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# BV Equivalence between Palatini Gravity and BF Theory in Three Dimensions


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# Introduction

General Relativity and Quantum Field Theory are two very different and until today incompatible theories. Nonetheless, both have a common point, since both are gauge theories.

Gauge theories are a class of quantum field theory, in which the Lagrangian is invariant under certain Lie groups of local transformations. Hence, over non-compact submanifolds of the configuration space the integrand is constant, which makes the path integral ill-defined. The solution given by Faddeev-Popov [14] and BRST (Becchi, Rouet, Stora and Tyutin; [2]), which is an approach of quantizing a field theory with a gauge symmetry, is to quotient out the orbits of the gauge group and to define the quotient space as the related configuration space. Another way of handling gauge theories in a symplectic Lagrangian formalism has been made by Batalin–Vilkovisky (BV) formalism [1], sometimes also called the antifield formalism, which is a generalization of BRST. In BV formalism we work in an extended configuration space, i.e. on the shifted cotangent space of the initial one, and view the path integral as an integral over some Lagrangian submanifold of this extended space. The aim of BV formalism is to handle with such theories, as mentioned at the begin, and their generalizations.

The advantage of using BV formalism instead of the BRST one is, since it is a theory of integration, one is allowed to treat much more general functions than in the BRST formalism, and the integral is well-defined independent of the precise form of these functions. So, BV formalism is very suitable to treat theories with open symmetries. [11]

The BV formalism together with the Batalin–Fradkin–Vilkovisky (BFV) formalism (the hamiltonian case of BV) has been used e.g. to compute the Chern–Simons invariants of manifolds with boundaries [8].

In the BV formalism, one can give a clear definition of an equivalence of (the BV counterpart) of a classical field theory. We will see the definition of strong equivalence in Chapter 3.

We will show in this thesis, that the 3-dimensional General Relativity, in the Cartan–Palatini formalism, is strongly equivalent to BF theory, which is a special case of topological field theory with freely selectable dimension.

The nontrivial part of this equivalence concerns the symmetries of the two theories: in the former they are diffeomorphisms and internal Lorentz' transformations, in the latter they are internal Poincaré transformations.

# Chapter 1

# Lagrangian field theories

## 1.1 The Principle of Least Action

The greatest goal in the field of physics has been to find one principle which would explain all natural phenomena and predict the behaviour of physical systems. That means one seeks one overarching framework in physics linking all physical aspects of the universe in a single paradigm. On the other side what others are trying, is to quantize General Relativity (GR), which is also called Quantum Gravity (QG). GR focuses on gravity which gives us an understanding for the universe in regions of large-scale and high-mass, i.e. stars, galaxies, etc. Quantum Field Theory (QFT), is a theory that focuses on gravitational forces at small scale and low mass objects, i.e. atoms, molecules etc. During several years physicists have shown in experiments that both theories make accurate predictions in their own domain. But they also found out that as GR and QFT are formulated currently, we have not found a way to understand GR in the quantum setting. This is the reason why one wants to find another way to understand these two theories. Although this aim has not been reached yet, one tries to unify the treatment of all classical theories from electromagnetism to GR by phrasing them as a variational problem. As a matter of fact, the standard model is phrased as a "quantization" of the Standard model action functional which is called the **Principle of Least Action**, and the main attempts to quantize the gravitational interaction also assume a variational formulation of GR. (In addition, there are approaches like string theory in which gravity is an emergent phenomenon.)

Leibniz wrote a first formulation of this in a letter in 1707 [13]. He wrote that *"...of all the worlds which could be created, the effective world is that which contains, along with all the inevitable bad, the maximum good"* [20]. Later this principle has been developed further by de Maupertuis, Euler and Lagrange.

Maupertuis [18] formulated the Principle of Least Action in a very understable way:

*"After mediating deeply on this topic, it occurred to me that light, upon passing from one medium to another, has to make a choice, whether to follow the path of shortest distance (the straight line) or the path of least time. But why should it prefer time over space? Light cannot travel both paths at once, yet how does it decide to take one path over another? Rather than taking either of these paths per se, light takes the path that offers a real advantage: light takes the path that minimizes its action. Now I have to define what I mean by "action". When a material body is transported from one point to another, it involves an action that depends on the speed of the*

*body and on the distance it travels. However, the action is neither the speed nor the distance taken separately; rather, it is proportional to the sum of the distances travelled each multiplied by the speed at which they were travelled. Hence, the action increases linearly with the speed of the body and with the distance travelled. This action is the true expense of Nature, which she manages to make as small as possible in the motion of light."* [18]

Euler [19] extended this idea and proposed his version of a principle. But later Lagrange was the one who formulated the principle in 1788 complete generally as a principle of stationary action for a general system of $n$ bodies interacting among themselves.

Nearly 50 years later, William Rowan Hamilton, showed the Principle of Least Action admits other representations and he made the connection between mechanics and optics. In this way, he came to one of the most used form of the Principle of Least Action as a variational principle of mechanics, which is called the Hamilton principle.

Initially the existence of such a principle was not more than a mathematical curiosity but after some time mathematicians and physicists realised that the Principle of Least Action takes a big role in mechanics but also in electrodynamics and thermodynamics. Later Feynman showed that this principle has also its valid place in quantum mechanics. One of the result has been attained by the fact that Einstein's theory of special relativity had shown that it occupied an outstanding position among the laws of physics because the *action* defined by Hamilton [12] is an invariant with regard to all Lorentz' transformations. Also the Einstein–Hilbert action is invariant under all general coordinate transformations, i.e. invariant under the action of the group of spacetime diffeomorphisms. This means that it does not depend on the reference frame of the observer. This short summary of the history of Principle of Least Action is an explanation for the search of such an ideal. [20]

## 1.2 Lagrangian formalism

To get the setting from a mechanical system to a field theoretical one, we use the Lagrangian formulation. But before stating about the Lagrangian, what is exactly a Field Theory (FT)?

### 1.2.1 Classical Field Theory

FT is a theory that studies fields which are usually sections of vector bundles or sheaves, e.g. gravitational or electromagnetic (EM) field.

Weather forecast is a daily example of an application of field theory, which we can grasp on. One assigns for the wind velocity over a location a vector to each point in space respresenting the direction of the movement of air at that particular point. So, the set of these vectors at a given time gives us a vector field. During the whole day the directions of the wind change and so also the directions of the vectors point.

This is for the intuitive explanation at the moment. More formally, we will see it when we take a look at the Lagrangian Formulation.

### 1.2.2 Lagrangian Formulation of a Mechanical System

Let the space of fields be maps from $I = [0, 1]$ to the spacetime manifold $R^N$ endowed with the euclidean metric. Let us restrict to systems for which the forces are conservative. These forces can be derived by differentiating a potential energy function $U(r_1, \ldots, r_N)$, i.e.

$$F_i = -\frac{\partial U}{\partial r_i} \ \forall\ 1 \le i \le N$$

Assuming the forces are conservative, i.e. the total energy is conserved, so

$$E = K + U = \frac{1}{2}\sum_{i=1}^{N} m_i \dot{r}_i^2 + U(r_1, \ldots, r_N)$$

where $K$ is the kinetic energy function $K(r_1, \ldots, r_N)$.

To check that this is true, we differentiate $E$ with respect to time:

$$\frac{dE}{dt} = \sum_{i=1}^{N} m_i \dot{r}_i \cdot \ddot{r}_i + \sum_{i=1}^{N} \frac{\partial U}{\partial r_i} \cdot \dot{r}_i = 0$$

We have used the Newton's law $\ddot{r}_i = F_i/m_i$ and the conservative force definition $F_i = -\partial U/\partial r_i$ to get this result.

The Lagrangian Formulation is an elegant way to formulate classical mechanics for conservative systems. The Lagrangian function $L$ is defined to be the difference between the kinetic and potential energies which is being expressed as a function of positions and velocities. Let $r = r_1, \ldots, r_N$ be the positions and $\dot{r} = \dot{r}_1, \ldots, \dot{r}_N$ be the corresponding velocities. Then L is defined as:

$$L(r, \dot{r}) = K - U = \frac{1}{2}\sum_{i=1}^{N} m_i \dot{r}_i^2 - U(r_1, \ldots, r_N)$$

The equations of motion (EOM) are given by the Euler–Lagrange equation (ELE):

Let us consider a general functional

$$S = \int_a^b L(r, \dot{r}, t)dt, \tag{1.1}$$

such that the values of function $r$ at the end points are fixed. Our goal is to find $r$ such that it minimizes (or maximizes) $S$ and it satisfies the boundary conditions. One can reduce this problem such that it is enough to find a function $r$ that makes the variation in $S$ to be equal to zero,

$$\delta S = 0. \tag{1.2}$$

Let us now derive a differential form equivalent to the variational form equation (1.2). One can calculate the variation in $S$ the following way:

$$\delta S = \delta \int_a^b L(r, \dot{r}, t) dt \tag{1.3}$$

$$= \int_a^b (\frac{\partial L}{\partial r}\delta r + \frac{\partial L}{\partial \dot{r}}\delta \dot{r}) dt \tag{1.4}$$

where $\delta \dot{r}$ is the variation of $\dot{r}$, which we can write as

$$\delta \dot{r} = \delta(\frac{dr}{dt}) = \frac{d(\delta r)}{dt} \tag{1.5}$$

Thus, we can write the second term on the right-hand side of equation (1.4) as

$$\int_a^b (\frac{\partial L}{\partial \dot{r}}\delta \dot{r}) dt = \int_a^b (\frac{\partial L}{\partial \dot{r}}\frac{d\delta r}{dt}) dt \tag{1.6}$$

$$= \int_a^b \frac{\partial L}{\partial \dot{r}} d\partial r \tag{1.7}$$

$$= \frac{\partial L}{\partial \dot{r}}\delta r\Big|_a^b - \int_a^b \delta r d(\frac{\partial L}{\partial \dot{r}}) \tag{1.8}$$

