QUANTUM MECHANICS FOR MATHEMATICIANS

PROF. DR. A. S. CATTANEO

FS 2011

Programm

(1) Classical mechanics: Principle of the least action. Lagrangian mechanics. Symmetries and Noether's theorem. The Kepler problem. The Legendre transform. Hamiltonian mechanics. The classical action. Poisson brackets. Canonical transformations and generating functions. Symplectic manifolds. Hamilton's and Liouville's representations. [1][Chapter 1]

(2) Hilbert spaces: Basic definitions. Bounded operators. Compact operators. The spectral theorem for self-adjoint compact operators. Trace-class and HilbertSchmidt operators. Closed operators. Symmetric and self-adjoint operators. Resolvent and spectrum. The spectrum of self-adjoint operators. The spectral theorem for (bounded and unbounded) self-adjoint operators. Stone's theorem. [2, 3]

(3) Basic principles of quantum mechanics: Observables, states, and dynamics. Heisenberg's uncertainty relations. Heisenberg's commutation relations. Coordinate and momentum representations. Free quantum particle. Harmonic oscillator. Stone–von Neumann theorem (without proof). [1][Chapter 2: Sections 1, 2]

(4) Schrödinger equation: The virial theorem. Angular momentum and $SO(3)$. Two-body problem. Hydrogen atom; hidden $SO(4)$ symmetry. [1][Chapter 3: Sections 1.3, 3, 4.1, 4.2, 5]

(5) Spin and identical particles: Spin operators and representation theory of $SU(2)$. Pauli Hamiltonian. Systems of identical particles. [1][Chapter 4: Sections 1, 2, 3.1]

(6) Feynman path integral: The fundamental solution of the Schrödinger equation. Trotter formula. Path integral. [1][Chapter 5: Sections 1.1, 1.2, 1.3]

References

[1] L. A. Takhtajan, *Quantum Mechanics for Mathematicians*, American Mathematical Society, 2008.

[2] N. P. Landsman, *Hilbert Spaces and Quantum Mechanics*, old version, http://www.math.kun.nl/~landsman/HSQM.pdf

[3] N. P. Landsman, *Hilbert Spaces and Quantum Mechanics*, http://www.math.kun.nl/~landsman/HSQM2006.pdf