LINEAR ALGEBRA II FOR PHYSICS

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Program

(1) Linear ODEs and Diagonalization of Endomorphisms.
   (a) Differential equations: linear ODEs with constant coefficients; systems of linear ODEs with constant coefficients; the matrix exponential and its evaluation.
   (b) Properties of the matrix exponential; computing the matrix exponential.
   (c) Diagonalization of matrices.
   (d) Solving real linear systems of ODEs via diagonalization.
   (e) The harmonic oscillator.
   (f) Linear independence of eigenvectors.
   (g) Eigenspaces.
   (h) The real case.
   (i) Diagonalization of endomorphisms.
   (j) The spectral decomposition.
   (k) The vibrating string.
   (l) Generalized eigenspaces and trigonalization.
(2) Inner Products.
   (a) The dot product and its properties.
   (b) Inner product and inner product spaces. Some examples.
   (c) Nondegeneracy of the inner product and induced isomorphisms.
   (d) The induced norm.
   (e) Cauchy–Schwarz and triangle inequalities.
   (f) Angles between vectors.
   (g) Square-integrable continuous functions.
   (h) Orthogonality. Orthogonal/orthonormal systems/bases.
   (i) Bessel's inequality.
   (j) Orthogonal projections.
   (k) The Gram–Schmidt process.
   (l) Positive-definite matrices and coframes.
   (m) Sylverster's criterion.
   (n) Orthogonal spaces and orthogonal complements.
   (o) Orthogonal sums.
   (p) Orthogonal operators.
   (q) Isometries.

(r) Orthogonal groups.
(s) Normal forms in SO(2), O(2), and SO(3).
(t) Symmetric and skew-symmetric operators.

(3) Hermitian products.
(a) The standard hermitian product.
(b) Hermitian spaces.
(c) The hermitian conjugate and self-adjoint matrices.
(d) Examples of hermitian spaces.
(e) The adjoint of an operator.
(f) The norm.
(g) Orthogonality.
(h) Orthogonal projections.
(i) Gram–Schmidt.
(j) Orthogonal complements.
(k) Unitary operators.
(l) Isometries.
(m) Unitary groups.
(n) Self-adjoint and anti-self-adjoint operators.
(o) Normal matrices and normal operators.
(p) Diagonalizability of normal matrices; spectral decomposition.
(q) Diagonalization of unitary matrices; properties of the eigenvalues and of the exponential map.
(r) Diagonalization of self-adjoint matrices and properties of the eigenvalues.
(s) Normal form of orthogonal matrices and properties of the exponential map.
(t) Diagonalization of real symmetric matrices and of real symmetric bilinear forms.
(u) Normal form of real skew-symmetric bilinear forms.