Since the values of $r$ are fixed at the end points, $\delta r = 0$ at $a$ and $b$.

$$\int_a^b (\frac{\partial L}{\partial \dot{r}}\delta \dot{r}) dt = -\int_a^b \delta r d(\frac{\partial L}{\partial \dot{r}}) \tag{1.9}$$

$$= -\int_a^b \delta r \frac{d}{dt}(\frac{\partial L}{\partial \dot{r}}) dt \tag{1.10}$$

Using this in equation (1.4)

$$\delta S = \int_a^b \delta r[\frac{\partial L}{\partial r} - \frac{d}{dt}(\frac{\partial L}{\partial \dot{r}})] dt \tag{1.11}$$

Then, $\delta S = 0$ is written as

$$\int_a^b \delta r[\frac{\partial L}{\partial r} - \frac{d}{dt}(\frac{\partial L}{\partial \dot{r}})] dt = 0 \tag{1.12}$$

Since (1.12) must hold for arbitrary $\delta r$, the only possible way is

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{r}_i}) - \frac{\partial L}{\partial r_i} = 0 \tag{1.13}$$

which is called the Euler–Lagrange equation.

The advantage of the Lagrangian formulation is that one can not just use it in the Cartesian coordinates, but in any set of coordinates.

### 1.2.3 Lagrangian Formulation in Field Theory

In field theory, we replace the independent variable $t \in [0,1]$ by an event in spacetime $s = (x, y, z, t)$ on a manifold $M$ and the dependent variables (before position and velocity) by $\varphi(x, y, z, t)$, such that ELE are obtained by:

$$\delta S = 0$$

where S is the action, i. e. a local functional of $\varphi_i(s)$, which is a section of some vector bundle or sheaf on spacetime, with their derivatives and s itself.

$$S[\varphi_i] = \int \mathcal{L}(\varphi_i(s), \frac{\partial \varphi_i(s)}{\partial s^\mu}, s^\mu) d^n s$$

where $s = \{s^\mu\}$ is the set of $n$ independent variables of the system.

In a field theoretical framework we do not call it anymore Lagrangian but Lagrangian density $\mathcal{L}$.

## 1.3 BV formalism

The BV formalism provides a cohomological reformulation of several important questions of QFT. Usually BV formalism are used for following kind of problems:

1. determination of gauge invariant operators
2. problem of consistent deformation of a theory

The latter means the following: Taking a free field theory and finding out which interaction terms are allowed to satisfy the consistency requirements, i.e. gauge invariance, is a typical problem in mathematical physics. If an interaction term is given, it is easy to check if the term is gauge invariant. But in some cases, one would like to begin from a free action principle and try to classify all possible interaction terms, which is much more difficult.

In mathematics, a BV algebra is a graded supercommutative algebra with unit 1 and a second-order nilpotent operator $\Delta$ of degree $-1$. However, the main definitions we will need for the rest of the paper will not use the full-fledged BV formalism but just its $-1$-symplectic framework, as follows:

**Definition 1.** *(see [7] and references therein) A BV manifold is a 4-tuple $(\mathcal{F}, \Omega, S, Q)$, consisting of a $(-1)$-symplectic graded manifold $(\mathcal{F}, \Omega)$, a cohomological vector field $Q \in \mathfrak{X}[1](\mathcal{F})$, i.e. such that $[Q, Q] = 0$ and a BV action, a function $S : \mathcal{F} \to \mathbb{R}$ such that $\iota_Q \Omega = \delta S$.*

**Definition 2.** *[7] A d-dimensional BV theory $\mathfrak{F}_M$ is the assignment to every closed d-dimensional manifold $M$ of a BV manifold $(\mathcal{F}_M, \Omega_M, S_M, Q_M)$, given in terms of local data.*

**Remark 3.** *[7] Observe that when a boundary is present the equation for the Hamiltonian function* $\iota_Q \Omega = \delta S$ *is likely to be spoiled by a boundary term. This is handled in the BV-BFV formalism of Cattaneo, Mnev and Reshetikhin [9]. However, because the whole construction is local, it still makes sense to consider the BV theory as if there were no boundary.*

The definition we will need to state the main result of this thesis is the following:

**Definition 4.** *[7] A strong equivalence between the BV theories*

$$\mathfrak{F}_M^{(1|2)} = (\mathcal{F}_M^{(1|2)}, \Omega_M^{(1|2)}, S_M^{(1|2)}, Q_M^{(1|2)}) \tag{1.14}$$

*is a graded symplectomorphism* $\Phi : (\mathcal{F}_M^{(1)}, \Omega_M^{(1)}) \to (\mathcal{F}_M^{(2)}, \Omega_M^{(2)})$ *preserving the BV action, i.e.* $\Phi^* S_M^{(2)} = S_M^{(1)}$

# Chapter 2

# GR and BF theory

## 2.1 GR Theory

In a mechanical system an action is defined as the integral over time of a Lagrangian density, from which the system's dynamics can be determined by the principle of least action. The same happens for more general physical theories like the theory of General Relativity, our current model for the gravitational interaction, which can be phrased in terms of a Lagrangian density and its associated variational problem. Here, we will consider the triadic formulation of GR (somestimes attributed to Cartan and Palatini). We will be concerned with the three-dimensional version of such theory.

In 3d let $P \to M$ be an $SO(2,1)$ bundle over an orientable manifold $M$ without boundary, i.e. $\partial M = 0$. Let $\mathcal{W} \to M$ be an associated vector bundle equipped with an orientation and a smooth fiberwise Minkowski metric $(\mathcal{W}, \eta)$. A triad is a bundle isomorphism $e : TM \to \mathcal{W}$ covering the identity. Thus, let us consider $e \in \Omega^1_{nd}(M, \mathcal{W})$, where the subscript $nd$ stands for non-degenerate, and the isomorphism $so(2,1) \simeq \Lambda^2 W \simeq W$ induced by the metric and the internal Hodge dual. Given a connection $\omega \in A_P$ on $P$, its curvature $F_\omega$ will be regarded as a $\bigwedge^2 W$-valued two-form.

Let us denote by $F_{GR} := \Omega^1_{nd}(M, \mathcal{W}) \times A_P$ the space of physical fields and consider the action functional

$$S^0_{GR}(\Lambda) = \int_M Tr[e \wedge F_\omega + \frac{\Lambda}{3} e^3] \tag{2.1}$$

where the trace denotes the pairing with volume form in $\Lambda^3 W$ and $\Lambda \in \mathbb{R}$ the cosmological constant.

The Euler–Lagrange equations are given by

$$F_\omega + \Lambda e^2 = 0$$
$$d_\omega e = 0$$

The second equation fixes $\omega = \omega(e)$ and pulls back to the Levi–Civita connection along $e$. Then, the first equation reduces to Einstein equation. One can show that $S^0_{GR}$ can be reduced to the standard Einstein–Hilbert action functional $S_{EH} = \int_M \sqrt{-|g|}R[g]$, where $g = e^*\eta$ is the metric [7].

We extend $S^0_{GR}$ in BV formalism, as explained in [7] (and references therein). Adapting the construction of [6] to three dimensions, we extend the above version to a BV manifold by declaring the following transformations

$$\begin{aligned} Q_{GR}(e) &= L^{\omega}_{\xi} e + [\chi, e] \\ Q_{GR}(\omega) &= \iota_\xi F_\omega + d_\omega \chi \\ Q_{GR}(\xi) &= L_\xi \xi \\ Q_{GR}(\chi) &= \frac{1}{2}([\chi,\chi] - \iota_\xi \iota_\xi F_\omega) \end{aligned}$$

where $L^{\omega}_{\xi} := [\iota_\xi, d_\omega]$ and $[\chi, e]$ denotes the action of the Lie Algebra on $V$. This defines the cohomological vector field [1] $Q_{GR}$ of degree 1, with $\xi \in \mathfrak{X}[1](M)$ and $\chi \in \Omega[1](M, adP)$. The space of BV fields is $\mathcal{F}_{GR} = T^*[-1](F_{GR} \times \mathfrak{X}[1](M) \times \Omega(M, adP))$. By denoting cotangent fields with a dagger, we will get the corresponding BV action by

$$S_{GR}(\Lambda) = \int_M Tr[e \wedge F_\omega + e^\dagger(L^\omega_\xi e + [\chi, e]) + A^\dagger(\iota_\xi F_\omega + d_\omega \chi) + \frac{1}{2}\chi^\dagger([\chi,\chi] - \iota_\xi\iota_\xi F_\omega) + \frac{1}{2}\iota_{[\xi,\xi]}\xi^\dagger] \tag{2.2}$$

