SYMPLECTIC AND POISSON GEOMETRY II

PROF. DR. A. S. CATTANEO

Sommersemester 2006

1. Graded linear algebra

(1) Superspaces, graded vector spaces, filtered vector spaces
(2) Suspensions
(3) Morphisms and graded morphisms
(4) Supertrace and superdeterminant
(5) Super, graded and filtered algebras
(6) Graded Lie algebras [GLAs], graded Poisson algebras [GPAs], n-Poisson algebras, Gerstenhaber algebras; examples
(7) The graded symmetric algebra
(8) The associated graded algebra of a filtered algebra
(9) Left and right derivations
(10) The Moyal product on a graded vector space with constant Poisson structure
(11) Differential GLAs [DGLAs] and GPAs [DGPAs]
(12) Graded coalgebras; graded coderivations
(13) L_∞-algebras (Stasheff's and Voronov's sign conventions)

2. The BRS method (after Kostant and Sternberg)

(1) Marsden–Weinstein reduction
(2) Koszul resolution
(3) Lie algebra cohomology
(4) The BRS differential and its cohomology
(5) Strategies for quantization
(6) The Clifford algebra as quantization of the exterior algebra of a vector space with scalar product
(7) The action of the orthogonal group
(8) Creation and annihilation operators
(9) Quantization of a finite-dimensional quadratic Lie algebra and the anomaly
(10) Anomaly-free quantization of a finite-dimensional Lie algebra plus its dual
(11) Modules of the Clifford algebra

(12) The infinite-dimensional case and the anomaly
(13) Deformation quantization description

3. The BVF method (after Stasheff)

(1) Coisotropic submanifolds of a Poisson manifold and their associated Lie algebroids
(2) Coisotropes in a Poisson algebra and their associated Lie–Rinehart algebras
(3) Lie algebroid (Lie–Rinehart) cohomology and reduction
(4) Koszul resolution
(5) The BVF existence and uniqueness theorem of a DGPA structure up to canonical transformations on the the BVF complex (using homological perturbation theory); the infinite-dimensional case
(6) BVF cohomology and reduction
(7) Strategies for quantization
(8) "Gauge fixing;" application to topological quantum mechanics, heat kernels and index theorems

4. Graded manifolds

(1) Definitions
(2) Graded vector fields
(3) The graded Euler vector field
(4) Cohomological vector fields, differential graded manifolds, reinterpretation of BRS and BVF methods; geometrical interpretation of L_∞-algebras
(5) Grassmann integration
(6) The Berezinian bundle
(7) The divergence operator
(8) Change of variables and the Berezinian of a transformation
(9) Graded differential forms; the de Rham differential
(10) Integral forms (see next Section)
(11) The GLA of multivector fields
(12) Graded manifolds of maps

5. The BRST method

(1) Integration of invariant functions on principal bundles and the Faddeev–Popov determinant
(2) Reinterpretation as integrals on graded manifolds (ghosts, antighosts, Lagrange multipliers)
(3) BRST cohomology and gauge-fixing-independent integration

(4) Infinite dimensions

6. Graded symplectic geometry

(1) Graded symplectic linear algebra; structure theorems
(2) Graded symplectic manifolds; Darboux coordinates
(3) Main facts on symplectic forms and Hamiltonian vector fields
(4) Normal form of odd symplectic manifolds
(5) Symplectic manifolds of degree -1 and their induced cohomology
(6) Reinterpretation of the Berezinian
(7) Integral forms (after Manin); integration on submanifolds
(8) The Legendre mapping
(9) The global version of the BVF method
(10) The L_∞-algebra of a coisotropic submanifold

7. The BV method

(1) Symplectic manifolds of degree -1 and the canonical BV operator
(2) BV cohomology and integration on Lagrangian submanifolds
(3) Berezinians and BV operators
(4) BV algebras
(5) The quantum master equation (QME)
(6) The classical master equation (CME)
(7) From the CME to the QME: anomalies
(8) The CME and symmetries
(9) Quantum, BRST, and classical observables
(10) Generating functions, nondegenerate Morse families, and the gauge fixing fermion; the extended action
(11) Reinterpretation of the BRST method
(12) The effective action and Zinn-Justin's theorem
(13) Application to functional integrals: renormalization of quantum field theories with symmetry

References

Textbooks.

(1) J. Zinn-Justin, *Quantum field theory and critical phenomena*, Second edition, International Series of Monographs on Physics **85**, Oxford Science Publications, (The Clarendon Press, Oxford University Press, New York, 1993).

(2) A. S. Cattaneo, B. Keller, C. Torossian and A. Bruguières, *Déformation, Quantification, Théorie de Lie*, Panoramas et Synthèse **20** (2005), viii+186 pages; Part III.

Papers.

(1) A. S. Cattaneo, "On the BV formalism," unpublished, http://www.math.unizh.ch/reports/07_05.pdf
(2) ———, "From topological field theory to deformation quantization and reduction," in *Proceedings of the International Congress of Mathematicians, Madrid, Spain, 2006*, (ed. M. Sanz-Solé, J. Soria, J. L. Varona, J. Verdera), **Vol. III**, 338–365 (European Mathematical Society, 2006); http://www.math.unizh.ch/fileadmin/user/asc/publikation/asc-v2.pdf
(3) H. M. Khudaverdian and Th. Th. Voronov, "Geometry of differential operators, odd Laplacians, and homotopy algebras," math.DG/0402292
(4) A. Rogers, "Gauge fixing and equivariant cohomology," hep-th/0505241
(5) B. Kostant and S. Sternberg, "Symplectic reduction, BRS cohomology, and infinite-dimensional Clifford algebras," Ann. Phys. **176**, 49–113 (1987).
(6) A. Schwarz, "Geometry of Batalin–Vilkovisky quantization," Commun. Math. Phys. **155**, 249–260 (1993).
(7) P. Severa, "On the origin of the BV operator on odd symplectic supermanifolds," math.DG/0506331
(8) J. Stasheff, "Homological reduction of constrained Poisson algebras," J. Diff. Geom. **45**, 221–240 (1997).