on negatively curved surfaces. Definition of *resonances* (as in the right figure).
- **Applications.** Ergodicity and speed of mixing of geodesic flows on negatively curved surfaces. Dynamical zeta functions (time-permitting).

Pre-requisites. Basic distribution theory and Fourier Analysis, which will be recalled at the beginning. Familiarity to basic differential geometry and functional analysis would be useful. We will try to keep the course self-contained.

References. I will provide LaTeXed lecture notes at my webpage.

Further information. Please contact me at mihajlo.cekic@math.uzh.ch (office Y27K04).