The 4-tuple $\mathfrak{F}_{GR} := (\mathcal{F}_{GR}, \Omega_{GR}, Q_{GR}, S_{GR})$ defines a BV theory. [5]

In order to prove later the equivalence between the BV theories $\mathcal{F}_{GR}$ and $\mathcal{F}_{BF}$, which will be defined in the following chapter, we will adapt the strong equivalence between the Cartan–Palatini BV formulation of gravity presented in [5] and the version suggested by Piguet [21] to three dimensions. Let us consider the following assignments

$$\begin{aligned} se' &= L_{\xi'} e' + [\chi', e'] \\ s\omega' &= L_{\xi'}\omega' + d_{\omega'}\chi' \\ s\xi' &= \frac{1}{2}[\xi', \xi'] \\ s\chi' &= L_{\xi'}\chi' + \frac{1}{2}[\chi', \chi'] \end{aligned}$$

which defines a cohomological vector field $s$ over

$$F_{PP} := \Omega^1(M, \mathcal{W}) \times \Omega^1(M, \Lambda^2\mathcal{W}) \times \mathfrak{X}[1](M) \times \Omega^0[1](M, adP) \ni (e', \omega', \xi', \chi') \tag{2.3}$$

Such an extension will be denoted by $S_{PP}$, where PP stands for Palatini–Piguet:

$$\begin{aligned} S_{PP}(\Lambda) = S^0_{GR}(\Lambda) + \int_M Tr\{e^{\dagger'}(L_{\xi'}e' + [\chi', e']) + \omega^{\dagger'}(L_{\xi'}\omega' + d_{\omega'}\chi')\} + \\ + Tr\{\chi^{\dagger'}(L_{\xi'}\chi' + \frac{1}{2}[\chi', \chi'])\} + \frac{1}{2}\iota_{[\chi',\chi']}\xi^{\dagger'} \end{aligned} \tag{2.4}$$

[1] Recall that this means: $[Q, Q] = 0$

**Proposition 5.** *(Cattaneo, Schiavina, Selliah [7]) The BV theory* $\mathfrak{F}_{PP} = (\mathcal{F}_{PP}, \Omega_{PP}, S_{PP}, s)$ *is strongly equivalent to* $\mathfrak{F}_{GR}$.

**Remark 6.** *Actually this is proven in [5]. [7] is a specialised version to the three-dimensional case.*

## 2.2 BF Theory

BF theory is a special kind of topological field theory which can be defined on a manifold $M$ of any dimension. The theory is called BF theory because the action contains a term given by the wedge-product of an $(n-2)$-form $B$ of the adjoint type times the curvature $F$ of a connection $A$, where $n = \dim(M)$. We consider the $SO(2,1)$ principal bundle $P \to M$ and the associated oriented Minkowski bundle $\mathcal{W} \to M$. To define BF theory, we consider the space of classical, i.e. 0-degree, fields $F_{BF} := \Omega^1(M, \Lambda^2\mathcal{W}) \times \mathcal{A}_P \ni (B, A)$ with the action functional

$$S^0_{BF} := \int_M < B, F_A > \equiv \int_M Tr[BF_A] \tag{2.5}$$

We can construct the degree-1 vector field *via* the assignment

$$\begin{aligned} Q_{BF}(B) &= d_A\tau + [c, B] \\ Q_{BF}(A) &= d_A c \\ Q_{BF}(\tau) &= [c, \tau] \\ Q_{BF}(c) &= \frac{1}{2}[c, c] \end{aligned}$$

over the space of BV fields

$$\mathcal{F}_{BF} := T^*[-1](F_{BF} \times \Omega^0(M, \mathcal{W}) \times \Omega^0(M, \Lambda^2\mathcal{W})) \ni (B, A, \tau, c) \tag{2.6}$$

The BV extended action is then

$$S_{BF} = \int_M Tr[BF_A + B^\dagger(d_A\tau + [c, B]) + A^\dagger d_A c + \tau^\dagger[c, \tau] + \frac{1}{2}c^\dagger[c, c]] \tag{2.7}$$

and the 4-tuple $\mathfrak{F}_{BF} := (\mathcal{F}_{BF}, \Omega_{BF}, Q_{BF}, S_{BF})$ defines a BV theory [7].

**Remark 7.** *BF theory and GR are obviously equivalent in degree 0, by sending* $B$ *to* $e$ *and* $A$ *to* $\omega$. *The symmetries can be shown to be equivalent "on shell", i.e. when the equations of motion are enforced, but in this thesis we will get a result which will establish a deeper equivalence, that holds off-shell.*

# Chapter 3

# Strong Equivalence of 3d Gravity and BF-Theory

**Theorem 8.** *$\mathfrak{F}_{GR}$ and $\mathfrak{F}_{BF}$ are strongly equivalent.*

**Remark 9.** *We will actually show that $\mathfrak{F}_{PP}$ and $\mathfrak{F}_{BF}$ are strongly equivalent. Thus, using Proposition* (5), *$\mathfrak{F}_{GR}$ and $\mathfrak{F}_{BF}$ are also strongly equivalent.*

## 3.1 Strategy

To prove this, we use the fields' total degrees from the definitions given above. The table is in the Appendix.

For better readability, let us define

$$\int (\ldots) := \int Tr[\ldots]$$

On one hand, we have the Palatini-Piguet action which we defined before:

$$S_{PP} = \int eF_\omega + e^\dagger L_\xi e + e^\dagger[\chi, e] + \omega^\dagger L_\xi \omega + \omega^\dagger d_\omega \chi + \chi^\dagger L_\xi \chi + \frac{1}{2}\chi^\dagger[\chi, \chi] + \frac{1}{2}\iota_{[\xi,\xi]}\xi^\dagger \tag{3.1}$$

On the other hand, we have the BF action, called $S_{BF}$:

$$S_{BF} = \int BF_A + B^\dagger d_A \tau + B^\dagger[c, B] + A^\dagger d_A c + \frac{1}{2}c^\dagger[c, c] + \tau^\dagger[c, \tau] \tag{3.2}$$

Our goal is to construct a canonical transformation $\phi$ between the spaces of fields, such that it preserves the respective actions, i.e. $\phi^* S_{BF} = S_{PP}$.

## 3.2 Proof

The proof will be divided in five steps:

1. Guessing the generating function $G$
2. Deriving the field transformation from $G$
3. Pulling back the action and fixing the parameters such that we get $\phi^* S_{BF} = S_{PP} + \text{extraterms}$
4. Showing that extraterms $= 0$
5. Composing with the symplectomorphism $\mathfrak{F}_{PP} \to \mathfrak{F}_{GR}$

**Step 1** Our ansatz for the generating function $G$ will depend on some parameters $a, b, y, z \in \mathbb{R}$, that we will be able to fix step by step, comparing $\phi^* S_{BF}$ with $S_{PP}$.

**Remark 10.** *The ansatz is in principle made by taking all possible combinations of the fields of a given degree, but for simplicity we use the parameters $a, b, y, z$ as follows:*

$$\begin{aligned} G &= (B^\dagger + b\iota_\xi \tau^\dagger)e + (c + b\iota_\xi A + a\iota_\xi\iota_\xi B^\dagger + y\iota_\xi\iota_\xi\iota_\xi\tau^\dagger)\chi^\dagger + (A + b\iota_\xi B^\dagger + z\iota_\xi\iota_\xi\tau^\dagger)\omega^\dagger \\ &= B^\dagger e + b\iota_\xi\tau^\dagger e + c\chi^\dagger + b\iota_\xi A\chi^\dagger + a\iota_\xi\iota_\xi B^\dagger\chi^\dagger + y\iota_\xi\iota_\xi\iota_\xi\tau^\dagger\chi^\dagger + A\omega^\dagger + b\iota_\xi B^\dagger\omega^\dagger + z\iota_\xi\iota_\xi\tau^\dagger\omega^\dagger \end{aligned} \tag{3.3}$$

**Step 2** We get a canonical transformation by introducing a generating function $G(\Phi, \Psi^\dagger)$ such that $\Psi = \frac{\delta G}{\delta \Psi^\dagger}$ and $\Phi^\dagger = \frac{\delta G}{\delta \Phi}$. In our case, $\Phi^\dagger = (A^\dagger, B^\dagger, c^\dagger, \tau^\dagger)$ and $\Psi = (\omega, e, \chi, \xi)$. But since we are working with graded Lie algebra, sign changes can occur and we have to take care of that. (Remark 25 of [7]) Using this, we can calculate the transformations for $A, B, c, \tau, A^\dagger, B^\dagger$ and $c^\dagger$. For now, we leave $\tau^\dagger$ as it is, observing that the following system of equations can be solved in virtue of the following fact:

**Lemma 11.** *The function $G$*

$$G = B^\dagger e + b\iota_\xi\tau^\dagger e + c\chi^\dagger + b\iota_\xi A\chi^\dagger + a\iota_\xi\iota_\xi B^\dagger\chi^\dagger + y\iota_\xi\iota_\xi\iota_\xi\tau^\dagger\chi^\dagger + A\omega^\dagger + b\iota_\xi B^\dagger\omega^\dagger + z\iota_\xi\iota_\xi\tau^\dagger\omega^\dagger \tag{3.4}$$

