

# GEOMETRY AND TOPOLOGY

PROF. DR. A. S. CATTANEO

HS 2019

## PROGRAM

- (1) Basic concepts in point set topology: continuity, Hausdorff spaces, bases, topological manifolds.
- (2) Subspaces, product spaces, disjoint union spaces, quotient spaces, adjunction spaces. Topological groups. Group actions.
- (3) Connected and compact spaces. Continuous functions and their properties.
- (4) Cell complexes and CW complexes.
- (5) Compact surfaces: connected sum, polygonal presentation, classification (part I), the Euler characteristic, orientability.
- (6) Homotopy. The fundamental group and its properties. Homotopy equivalence.
- (7) The circle: lifting properties, winding number, fundamental group.
- (8) The Seifert–Van Kampen theorem. Applications. Fundamental group of surfaces. Classification of surfaces (part II).
  
- (9) Parametrized curves in  $\mathbb{R}^n$ : arc length, local theory of plane and of space curves.
- (10) Surfaces in  $\mathbb{R}^n$ : definition, properties, and first examples. Graphs. Surfaces of revolution. Surfaces defined by equations. Coordinate recognition.
- (11) The tangent plane. The first fundamental form. Isothermal parametrizations. Ruled surfaces. Area.
- (12) Smooth maps between surfaces. The differential (or derivative) of a smooth map. Local isometries. Conformal maps.
- (13) Orientation. The Weingarten map. The second fundamental form. Matrix representations. Gaussian and mean curvature. Principal curvatures and directions. Curves on surfaces: normal and geodesic curvature. Umbilics. Special families of curves. Elliptic, hyperbolic, parabolic and planar points.
- (14) Christoffel symbols. The Theorema Egregium. The Codazzi–Mainardi equations.
- (15) Geodesics. ODE for geodesics. Stationary length.
- (16) Regular regions, interior angles. Gauss–Bonnet theorem for a triangle. General Gauss–Bonnet theorem.

## REFERENCES

- [1] JOHN M. LEE, *Introduction to Topological Manifolds*, Chapters 1-10, Springer, 2011.
- [2] L. M. WOODWARD and J. BOLTON, *A First Course in Differential Geometry*, Chapters 1-8, Cambridge University Press, 2019.