

# LINEAR ALGEBRA II FOR PHYSICS

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## PROGRAM

- (1) Linear ODEs and Diagonalization of Endomorphisms.
  - (a) Differential equations: linear ODEs with constant coefficients; systems of linear ODEs with constant coefficients; the matrix exponential and its evaluation.
  - (b) Properties of the matrix exponential; computing the matrix exponential.
  - (c) Diagonalization of matrices.
  - (d) Solving real linear systems of ODEs via diagonalization.
  - (e) The harmonic oscillator.
  - (f) Linear independence of eigenvectors.
  - (g) Eigenspaces.
  - (h) The real case.
  - (i) Diagonalization of endomorphisms.
  - (j) The spectral decomposition.
  - (k) The vibrating string.
  - (l) Generalized eigenspaces and trigonalization.
- (2) Inner Products.
  - (a) The dot product and its properties.
  - (b) Inner product and inner product spaces. Some examples.
  - (c) Nondegeneracy of the inner product and induced isomorphisms.
  - (d) The induced norm.
  - (e) Cauchy–Schwarz and triangle inequalities.
  - (f) Angles between vectors.
  - (g) Square-integrable continuous functions.
  - (h) Orthogonality. Orthogonal/orthonormal systems/bases.
  - (i) Bessel’s inequality.
  - (j) Orthogonal projections.
  - (k) The Gram–Schmidt process.
  - (l) Positive-definite matrices and coframes.
  - (m) Sylvester’s criterion.
  - (n) Orthogonal spaces and orthogonal complements.
  - (o) Orthogonal sums.
  - (p) Orthogonal operators.
  - (q) Isometries.

- (r) Orthogonal groups.
  - (s) Normal forms in  $SO(2)$ ,  $O(2)$ , and  $SO(3)$ .
  - (t) Symmetric and skew-symmetric operators.
- (3) Hermitian products.
- (a) The standard hermitian product.
  - (b) Hermitian spaces.
  - (c) The hermitian conjugate and self-adjoint matrices.
  - (d) Examples of hermitian spaces.
  - (e) The adjoint of an operator.
  - (f) The norm.
  - (g) Orthogonality.
  - (h) Orthogonal projections.
  - (i) Gram–Schmidt.
  - (j) Orthogonal complements.
  - (k) Unitary operators.
  - (l) Isometries.
  - (m) Unitary groups.
  - (n) Self-adjoint and anti-self-adjoint operators.
  - (o) Normal matrices and normal operators.
  - (p) Diagonalizability of normal matrices; spectral decomposition.
  - (q) Diagonalization of unitary matrices; properties of the eigenvalues and of the exponential map.
  - (r) Diagonalization of self-adjoint matrices and properties of the eigenvalues.
  - (s) Normal form of orthogonal matrices and properties of the exponential map.
  - (t) Diagonalization of real symmetric matrices and of real symmetric bilinear forms.
  - (u) Normal form of real skew-symmetric bilinear forms.