*defines a symplectomorphism. The system of equations*

$$\Psi = \frac{\delta G}{\delta \Psi^\dagger} \tag{3.5}$$

$$\Phi^\dagger = \frac{\delta G}{\delta \Phi} \tag{3.6}$$

*is invertible.*

*Proof.* The transformations are:

$$B = \frac{\delta G}{\delta B^\dagger} = e + a\iota_\xi\iota_\xi\chi^\dagger - b\iota_\xi\omega^\dagger \tag{3.7}$$

$$\tau = \frac{\delta G}{\delta \tau^\dagger} = -b\iota_\xi e - y\iota_\xi\iota_\xi\iota_\xi\chi^\dagger + z\iota_\xi\iota_\xi\omega^\dagger \tag{3.8}$$

$$\omega = \frac{\delta G}{\delta \omega^\dagger} = A + b\iota_\xi B^\dagger + z\iota_\xi\iota_\xi\tau^\dagger \tag{3.9}$$

$$A^\dagger = \frac{\delta G}{\delta A} = -b\iota_\xi\chi^\dagger + \omega^\dagger \tag{3.10}$$

$$\chi = \frac{\delta G}{\delta \chi^\dagger} = c + b\iota_\xi A + a\iota_\xi\iota_\xi B^\dagger + y\iota_\xi\iota_\xi\iota_\xi\tau^\dagger \tag{3.11}$$

$$c^\dagger = \frac{\delta G}{\delta c} = \chi^\dagger \tag{3.12}$$

$$e^\dagger = \frac{\delta G}{\delta e} = B^\dagger + b\iota_\xi\tau^\dagger \Rightarrow B^\dagger = e^\dagger - b\iota_\xi\tau^\dagger \tag{3.13}$$

Note that in the case of $\xi^\dagger$ the derivative will have a negative sign in front of it. This comes from the sign convention for odd generating functions.

$$\begin{aligned}\xi^\dagger = -\frac{\delta G}{\delta \xi_i} &= -(-b\tau^\dagger e_i - bA\chi_i^\dagger + 2aB^\dagger\iota_\xi\chi_i^\dagger - 3y\tau^\dagger\iota_\xi\iota_\xi\chi_i^\dagger - bB^\dagger\omega_i^\dagger + 2z\tau^\dagger\iota_\xi\omega_i^\dagger) \\ &= b\tau^\dagger e_i + bA\chi_i^\dagger - 2aB^\dagger\iota_\xi\chi_i^\dagger + 3y\tau^\dagger\iota_\xi\iota_\xi\chi_i^\dagger + bB^\dagger\omega_i^\dagger - 2z\tau^\dagger\iota_\xi\omega_i^\dagger \end{aligned} \tag{3.14}$$

Since (3.9) $= A + b\iota_\xi(e^\dagger - b\iota_\xi\tau^\dagger) + z\iota_\xi\iota_\xi\tau^\dagger = A + b\iota_\xi e^\dagger - b^2\iota_\xi\iota_\xi\tau^\dagger + z\iota_\xi\iota_\xi\tau^\dagger = A + b\iota_\xi e^\dagger + (z - b^2)\iota_\xi\iota_\xi\tau^\dagger$, from this it follows:

$$A = \omega - b\iota_\xi e^\dagger - (z - b^2)\iota_\xi\iota_\xi\tau^\dagger \tag{3.15}$$

Since

$$\begin{aligned}(3.11) = c + b\iota_\xi A + a\iota_\xi\iota_\xi(e^\dagger - b\iota_\xi\tau^\dagger) + y\iota_\xi\iota_\xi\iota_\xi\tau^\dagger &= c + b\iota_\xi A + a\iota_\xi\iota_\xi e^\dagger - ab\iota_\xi\iota_\xi\iota_\xi\tau^\dagger + y\iota_\xi\iota_\xi\iota_\xi\tau^\dagger \\ &= c + b\iota_\xi A + a\iota_\xi\iota_\xi e^\dagger + (y - ab)\iota_\xi\iota_\xi\iota_\xi\tau^\dagger\end{aligned}$$

$$\begin{aligned}\Rightarrow c = \chi - b\iota_\xi A - a\iota_\xi\iota_\xi e^\dagger - (y - ab)\iota_\xi\iota_\xi\iota_\xi\tau^\dagger &= \chi - b\iota_\xi(\omega - b\iota_\xi e^\dagger - (z - b^2)\iota_\xi\iota_\xi\tau^\dagger) - a\iota_\xi\iota_\xi e^\dagger - (y - ab)\iota_\xi\iota_\xi\iota_\xi\tau^\dagger \\ &= \chi - b\iota_\xi\omega + b^2\iota_\xi\iota_\xi e^\dagger + b(z - b^2)\iota_\xi\iota_\xi\iota_\xi\tau^\dagger - a\iota_\xi\iota_\xi e^\dagger - (y - ab)\iota_\xi\iota_\xi\iota_\xi\tau^\dagger \\ &= \chi - b\iota_\xi\omega + (b^2 - a)\iota_\xi\iota_\xi e^\dagger + (b(z - b^2) - (y - ab))\iota_\xi\iota_\xi\iota_\xi\tau^\dagger\end{aligned}$$

From this it follows:

$$c = \chi - b\iota_\xi\omega + (b^2 - a)\iota_\xi\iota_\xi e^\dagger + (b(z - b^2) - (y - ab))\iota_\xi\iota_\xi\iota_\xi\tau^\dagger \tag{3.16}$$

This problem reduces to checking whether the following matrix is invertible:

$$\begin{pmatrix} 1 & b\iota_\xi \\ (a\iota_\xi\chi_a^\dagger - b\omega_a^\dagger) & -be_a \end{pmatrix} \begin{pmatrix} B^\dagger \\ \tau^\dagger \end{pmatrix} = \begin{pmatrix} e^\dagger \\ \xi_a^\dagger \end{pmatrix} \tag{3.17}$$

since $e^\dagger = \frac{\delta G}{\delta e} = B^\dagger + b\iota_\xi\tau^\dagger$ and $\xi_a^\dagger = \frac{\delta G}{\delta \xi^a} = B^\dagger(a\iota_\xi\chi_a^\dagger - b\omega_a^\dagger) - b\tau^\dagger e_a$.

We claim that we can separate the first matrix on the left side into one being a sum of a diagonal and a nilpotent matrix.

$$\begin{pmatrix} 1 & 0 \\ 0 & -be_a \end{pmatrix} + \begin{pmatrix} 0 & b\iota_\xi \\ (a\iota_\xi\chi_a^\dagger - b\omega_a^\dagger) & 0 \end{pmatrix} = D + N \tag{3.18}$$

where D stands for the diagonal matrix and N for the nilpotent one. To prove the nilpotent property, one calculates the powers of N and sees that it is nilpotent of order 4. Since $e$ is non-degenerate, $D$ is also non-degenerate and so we can find the inverse of $D+N$ by means of this formula:

$$(D+N)^{-1} = D^{-1}(1 - (ND^{-1}) + (ND^{-1})^2 + (ND^{-1})^3 + (ND^{-1})^4) \tag{3.19}$$

This proves the invertibility of the matrix and so $G$ is the generating function of a symplectomorphism.

□

Instead of inverting the system, we use these formulas:

$$\phi(A) = \omega - b\iota_\xi e^\dagger - (z - b^2)\iota_\xi\iota_\xi\tau^\dagger \tag{3.20}$$

$$\phi(B) = e + a\iota_\xi\iota_\xi\chi^\dagger - b\iota_\xi\omega^\dagger \tag{3.21}$$

$$\phi(c) = \chi - b\iota_\xi\omega + (b^2 - a)\iota_\xi\iota_\xi e^\dagger + (b(z - b^2) - (y - ab))\iota_\xi\iota_\xi\iota_\xi\tau^\dagger \tag{3.22}$$

$$\phi(\tau) = -b\iota_\xi e - y\iota_\xi\iota_\xi\iota_\xi\chi^\dagger + z\iota_\xi\iota_\xi\omega^\dagger \tag{3.23}$$

$$\phi(A^\dagger) = \omega^\dagger - b\iota_\xi\chi^\dagger \tag{3.24}$$

$$\phi(B^\dagger) = e^\dagger - b\iota_\xi\tau^\dagger \tag{3.25}$$

$$\phi(c^\dagger) = \chi^\dagger \tag{3.26}$$

$$\phi(\tau^\dagger) \text{ is left unsolved at the moment.} \tag{3.27}$$

The right combination of parameters will eliminate $\tau^\dagger$ from all expressions.

The previous lemma yields an almost-explicit expression for the symplectomorphism $\phi$ such that $\phi^*\Omega_{BF} = \Omega_{GR}$. Back to the actions, our goal is to fix the parameters of $G$ by comparing $\phi^*S_{BF}$ with $S_{PP}$. To make the calculation easier, we will forget about the Lie-Brackets in (3.1) at the moment.

**Step 3** Now, we calculate the pullback $\phi^* S_{BF}$.

$$\phi^* S_{BF} = \int (e + a\iota_\xi\iota_\xi\chi^\dagger - b\iota_\xi\omega^\dagger)F_{\omega - b\iota_\xi e^\dagger - (z-b^2)\iota_\xi\iota_\xi\tau^\dagger} + (e^\dagger - b\iota_\xi\tau^\dagger)d_{\omega - b\iota_\xi e^\dagger - (z-b^2)\iota_\xi\iota_\xi\tau^\dagger}(-b\iota_\xi e - y\iota_\xi\iota_\xi\iota_\xi\chi^\dagger + z\iota_\xi\iota_\xi\omega^\dagger)$$
$$+ (\omega^\dagger - b\iota_\xi\chi^\dagger)d_{\omega - b\iota_\xi e^\dagger - (z-b^2)\iota_\xi\iota_\xi\tau^\dagger}(\chi - b\iota_\xi\omega + (b^2 - a)\iota_\xi\iota_\xi e^\dagger + (b(z-b^2) - (y-ab))\iota_\xi\iota_\xi\iota_\xi\tau^\dagger)$$
$$+ (e^\dagger - b\iota_\xi\tau^\dagger)[\chi - b\iota_\xi\omega + (b^2-a)\iota_\xi\iota_\xi e^\dagger + (b(z-b^2) - (y-ab))\iota_\xi\iota_\xi\iota_\xi\tau^\dagger, e + a\iota_\xi\iota_\xi\chi^\dagger - b\iota_\xi\omega^\dagger]$$
$$+ \frac{1}{2}\chi^\dagger[\chi - b\iota_\xi\omega + (b^2-a)\iota_\xi\iota_\xi e^\dagger + (b(z-b^2) - (y-ab))\iota_\xi\iota_\xi\iota_\xi\tau^\dagger, \chi - b\iota_\xi\omega + (b^2-a)\iota_\xi\iota_\xi e^\dagger$$
$$+ (b(z-b^2) - (y-ab))\iota_\xi\iota_\xi\iota_\xi\tau^\dagger] + \tau^\dagger[\chi - b\iota_\xi\omega + (b^2-a)\iota_\xi\iota_\xi e^\dagger + (b(z-b^2) - (y-ab))\iota_\xi\iota_\xi\iota_\xi\tau^\dagger, -b\iota_\xi e - y\iota_\xi\iota_\xi\iota_\xi\chi^\dagger$$
$$+ z\iota_\xi\iota_\xi\omega^\dagger]$$

For the time being, we will discard lines 3 to 6, i.e. all Lie brackets, and come back to them later on. Using the computations done in the Appendix, we get:

$$\begin{aligned}\phi^* S_{BF} = \int &(e + a\iota_\xi\iota_\xi\chi^\dagger - b\iota_\xi\omega^\dagger)(d\omega - bd\iota_\xi e^\dagger - (z-b^2)d\iota_\xi\iota_\xi\tau^\dagger) + (e^\dagger - b\iota_\xi\tau^\dagger)(-bd\iota_\xi e - yd\iota_\xi\iota_\xi\iota_\xi\chi^\dagger + zd\iota_\xi\iota_\xi\omega^\dagger)\\
&+ (\omega^\dagger - b\iota_\xi\chi^\dagger)(d\chi - bd\iota_\xi\omega + (b^2-a)d\iota_\xi\iota_\xi e^\dagger + (b(z-b^2) - (y-ab))d\iota_\xi\iota_\xi\iota_\xi\tau^\dagger)\\
= \int &ed\omega - bed\iota_\xi e^\dagger - (z-b^2)ed\iota_\xi\iota_\xi\tau^\dagger + a\iota_\xi\iota_\xi\chi^\dagger d\omega - ab\iota_\xi\iota_\xi\chi^\dagger d\iota_\xi e^\dagger - a(z-b^2)\iota_\xi\iota_\xi\chi^\dagger d\iota_\xi\iota_\xi\tau^\dagger - b\iota_\xi\omega^\dagger d\omega\\
&+ b^2\iota_\xi\omega^\dagger\iota_\xi e^\dagger + b(z-b^2)\iota_\xi\omega^\dagger d\iota_\xi\iota_\xi\tau^\dagger - be^\dagger d\iota_\xi e - ye^\dagger d\iota_\xi\iota_\xi\iota_\xi\chi^\dagger + ze^\dagger d\iota_\xi\iota_\xi\omega^\dagger + b^2\iota_\xi\tau^\dagger d\iota_\xi e\\
&+ by\iota_\xi\tau^\dagger d\iota_\xi\iota_\xi\iota_\xi\chi^\dagger - bz\iota_\xi\tau^\dagger d\iota_\xi\iota_\xi\omega^\dagger + \omega^\dagger d\chi - b\omega^\dagger d\iota_\xi\omega + (b^2-a)\omega^\dagger d\iota_\xi\iota_\xi e^\dagger\\
&+ (b(z-b^2) - (y-ab))\omega^\dagger d\iota_\xi\iota_\xi\iota_\xi\tau^\dagger - b\iota_\xi\chi^\dagger d\chi + b^2\iota_\xi\chi^\dagger d\iota_\xi\omega - b(b^2-a)\iota_\xi\chi^\dagger d\iota_\xi\iota_\xi e^\dagger - b(b(z-b^2) - (y-ab))\iota_\xi\chi^\dagger d\iota_\xi\iota_\xi\iota_\xi\tau^\dagger\end{aligned} \tag{3.28}$$

Some of the parameters can be guessed already by looking at the previous computation. For example, by looking all summands which contain a combination of $\omega$ and $\omega^\dagger$, we can already guess that $b$ is going to be equal to one. The strategy to fix the remaining ones will be that of reconstructing the term $\frac{1}{2}\iota_{[\xi,\xi]}\xi^\dagger$ in (3.1) from what is left in (3.28).

$$\frac{1}{2}\iota_{[\xi,\xi]}\xi_i^\dagger = \frac{1}{2}[b\tau^\dagger e_i + bA\chi_i^\dagger - 2aB^\dagger\iota_\xi\chi_i^\dagger + 3y\tau^\dagger\iota_\xi\iota_\xi\chi_i^\dagger + bB^\dagger\omega_i^\dagger - 2z\tau^\dagger\iota_\xi\omega_i^\dagger][\xi,\xi]^i$$

$$\begin{aligned}\frac{1}{2}\iota_{[\xi,\xi]}\xi^\dagger &= \frac{1}{2}[-b\tau^\dagger\iota_{[\xi,\xi]}e - bA\iota_{[\xi,\xi]}\chi^\dagger + 2aB^\dagger\iota_{[\xi,\xi]}\iota_\xi\chi^\dagger - 3y\tau^\dagger\iota_{[\xi,\xi]}\iota_\xi\iota_\xi\chi^\dagger - bB^\dagger\iota_{[\xi,\xi]}\omega^\dagger + 2z\tau^\dagger\iota_{[\xi,\xi]}\iota_\xi\omega^\dagger]\\
&= \frac{1}{2}[-b\tau^\dagger\iota_{[\xi,\xi]}e - b\omega\iota_{[\xi,\xi]}\chi^\dagger + b^2\iota_\xi e^\dagger\iota_{[\xi,\xi]}\chi^\dagger + b(z-b^2)\iota_\xi\iota_\xi\tau^\dagger\iota_{[\xi,\xi]}\chi^\dagger + 2ae^\dagger\iota_{[\xi,\xi]}\iota_\xi\chi^\dagger - 2ab\iota_\xi\tau^\dagger\iota_{[\xi,\xi]}\iota_\xi\chi^\dagger - 3y\tau^\dagger\iota_{[\xi,\xi]}\iota_\xi\iota_\xi\chi^\dagger\\
&\qquad - be^\dagger\iota_{[\xi,\xi]}\omega^\dagger + b^2\iota_\xi\tau^\dagger\iota_{[\xi,\xi]}\omega^\dagger + 2z\tau^\dagger\iota_{[\xi,\xi]}\iota_\xi\omega^\dagger]\\
&= \frac{1}{2}[-b\tau^\dagger\iota_{[\xi,\xi]}e - b\omega\iota_{[\xi,\xi]}\chi^\dagger - be^\dagger\iota_{[\xi,\xi]}\omega^\dagger + (2a-b^2)e^\dagger\iota_\xi\iota_{[\xi,\xi]}\chi^\dagger + (b(z-b^2) + 2ab - 3y)\tau^\dagger\iota_\xi\iota_\xi\iota_{[\xi,\xi]}\chi^\dagger +\\
&\qquad + (2z-b^2)\tau^\dagger\iota_\xi\iota_{[\xi,\xi]}\omega^\dagger]\end{aligned} \tag{3.29}$$

After we have calculated the needed parts, now we calculate the parameters $a, b, y, z$ by comparing the summands in (3.29) with the summands in (3.28).

**Claim 1** The parameter b is equal to 1.

*Proof.*

$$\begin{aligned}\int -bed\iota_\xi e^\dagger - be^\dagger d\iota_\xi e &= -bd\iota_\xi e^\dagger e - be^\dagger d\iota_\xi e \text{ , since } e \text{ has even total degree} \\ &= \int -b\iota_\xi e^\dagger de - be^\dagger d\iota_\xi e \text{ , because of Stoke's Theorem} \\ &= \int be^\dagger \iota_\xi de - be^\dagger d\iota_\xi e = \int be^\dagger(\iota_\xi de - d\iota_\xi e) = \int be^\dagger L_\xi e \Leftrightarrow b = 1\end{aligned}$$

□

**Claim 2** The parameter z is equal to $\frac{1}{2}$.

*Proof.*

$$\begin{aligned}\int -(z-b^2)ed\iota_\xi\iota_\xi\tau^\dagger + b^2\iota_\xi\tau^\dagger d\iota_\xi e &= \int -(z-b^2)d\iota_\xi\iota_\xi\tau^\dagger e + b^2\iota_\xi\tau^\dagger d\iota_\xi e = \int -(z-b^2)\iota_\xi\iota_\xi\tau^\dagger de - b^2\tau^\dagger\iota_\xi d\iota_\xi e \\ &= \int -(z-b^2)\tau^\dagger\iota_\xi\iota_\xi de - b^2\tau^\dagger\iota_\xi d\iota_\xi e = \int -\frac{1}{2}\tau^\dagger\iota_{[\xi,\xi]}e\end{aligned}$$

The second equality holds, because of Stoke's Theorem, i.e.

$$\int_M d(\iota_\xi\iota_\xi\tau^\dagger e) = \int_{\delta M} d\iota_\xi\iota_\xi\tau^\dagger e + (-1)^{|d||\iota_\xi\iota_\xi\tau^\dagger|}\iota_\xi\iota_\xi\tau^\dagger de = \int_{\delta M} d\iota_\xi\iota_\xi\tau^\dagger e - \iota_\xi\iota_\xi\tau^\dagger de = 0$$

Using $b = 1 \Rightarrow z = \frac{1}{2}$ □

**Claim 3** The parameter a is equal to $\frac{1}{2}$.

*Proof.*

$$\begin{aligned}\int a\iota_\xi\iota_\xi\chi^\dagger d\omega + b^2\iota_\xi\chi^\dagger d\iota_\xi\omega &= \int a\chi^\dagger\iota_\xi\iota_\xi d\omega - b^2\chi^\dagger\iota_\xi d\iota_\xi\omega = \int \frac{1}{2}\chi^\dagger\iota_\xi\iota_\xi d\omega - \chi^\dagger\iota_\xi d\iota_\xi\omega \\ &= \int -(\chi^\dagger\iota_\xi d\iota_\xi\omega - \frac{1}{2}\chi^\dagger\iota_\xi\iota_\xi d\omega) = \int -(\frac{1}{2}\chi^\dagger\iota_{[\xi,\xi]}\omega + \frac{1}{2}\chi^\dagger d\iota_\xi\iota_\xi\omega) = \int -\frac{1}{2}\chi^\dagger\iota_{[\xi,\xi]}\omega\end{aligned}$$

From the first to second equality, we use $b^2 = 1$ and $a = \frac{1}{2}$ and in the second last equality $\frac{1}{2}\chi^\dagger d\iota_\xi\iota_\xi\omega$ vanishes, since $\omega$ is a 1-form. $\Rightarrow a = \frac{1}{2}$ □

**Claim 4** The parameter y is equal to $\frac{1}{6}$.

*Proof.*

$$\begin{aligned}\int -ab\iota_\xi\iota_\xi\chi^\dagger d\iota_\xi e^\dagger - ye^\dagger d\iota_\xi\iota_\xi\iota_\xi\chi^\dagger - b(b^2-a)\iota_\xi\chi^\dagger d\iota_\xi\iota_\xi e^\dagger &= \int -abd\iota_\xi e^\dagger\iota_\xi\iota_\xi\chi^\dagger - ye^\dagger d\iota_\xi\iota_\xi\iota_\xi\chi^\dagger - b(b^2-a)d\iota_\xi\iota_\xi e^\dagger\iota_\xi\chi^\dagger \\ &= \int -ab\iota_\xi e^\dagger d\iota_\xi\iota_\xi\chi^\dagger - ye^\dagger d\iota_\xi\iota_\xi\iota_\xi\chi^\dagger - b(b^2-a)\iota_\xi\iota_\xi e^\dagger d\iota_\xi\chi^\dagger \\ &= \int abe^\dagger\iota_\xi d\iota_\xi\iota_\xi\chi^\dagger - ye^\dagger d\iota_\xi\iota_\xi\iota_\xi\chi^\dagger - b(b^2-a)e^\dagger\iota_\xi\iota_\xi d\iota_\xi\chi^\dagger \\ &= \int \frac{1}{2}e^\dagger\iota_\xi d\iota_\xi\iota_\xi\chi^\dagger - \frac{1}{6}e^\dagger d\iota_\xi\iota_\xi\iota_\xi\chi^\dagger - \frac{1}{2}e^\dagger\iota_\xi\iota_\xi d\iota_\xi\chi^\dagger = \int -\frac{1}{2}e^\dagger\iota_\xi L_\xi\iota_\xi\chi^\dagger - \frac{1}{6}e^\dagger d\iota_\xi\iota_\xi\iota_\xi\chi^\dagger \\ &= \int -(\frac{1}{2}e^\dagger\iota_\xi L_\xi\iota_\xi\chi^\dagger + \frac{1}{6}e^\dagger d\iota_\xi\iota_\xi\iota_\xi\chi^\dagger) = 0\end{aligned}$$

This term vanishes by the following lemma:

**Lemma 12.** *Remark 9 of [7]*

*From the fact that* $\iota_{[\xi,\xi]}\iota_\xi\alpha = \iota_\xi\iota_{[\xi,\xi]}\alpha$, *in the case* $\alpha \in \Omega^{top}(M)$ *and* $dim(M) = 3$,*we deduce*

$$\iota_\xi L_\xi^\omega\iota_\xi\alpha = -\frac{1}{3}d_\omega\iota_\xi^3\alpha. \tag{3.30}$$

*Similarly, from* $\iota_\xi^2\iota_{[\xi,\xi]}\alpha = \iota_\xi\iota_{[\xi,\xi]}\iota_\xi\alpha = \iota_{[\xi,\xi]}\iota_\xi^2\alpha$, *when* $\alpha$ *is a top-form we have*

$$\iota_\xi^2 d_\omega\iota_\xi^2\alpha = \frac{4}{3}\iota_\xi d_\omega\iota_\xi^3\alpha. \tag{3.31}$$

$\Rightarrow y = \frac{1}{6}$ in Claim 4. ☐

So the generating function G reads:

$$G = (B^\dagger + \iota_\xi\tau^\dagger)e + (c + \iota_\xi A + \frac{1}{2}\iota_\xi\iota_\xi B^\dagger + \frac{1}{6}\iota_\xi\iota_\xi\iota_\xi\tau^\dagger)\chi^\dagger + (A + \iota_\xi B^\dagger + \frac{1}{2}\iota_\xi\iota_\xi\tau^\dagger)\omega^\dagger \tag{3.32}$$

**Step 4** Now, we will get to $\phi^* S_{BF} = S_{PP} + extraterms$. We will check if these extra terms would vanish using the parameters we have found above.

**Claim 1:** All the summands containing a combination of $\chi^\dagger$ and $\tau^\dagger$ will vanish.

*Proof.* First we collect all the terms in (3.28).

$$\int -a(z-b^2)\iota_\xi\iota_\xi\chi^\dagger d\iota_\xi\iota_\xi\tau^\dagger + by\iota_\xi\tau^\dagger d\iota_\xi\iota_\xi\iota_\xi\chi^\dagger - b[b(z-b^2) - (y-ab)]\iota_\xi\chi^\dagger d\iota_\xi\iota_\xi\iota_\xi\tau^\dagger = 0$$

This term vanishes by Lemma 12, i.e. $\iota_\xi^2 d_\omega\iota_\xi^2\alpha = \frac{4}{3}\iota_\xi d_\omega\iota_\xi^3\alpha$, with $\alpha$ a top form. Analogously, the same argument works on (3.29). ☐

**Claim 2:** All the summands in (3.28) containing a combination of $\omega^\dagger$ and $\omega$ coincide with $\omega^\dagger L_\xi \omega$ in (3.1) .

*Proof.*

$$\int -b\iota_\xi\omega^\dagger d\omega - b\omega^\dagger d\iota_\xi\omega = \int -\iota_\xi\omega^\dagger d\omega - \omega^\dagger d\iota_\xi\omega = \int \omega^\dagger\iota_\xi d\omega - \omega^\dagger d\iota_\xi\omega = \int \omega^\dagger L_\xi\omega$$

□

**Claim 3:** All the summands in (3.28) containing a combination of $\omega^\dagger$ and $e^\dagger$ coincide with $-\frac{1}{2}e^\dagger\iota_{[\xi,\xi]}\omega^\dagger$ in (3.29) .

*Proof.*

$$\begin{aligned}\int b^2\iota_\xi\omega^\dagger d\iota_\xi e^\dagger + ze^\dagger d\iota_\xi\iota_\xi\omega^\dagger + (b^2-a)\omega^\dagger d\iota_\xi\iota_\xi e^\dagger &= \int b^2 d\iota_\xi e^\dagger\iota_\xi\omega^\dagger + ze^\dagger d\iota_\xi\iota_\xi\omega^\dagger + (b^2-a)d\iota_\xi\iota_\xi e^\dagger\omega^\dagger \\ = \int d\iota_\xi e^\dagger\iota_\xi\omega^\dagger + \frac{1}{2}e^\dagger d\iota_\xi\iota_\xi\omega^\dagger + \frac{1}{2}d\iota_\xi\iota_\xi e^\dagger\omega^\dagger &= \int \iota_\xi e^\dagger d\iota_\xi\omega^\dagger + \frac{1}{2}e^\dagger d\iota_\xi\iota_\xi\omega^\dagger + \frac{1}{2}\iota_\xi\iota_\xi e^\dagger d\omega^\dagger \\ = \int -e^\dagger\iota_\xi d\iota_\xi\omega^\dagger + \frac{1}{2}e^\dagger d\iota_\xi\iota_\xi\omega^\dagger + \frac{1}{2}e^\dagger\iota_\xi\iota_\xi d\omega^\dagger &= \int -\frac{1}{2}e^\dagger\iota_{[\xi,\xi]}\omega^\dagger\end{aligned}$$

□

**Claim 4:** All the summands containing a combination of $\omega^\dagger$ and $\tau^\dagger$ will vanish.

*Proof.* First we collect all the terms in (3.28).

$$\begin{aligned}&\int -bz\iota_\xi\tau^\dagger d\iota_\xi\iota_\xi\omega^\dagger + [b(z-b^2)-(y-ab)]\omega^\dagger d\iota_\xi\iota_\xi\iota_\xi\tau^\dagger + b(z-b^2)\iota_\xi\omega^\dagger d\iota_\xi\iota_\xi\tau^\dagger \\ &= \int -\frac{1}{2}\iota_\xi\tau^\dagger d\iota_\xi\iota_\xi\omega^\dagger - \frac{1}{6}d\iota_\xi\iota_\xi\iota_\xi\tau^\dagger\omega^\dagger - \frac{1}{2}d\iota_\xi\iota_\xi\tau^\dagger\iota_\xi\omega^\dagger \\ &= \int -\frac{1}{2}\iota_\xi\iota_\xi d\iota_\xi\tau^\dagger\omega^\dagger + \frac{1}{2}\iota_\xi d\iota_\xi\iota_\xi\tau^\dagger\omega^\dagger - \frac{1}{6}d\iota_\xi\iota_\xi\iota_\xi\tau^\dagger\omega^\dagger \\ &= \int -\frac{1}{2}\iota_\xi L_\xi\iota_\xi\tau^\dagger\omega^\dagger - \frac{1}{6}d\iota_\xi\iota_\xi\iota_\xi\tau^\dagger\omega^\dagger = 0\end{aligned}$$

Using the same Remark 9 in [7] as we already used before. □

Until now we have ignored nearly all Lie brackets. Since in $S_{PP}$ no other Lie-Brackets unless the ones with $\chi$ appear, we have to check if the brackets in $\phi^* S_{BF}$ vanish. One checks that all brackets that contain any contraction with $\iota_\xi$ will vanish, but other brackets like $[\chi,\chi]$ will have to remain. We will explicitly report the computations for some of the easier and more complex terms in the computations section in the Appendix. The remaining ones will follow from analogous arguments.

So, the parameters $b=1, a=\frac{1}{2}, z=\frac{1}{2}, y=\frac{1}{6}$ work fine and (3.32) is correct.

**Step 5:** Since we know the strong equivalence between $\mathfrak{F}_{PP}$ and $\mathfrak{F}_{GR}$ by Proposition (5) in Chapter 2, one can compose the two generating functions and find the one for the composite symplectomorphism. For more details, I would recommend to read [7].

## 3.3 Conclusion

Our goal was to show that there exists a strong equivalence between the BV theories $\mathfrak{F}_{GR}$ and $\mathfrak{F}_{BF}$, i.e. there is a graded symplectomorhpism $\Phi$ preserving the BV action, so $\Phi^* S_{BF} = S_{GR}$. We have proven this by dividing the proof into five steps. First we guessed the generating function $G$ with some open parameters for which we did not know the actual values, and then we derived the field transformation from it. As third step, we calculated the pullback and found out the actual parameters by comparing the summands. Then, we have checked if the extra terms really vanish. Finally, we completed the proof using a Proposition from Chapter 2 and telling that one can compose the two generating function to find the composite one. This proves the theorem at the begin of this chapter, saying that there exists a canonical transformation between the symplectic spaces $\mathcal{F}_{GR}$ and $\mathcal{F}_{BF}$, which also preserves the action functional.

Since we showed the strong equivalence between the two theories, it gives a strong clue that their boundary theories are linked, too. For this, one could use Alexandrov, Kontsevich, Schwarz, Zaboronsky (AKSZ) theory to reduce the interaction canonically. Moreover, one would restrict our field to their boundary $\partial M$ and check whether the symplectomorphism reduces to the spaces of boundary fields.

Considering the way Cattaneo, Mnev and Reshetikhin paved for quantisation of BF theory in the BV formalism, the quantisation of GR in the three-dimensional case should also be accessible using the same techniques. Therefore, it can be compared with existing results in the literature, like [23].

# Chapter 4

# Appendix

## 4.1 Fields' Total Degrees

| FIELD | FORM DEGREE $\Omega^\bullet(M)$ | VECTOR DEGREE $\Lambda^\bullet V$ | GHOST $[\bullet]$ | TOTAL DEGREE |
|---|---|---|---|---|
| $\omega$ | 1 | 2 | 0 | 3 |
| $e$ | 1 | 1 | 0 | 2 |
| $\chi$ | 0 | 2 | 1 | 3 |
| $\xi$ | - | - | 1 | 1 |
| $\omega^\dagger$ | 2 | 1 | -1 | 2 |
| $e^\dagger$ | 2 | 2 | -1 | 3 |
| $\chi^\dagger$ | 3 | 1 | -2 | 2 |
| $\xi^\dagger$ | 1⊗3 | - | -2 | 2 |
| A | 1 | 2 | 0 | 3 |
| B | 1 | 1 | 0 | 2 |
| c | 0 | 2 | 1 | 3 |
| $\tau$ | 0 | 1 | 1 | 2 |
| $A^\dagger$ | 2 | 1 | -1 | 2 |
| $B^\dagger$ | 2 | 2 | -1 | 3 |
| $c^\dagger$ | 3 | 2 | -2 | 3 |
| $\tau^\dagger$ | 3 | 2 | -2 | 3 |

## 4.2 Properties

1. $[A,B] = -(-1)^{|A||B|}[B,A]$

2. $[A,B] = AB - (-1)^{|A||B|}BA$

3. $[c,[A,B]] = [[c,A]B] + (-1)^{|c||A|}[A,[c,B]]$

4. $d_A[c,\tau] = [d_Ac,\tau] + (-1)^{|d_A||c|}[c,d_A\tau] = [d_Ac,\tau] - [c,d_A\tau]$

5. $d_A\bullet = d\bullet + [A,\bullet]$

6. ON FORMS: $L^\omega_\xi e = [\iota_\xi, d_\omega]e = \iota_\xi d_\omega e - d_\omega\iota_\xi e$, because $|\xi| = 1$.

7. $|\iota_\xi| = |\xi| - 1$, $|d_Ac| = |c| + 1$, $|\delta A| = |A| + 1$, $|d| = |\delta| = 1$.

8. $\iota_{\frac{1}{2}[\xi,\xi]} = \frac{1}{2}\iota_{[\xi,\xi]} = \frac{1}{2}[L_\xi,\iota_\xi] = \iota_\xi d\iota_\xi - \frac{1}{2}d\iota_\xi\iota_\xi - \frac{1}{2}\iota_\xi\iota_\xi d$

9. $F_\omega = d_\omega\omega - \frac{1}{2}[\omega,\omega] = d\omega + \frac{1}{2}[\omega,\omega]$

10. $\iota_\xi[\omega,\omega] = [\iota_\xi\omega,\omega] + [\omega,\iota_\xi\omega]$

11. if $\alpha\wedge\beta \in \Omega^{top+1}(M) \Rightarrow 0 = \iota_\xi(\alpha\wedge\beta) = \iota_\xi\alpha\wedge\beta + (-1)^{(|\xi|-1)|\alpha|}\alpha\wedge\iota_\xi\beta \Rightarrow \iota_\xi\alpha\wedge\beta = (-1)^{(|\xi|-1)|\alpha|+1}\alpha\wedge\iota_\xi\beta$
 $\longrightarrow$ if $|\xi| = 1 \Rightarrow \iota_\xi\alpha\wedge\beta = -\alpha\wedge\iota_\xi\beta$

12. $\alpha\wedge\beta \in \Omega^{top+1}(M) \Rightarrow \iota_\xi\iota_\xi\alpha\beta = \alpha\iota_\xi\iota_\xi\beta$

13. $\int_M d_\omega(\iota_\xi ee^\dagger) = \int_{\delta M}\iota_\xi ee^\dagger$, therefore if the boundary $\partial M$ is empty $\Rightarrow \int_M d_\omega(\iota_\xi ee^\dagger) = 0$

## 4.3 Computations

**Pullback:**

1.

$$\int F_{\omega-b\iota_\xi e^\dagger-(z-b^2)\iota_\xi\iota_\xi\tau^\dagger} = \int d(\omega-b\iota_\xi e^\dagger-(z-b^2)\iota_\xi\iota_\xi\tau^\dagger)+\frac{1}{2}[\omega-b\iota_\xi e^\dagger-(z-b^2)\iota_\xi\iota_\xi\tau^\dagger,\omega-b\iota_\xi e^\dagger-(z-b^2)\iota_\xi\iota_\xi\tau^\dagger]$$
$$= \int d\omega - bd\iota_\xi e^\dagger - (z-b^2)d\iota_\xi\iota_\xi\tau^\dagger + \frac{1}{2}[\omega,\omega] + \frac{1}{2}[b\iota_\xi e^\dagger, b\iota_\xi e^\dagger] + \frac{1}{2}[(z-b^2)\iota_\xi\iota_\xi\tau^\dagger,(z-b^2)\iota_\xi\iota_\xi\tau^\dagger] - \frac{1}{2}[\omega,b\iota_\xi e^\dagger]$$
$$-\frac{1}{2}[\omega,(z-b^2)\iota_\xi\iota_\xi\tau^\dagger] - \frac{1}{2}[b\iota_\xi e^\dagger,\omega] + \frac{1}{2}[b\iota_\xi e^\dagger,(z-b^2)\iota_\xi\iota_\xi\tau^\dagger] - \frac{1}{2}[(z-b^2)\iota_\xi\iota_\xi\tau^\dagger,\omega] + \frac{1}{2}[(z-b^2)\iota_\xi\iota_\xi\tau^\dagger, b\iota_\xi e^\dagger]$$
$$= \int F_\omega - bd_\omega\iota_\xi e^\dagger - (z-b^2)d_\omega\iota_\xi\iota_\xi\tau^\dagger + \frac{1}{2}[...,...]$$

We forget all Lie-Brackets, for the moment.

2.

$$\int d_{\omega-b\iota_\xi e^\dagger-(z-b^2)\iota_\xi\iota_\xi\tau^\dagger}(-b\iota_\xi e - y\iota_\xi\iota_\xi\iota_\xi\chi^\dagger + z\iota_\xi\iota_\xi\omega^\dagger) = \int d(-b\iota_\xi e - y\iota_\xi\iota_\xi\iota_\xi\chi^\dagger + z\iota_\xi\iota_\xi\omega^\dagger)$$
$$+ [\omega - b\iota_\xi e^\dagger - (z-b^2)\iota_\xi\iota_\xi\tau^\dagger, -b\iota_\xi e - y\iota_\xi\iota_\xi\iota_\xi\chi^\dagger + z\iota_\xi\iota_\xi\omega^\dagger] = \int -bd\iota_\xi e - yd\iota_\xi\iota_\xi\iota_\xi\chi^\dagger + zd\iota_\xi\iota_\xi\omega^\dagger + [...,...]$$

**Brackets in $\phi^* S_{BF}$:**

1. (a) $\int a\iota_\xi\iota_\xi\chi^\dagger \cdot \frac{1}{2}[(z-b^2)\iota_\xi\iota_\xi\tau^\dagger, (z-b^2)\iota_\xi\iota_\xi\tau^\dagger] = \int \frac{1}{16}\iota_\xi\iota_\xi\chi^\dagger[\iota_\xi\iota_\xi\tau^\dagger, \iota_\xi\iota_\xi\tau^\dagger] = 0$

   (b) $\int -b\iota_\xi\chi^\dagger[-(z-b^2)\iota_\xi\iota_\xi\tau^\dagger, (b(z-b^2)-(y-ab))\iota_\xi\iota_\xi\iota_\xi\tau^\dagger] = \int -\iota_\xi\chi^\dagger[\frac{1}{2}\iota_\xi\iota_\xi\tau^\dagger, -\frac{1}{6}\iota_\xi\iota_\xi\iota_\xi\tau^\dagger]$

   $= \int \frac{1}{12}\iota_\xi\chi^\dagger[\iota_\xi\iota_\xi\tau^\dagger, \iota_\xi\iota_\xi\iota_\xi\tau^\dagger] = 0$

   This vanishes by Remark 9 in [7].

2. (a) $\int e \cdot \frac{1}{2}[b\iota_\xi e^\dagger, b\iota_\xi e^\dagger] = \int \frac{1}{2}e[\iota_\xi e^\dagger, \iota_\xi e^\dagger]$

   (b) $\int e^\dagger[(b^2-a)\iota_\xi\iota_\xi e^\dagger, e] = \int \frac{1}{2}e^\dagger[\iota_\xi\iota_\xi e^\dagger, e]$

   (c) $\int e^\dagger[-b\iota_\xi e^\dagger, -b\iota_\xi e] = \int e^\dagger[\iota_\xi e^\dagger, \iota_\xi e]$

   $\Rightarrow \int \frac{1}{2}e[\iota_\xi e^\dagger, \iota_\xi e^\dagger] + \frac{1}{2}e^\dagger[\iota_\xi\iota_\xi e^\dagger, e] + e^\dagger[\iota_\xi e^\dagger, \iota_\xi e] = \int \frac{1}{2}[\iota_\xi e^\dagger, \iota_\xi e^\dagger]e + \frac{1}{2}[e^\dagger, \iota_\xi\iota_\xi e^\dagger]e + [e^\dagger, \iota_\xi e^\dagger]\iota_\xi e$

   $= \int \frac{1}{2}[\iota_\xi e^\dagger, \iota_\xi e^\dagger]e + \frac{1}{2}[e^\dagger, \iota_\xi\iota_\xi e^\dagger]e - \iota_\xi[e^\dagger, \iota_\xi e^\dagger]e = \int -\frac{1}{2}[\iota_\xi e^\dagger, \iota_\xi e^\dagger]e - \frac{1}{2}[e^\dagger, \iota_\xi\iota_\xi e^\dagger]e = 0,$

   since $0 = \iota_\xi\iota_\xi[e^\dagger, e^\dagger] = 2[\iota_\xi e^\dagger, \iota_\xi e^\dagger] + 2[\iota_\xi\iota_\xi e^\dagger, e^\dagger] \Leftrightarrow [\iota_\xi\iota_\xi e^\dagger, e^\dagger] = -[\iota_\xi e^\dagger, \iota_\xi e^\dagger]$ is a 4-form which vanishes because we are working in dimension 3.

3. (a) $\int -b\iota_\xi\tau^\dagger[\chi, -b\iota_\xi\omega^\dagger] = \int -\iota_\xi\tau^\dagger[\chi, -\iota_\xi\omega^\dagger] = \int \iota_\xi\tau^\dagger[\chi, \iota_\xi\omega^\dagger] = \int -\tau^\dagger\iota_\xi[\chi, \iota_\xi\omega^\dagger]$

   (b) $\int \omega^\dagger[-(z-b^2)\iota_\xi\iota_\xi\tau^\dagger, \chi] = \int \omega^\dagger[\frac{1}{2}\iota_\xi\iota_\xi\tau^\dagger, \chi] = \int \frac{1}{2}\omega^\dagger[\iota_\xi\iota_\xi\tau^\dagger, \chi] = \int \frac{1}{2}[\iota_\xi\iota_\xi\tau^\dagger, \chi]\omega^\dagger = \int \frac{1}{2}\iota_\xi\iota_\xi\tau^\dagger[\chi, \omega^\dagger]$

   $= \int \frac{1}{2}\tau^\dagger\iota_\xi\iota_\xi[\chi, \omega^\dagger]$

   (c) $\int \tau^\dagger[\chi, z\iota_\xi\iota_\xi\omega^\dagger] = \int \tau^\dagger[\chi, \frac{1}{2}\iota_\xi\iota_\xi\omega^\dagger] = \int \frac{1}{2}\tau^\dagger[\chi, \iota_\xi\iota_\xi\omega^\dagger]$

   $\Rightarrow \int -\tau^\dagger\iota_\xi[\chi, \iota_\xi\omega^\dagger] + \frac{1}{2}\tau^\dagger\iota_\xi\iota_\xi[\chi, \omega^\dagger] + \frac{1}{2}\tau^\dagger[\chi, \iota_\xi\iota_\xi\omega^\dagger] = \int -\tau^\dagger([\iota_\xi\chi, \iota_\xi\omega^\dagger] + (-1)^{|\iota_\xi\chi||\iota_\xi\omega^\dagger|}[\chi, \iota_\xi\iota_\xi\omega^\dagger])$

   $+\frac{1}{2}\tau^\dagger([\iota_\xi\iota_\xi\chi, \omega^\dagger] + (-1)^{|\iota_\xi\chi||\iota_\xi\omega^\dagger|}2\cdot[\iota_\xi\chi, \iota_\xi\omega^\dagger] + (-1)^{|\iota_\xi\chi||\iota_\xi\omega^\dagger|}[\chi, \iota_\xi\iota_\xi\omega^\dagger] + \frac{1}{2}\tau^\dagger[\chi, \iota_\xi\iota_\xi\omega^\dagger]$

   $= \int -\tau^\dagger([\iota_\xi\chi, \iota_\xi\omega^\dagger] + [\chi, \iota_\xi\iota_\xi\omega^\dagger]) + \frac{1}{2}\tau^\dagger([\iota_\xi\iota_\xi\chi, \omega^\dagger] + 2\cdot[\iota_\xi\chi, \iota_\xi\omega^\dagger] + [\chi, \iota_\xi\iota_\xi\omega^\dagger] + \frac{1}{2}\tau^\dagger[\chi, \iota_\xi\iota_\xi\omega^\dagger].$

   Since $\chi$ is a 0-form there does not exist a contraction of it. So,

   $= \int -\tau^\dagger[\chi, \iota_\xi\iota_\xi\omega^\dagger] + \frac{1}{2}\tau^\dagger[\chi, \iota_\xi\iota_\xi\omega^\dagger] + \frac{1}{2}\tau^\dagger[\chi, \iota_\xi\iota_\xi\omega^\dagger] = 0$

   Which is what we wanted.

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